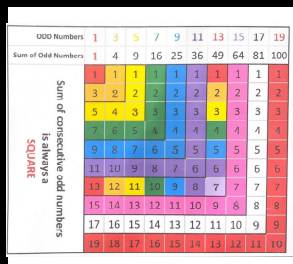
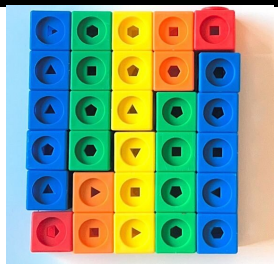


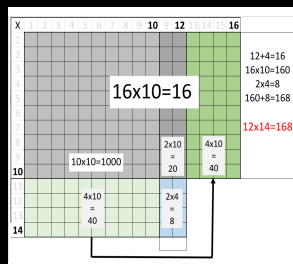
Math Circle 2023-2024 Rooz, Adam, and Dar



1. 5x5 Grid and Secrets of Odd Numbers



2. Cumulative Sum of Integers - Step Squad



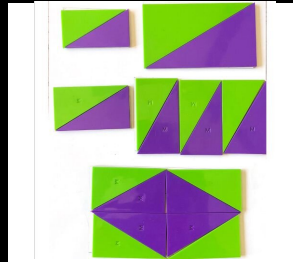
3. Mental Multiplication 11-19



4. Magical Mind Reader Binary Numbers



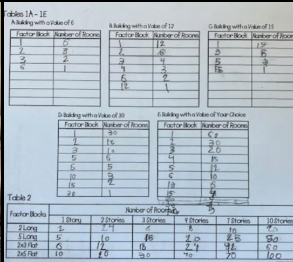
5. 10==>1 Machine Base Ten Arithmetic



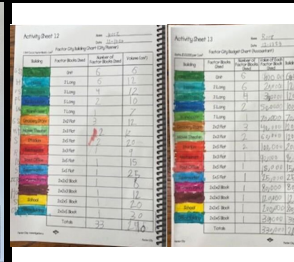
6. Hratio Rectangles Ratio Exploration



7. Factor City 1



8. Factor City 2



9. Factor City 3



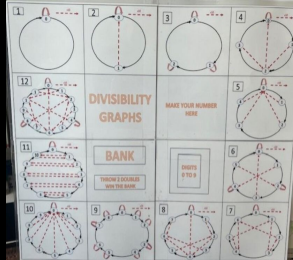
10. Factor City 4



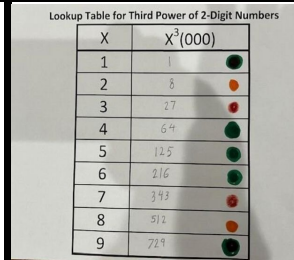
11. Divisibility Game 1



12. Divisibility Game 22



13. Divisibility Game 3 (Graph)



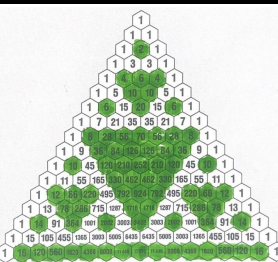
14. Mental Calculations



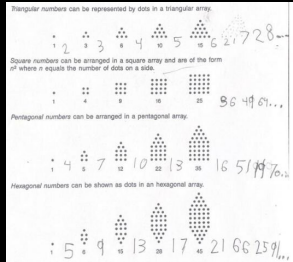
15. Mathemagic



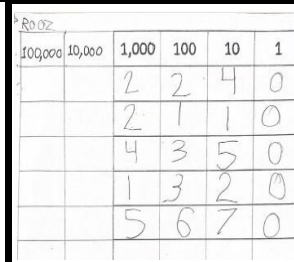
16. Triango!



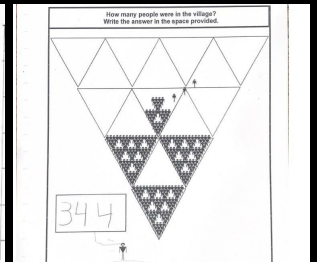
17. Visual Patterns in Pascal's Triangle 1



18. Visual Patterns in Pascal's Triangle 2



19. Super Cards - Super Match game



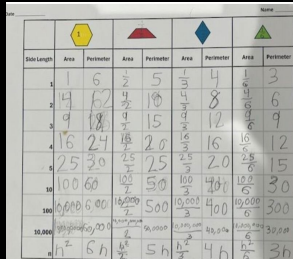
20. Super Arithmetic



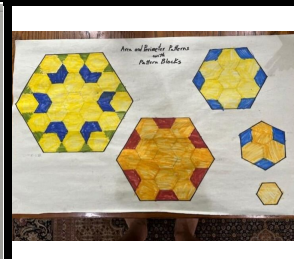
21. Babylonian Fraction game



22. Area & Perimeter with Pattern Blocks



23. Coloring Patterns



24. Preparing for our Class Demo



25. Graduation and Celebration

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Math Circle 1 8/16/2023 3:30-4:25 pm

Rooz, Cyrus, and Dar (Roоз was napping but Adam and I woke him up so he was a bit sleepy)

Dar showed some of the activities that he had done last year and told them about divisibility rules but they were not so sure about it. So we started with divisibility by 3 and Rooz came up with the rule of adding the digits and Adam liked it (I am not sure if he had seen it before) Then I asked about divisibility rule for 2 and Adam said it is easy and the number has to be even. We skipped 4 and did five and ended up with numbers ending with 0 and 5. Next we did 6 and I explained that it had to be divisible by 2 and 3 and we looked at some examples. Rooz also said that for 9 we add the digits and if the sum is divisible by 9 then the number is divisible by 9. They were getting a bit restless (in particular Rooz) so we moved on and I said that they will learn all the divisibility rules up to 12 and then 20 and even up to 100 if they wanted and it was not that difficult and we will play some fun games to master 1 to 12 first. Then I moved on to the main activity.

I introduced the 5x5 grid problem as hotel with 25 rooms where all the adjacent (Adam explained what it meant) rooms are connected and at a signal all the guest have to move to an adjoining room. I had made a 5x5 checkerboard grid and used red and white poker chips to demonstrate the guest with red guest on red squares and white guest on white squares (so that at the end the red guest should all end up in white rooms and the opposite for white guest. Adam said are we going to play checkers and I said well sort of.

I asked them to try to move the guest and discuss the problem and see if they thought it was possible or not. I thought I could leave the room and it would take them a few minutes to find a solutions. To my surprise Adam almost immediately said they cannot and when I asked him why he correctly pointed to a red guest and said he cannot move. To engage them to work together I said maybe they can work on it a bit more and move the guests and they started on it. I left the room and came back in a few minutes, and they said they were sure it could not be done and showed some of the moves and there was also a red guest that ends up on a red square. I asked them how many guests in total and tried to get them to say that there are 13 red guest and 12 white rooms so not all the red guests could move to a white room. They went back and forth and finally they said that there are 13 red guest and only 12 guests so it could not work. They were both engaged. ;=>

Then I gave them 8 small 5x5 grids and asked them to see if they could start in any square and move horizontally or vertically and go to all cells without going to any cell twice. They worked on it and discussed and concluded that on the previous red cells (grey cells here) you could do it and not on the white cells. It took them a while and Rooz and then Adam both marked the cells with P for Possible and I fo Impossible correctly.

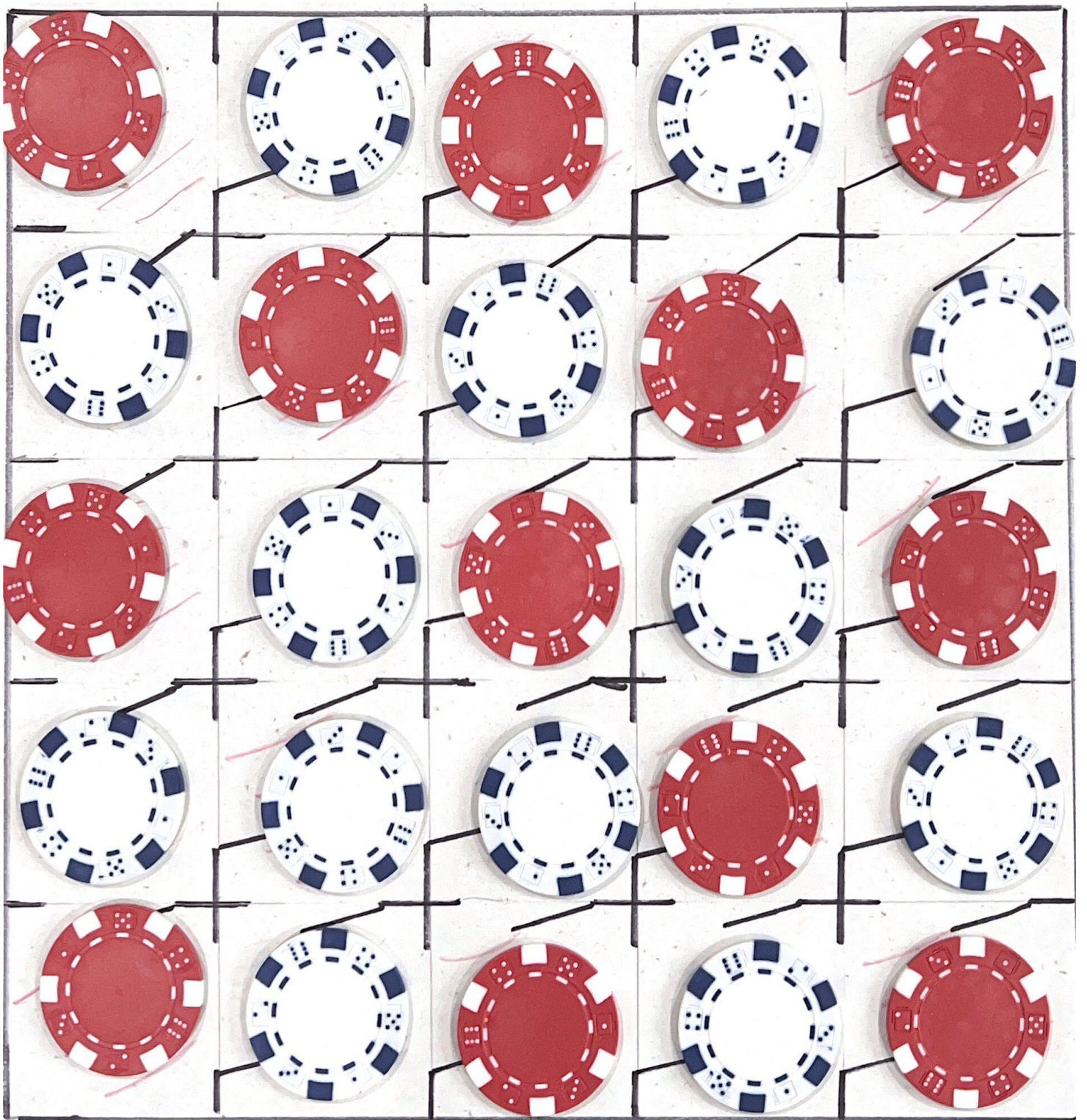
I asked them about the sum of odd numbers, and we started with 1 then I had them use the same big grid to see the sum of consecutive numbers. I had them work together so $1+3$ was 4 then plus 5 was 9 then plus 7 was 16 and plus 9 was 25. I asked them to take turns and place chips of different color on the big board on a 5x5 grid starting with 1 to see if they could see the squares. They did it and liked it (Roоз had seen this before). I also gave them the laminated sheet with the text on the back and the Images on the front and asked them to read the text. Rooz started immediately and did a great job reading it. He had seen the page before but I do not think he had read it before.

Then I tried to have them find the Galileo's ratio of $1/3$ (I had them first find 3 and we never got to $1/3$ as Rooz ran out in the middle of this) I showed Adam that $1+3$ will make 4 and a square and $5+7$ will make 12 divided the two numbers $12/4$ to get 3. I continued and now got 9 and 27. Adam said wow this will continue as he could see that there were 3 square to 1 square every time. A great observation.

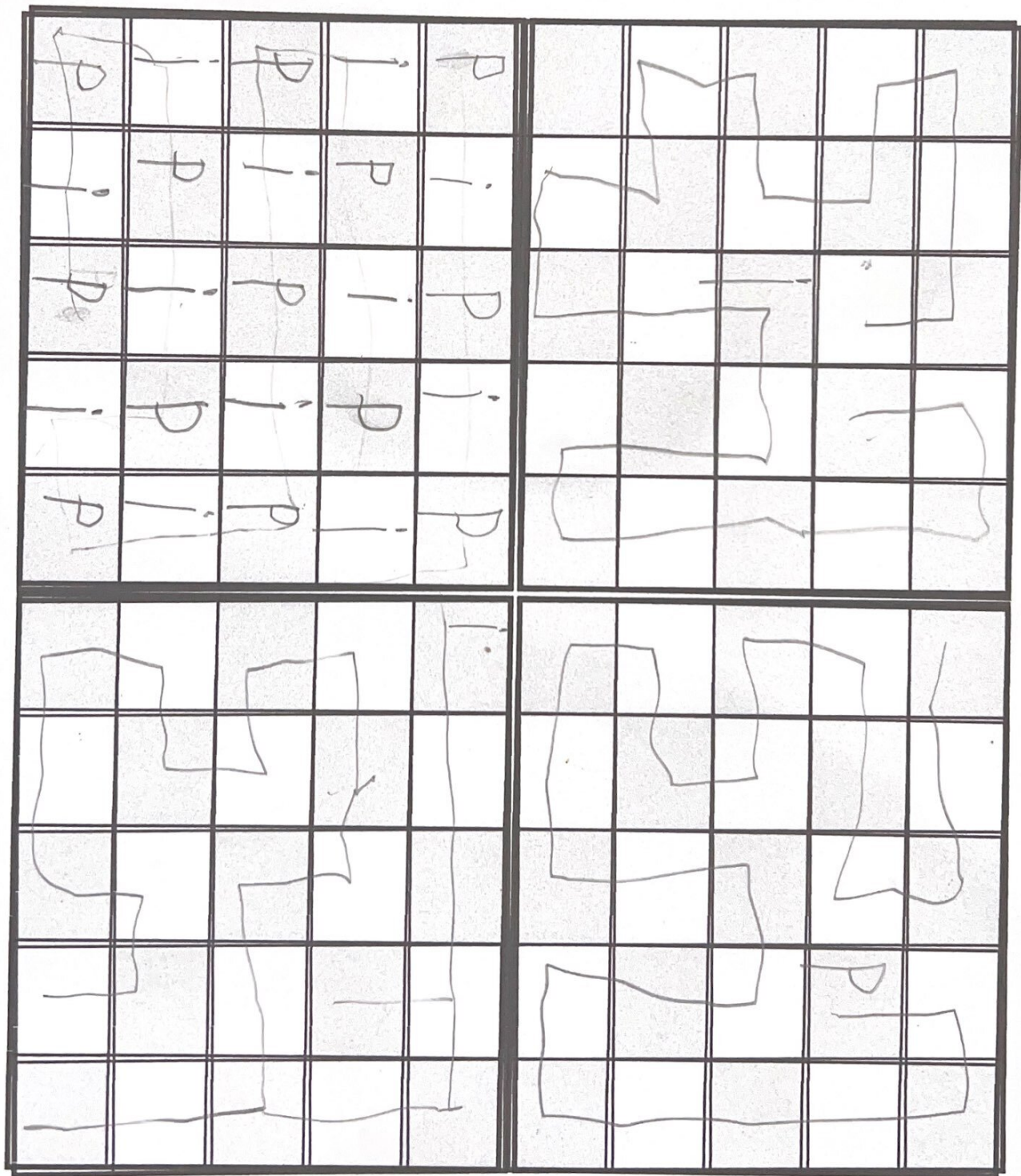
I had high hopes for the Math Circle for Rooz and Adam and it went well. Rooz was a bit tired and did not listen to me all the time but made some great comments on divisibility and some of the other work. Adam is a delight and very bright (same as Rooz) and I am sure will have a good effect on Rooz to focus more on what we do.;=> It was such a joy to see them work together.

References

For the first sessions I wanted to propose this problem that they can do with a 5x5 board and backgammon pieces. [Thinking 5x5](#) and depending on where they get maybe cover some of the other related problems (Pages 12-17 form the Power of Mathematical Visualization that is attached) There is also the original problem that James Tanton discusses in his first lecture which is vey similar to the problem in the 5x5 Thinking. [The Power of mathematical Visualization.](#)







The Secret of Odd Numbers

One is **one** times **one** a square, One is also a cube!

One plus three is **four** and four is **two** times **two** a square

Four plus five is **nine** and nine is **three** times **three** a square

Nine plus seven is **sixteen** and sixteen is **four** times **four** a square

Sixteen plus nine is **twenty five** that is **five** times **five** a square

Twenty five plus eleven is **thirty six** that is **six** times **six** a square

Thirty six plus thirteen is **forty nine** that is **seven** times **seven** a square

Forty nine plus fifteen is **sixty four** that is **eight** times **eight** a square

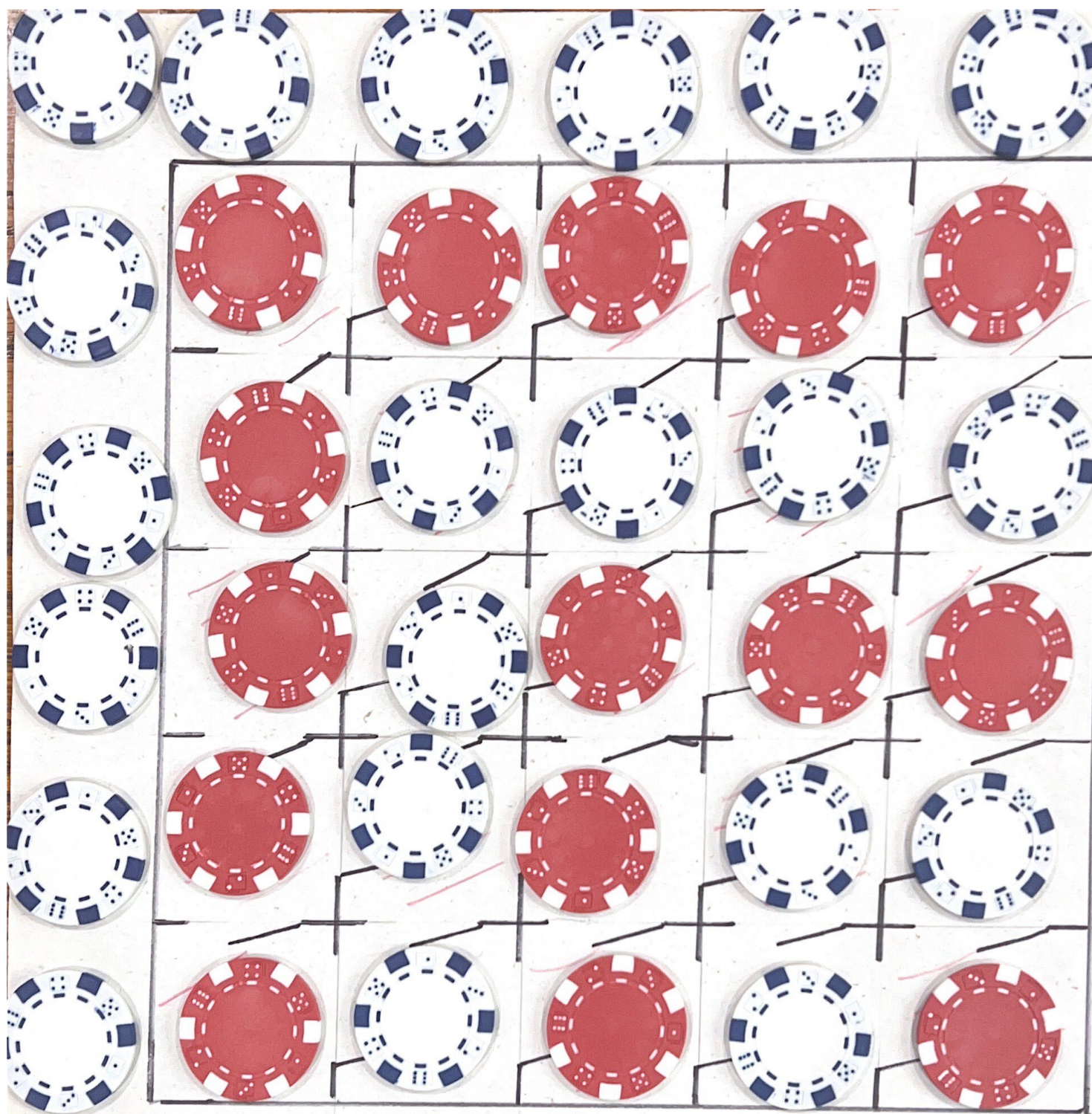
Sixty four is a also a cube as it is **four** times **four** times **four**!

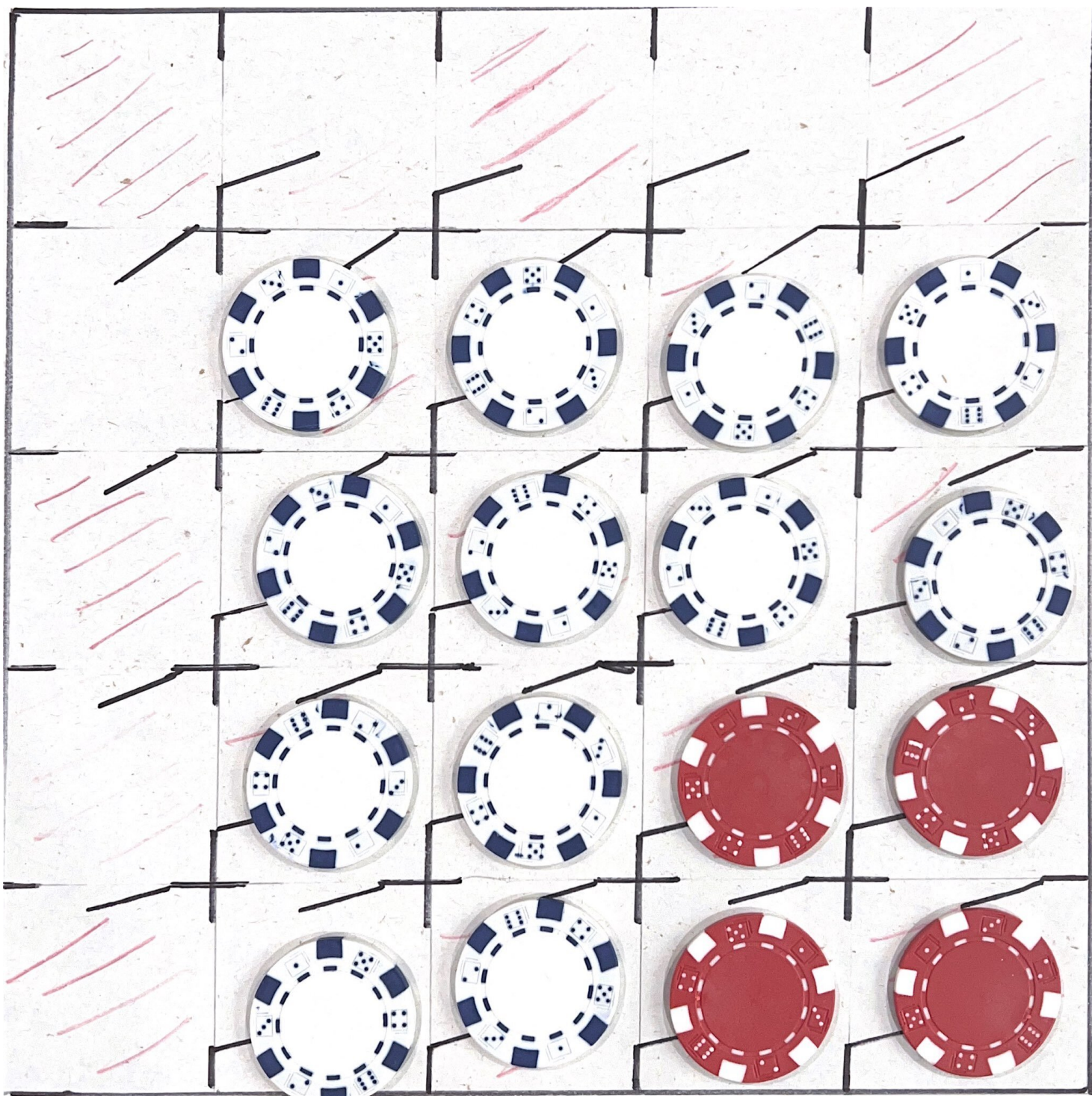
Sixty four plus seventeen is **eighty one** that is **nine** times **nine** a square
eighty one plus nineteen is **one hundred** that is **ten** times **ten** a square

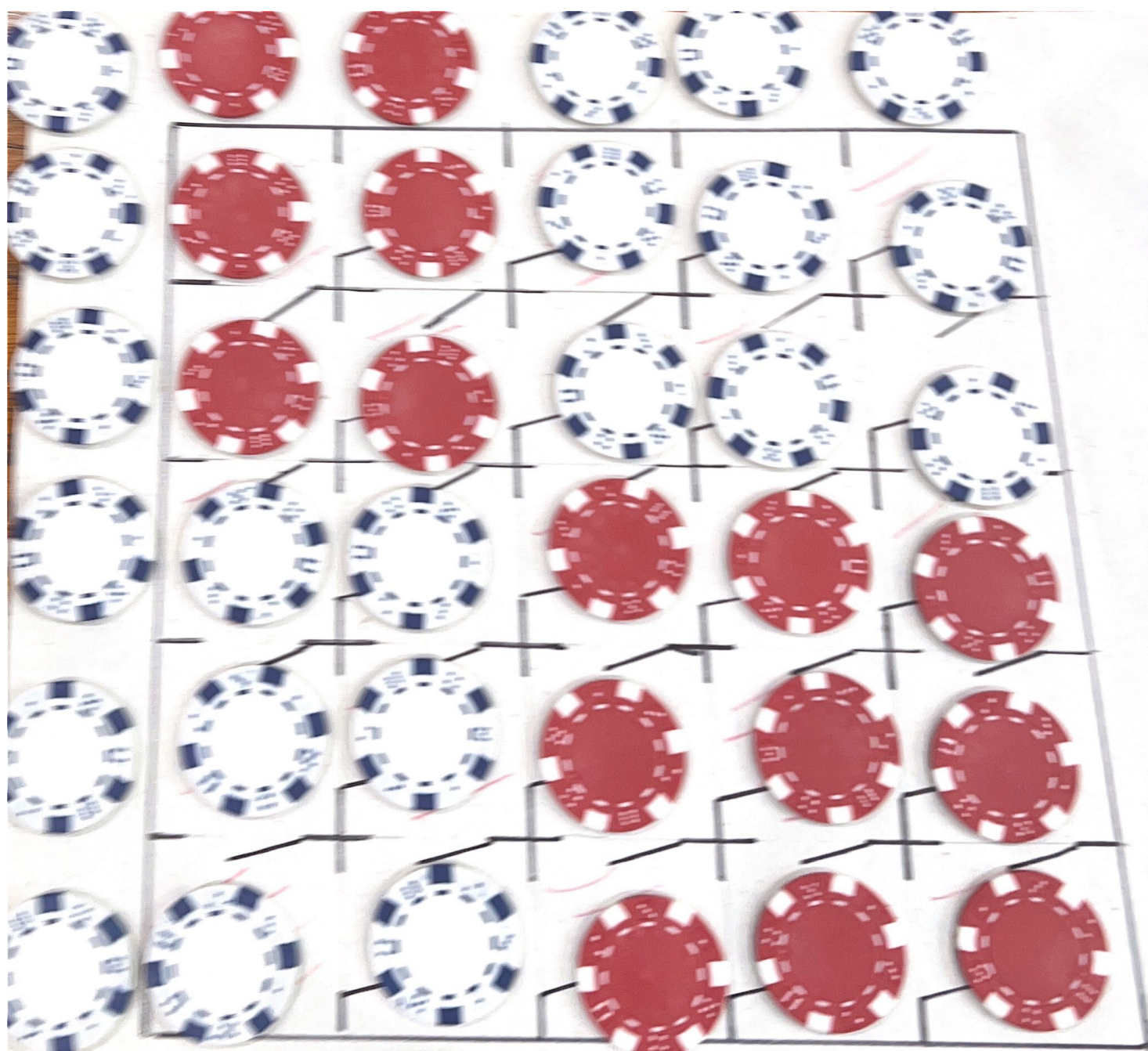
(Turn the page over for a picture)

ODD Numbers	1	3	5	7	9	11	13	15	17	19
Sum of Odd Numbers	1	4	9	16	25	36	49	64	81	100
	1	1	1	1	1	1	1	1	1	1
		1	1	1	1	1	1	1	1	1
	3	2	2	2	2	2	2	2	2	2
	5	4	3	3	3	3	3	3	3	3
	7	6	5	4	4	4	4	4	4	4
	9	8	7	6	5	5	5	5	5	5
	11	10	9	8	7	6	6	6	6	6
	13	12	11	10	9	8	7	7	7	7
	15	14	13	12	11	10	9	8	8	8
	17	16	15	14	13	12	11	10	9	9
	19	18	17	16	15	14	13	12	11	10

Sum of consecutive odd numbers is always a SQUARE







Math Circle 8/24/2022 3:30 - 4:05 pm

Adam, Rooz and Dar

I told them about Karl Fredrich Gauss that found the sum of 1 to 100 when he was about their age <and almost as smart ;=> I asked them what do they think it was and gave them a sheet that had the numbers 1 to a 100 on two rows. I left the room and came back after a few minutes. I was happy to hear from behind the door that they were discussing strategy of what to do. See the figure for Rooz' work Adam's was cleaner, but the purpose was for them to see that it was hard by brute force.

Then I told them that they will learn about a formula where they can find this sum and any other sum up to an number. Rooz would not give up and kept adding the numbers together. Then I had them make the sums from 1 to 5 (they call this step squad from Number Blocks that they both have watched see figure). At each step I asked them to put the pieces together to make a rectangle which was n by $n+1$ with manipulatives to see that for 2 the rectangle was $2 \times 3 = 6$ so $1+2=6/2$ and so on until for 5 the rectangle was $5 \times 6 = 30$ and $1+2+\dots+5=30/2=15$ and then they went to the sheet I had prepared for them and wrote down the values for the sum when $n= 1, 2, 3, 4, 5, 10, 17, 20$ and 100 (see figure). They were very much engaged, and I kept my trap shut well and only asked questions and gave them tasks and no real hints. At this point I think they had a good idea that $n(n+1)/2$ was the sum of 1 through n always. Then I showed them the method that Gauss had supposedly used by adding $100+1, 99+2, \dots, 50+51$ and to see there will be 50 of them so $50 \times 101 = 5050$ (see figure). I think they liked this better even though it was not something that they could do easily with all n . But later that night when I asked about the sum of 1 to 100 from Rooz in front of Aryan he used this second method and not the formula!

When we got to $n=17$ and they had to find 17×18 I asked them if they knew the trick for multiplying numbers between 11 to 19. Adam said he knew, and I asked him for 12×13 which he did with his method which was cool. I told them that there is a method that is quicker and easy to do in their head. I explained that to do 12×13 you can take the 3 from 13 and add it to 12 to get 15 and then multiply it by the 10 that is left from 13 to get 150 and finally multiply 2 by 3 to get 6 and add it to 150 to get 156. He liked this. Rooz had seen this method before but had never completely mastered it. They both liked it and then I would have them take turns and do it for several pairs like 14×15 and finally even 18×19 and 19×19 . What was nice was that they would both repeat the steps loud (something that I could not get Cyrus or Marc to do when they first saw this method.) So, if something went wrong I could say are you sure, and they would correct it. For 19×19 Adam was giving big answers so I helped him go back to the method and he got it and Rooz agreed as he was adding 81 to 280. They both enjoyed this like so many kids that I have thought this method to. I learned this formula from my 4th grade teacher in Iran and I think it was what made me interested in mathematics! I gave them the sheets for all the multiplications of numbers from 11 to 19 and asked Adam to test his parents at home. They always love to do things that their parents cannot do! I also told them that they would see a lot more methods for multiplying 2 digit numbers in their head. Adam told me that he knew a method for numbers whose 1s digit added to 9 and we discussed it a bit.

It was a really great session, and we all enjoyed it even though it was only 35 minutes I though that was all they could handle, and the Ping Pong Ball machine was calling them and Cyrus. Cyrus came in (as his Piano got cancelled because Ava had gotten sick) in the middle of the session and helped. He really liked that and so did Adam and Rooz.

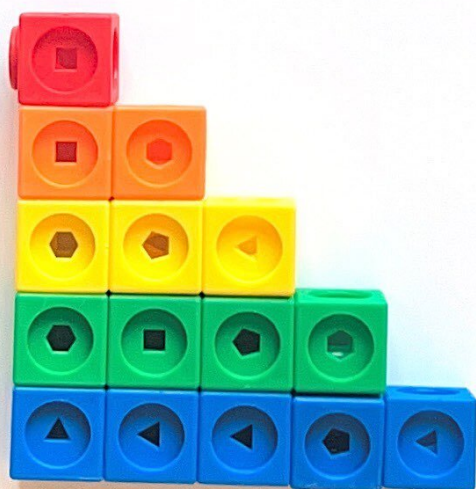
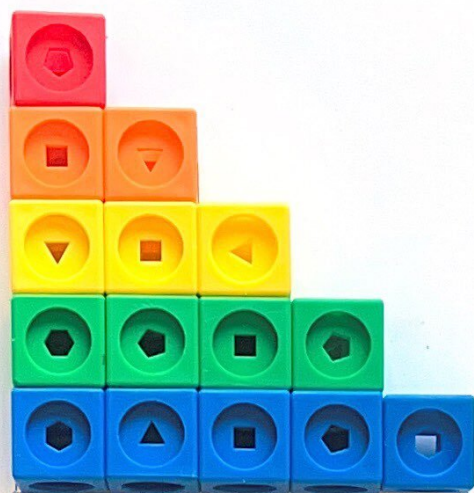
In the future I think their session should be kept to at most 45 minutes with a 5-minute break in the middle.

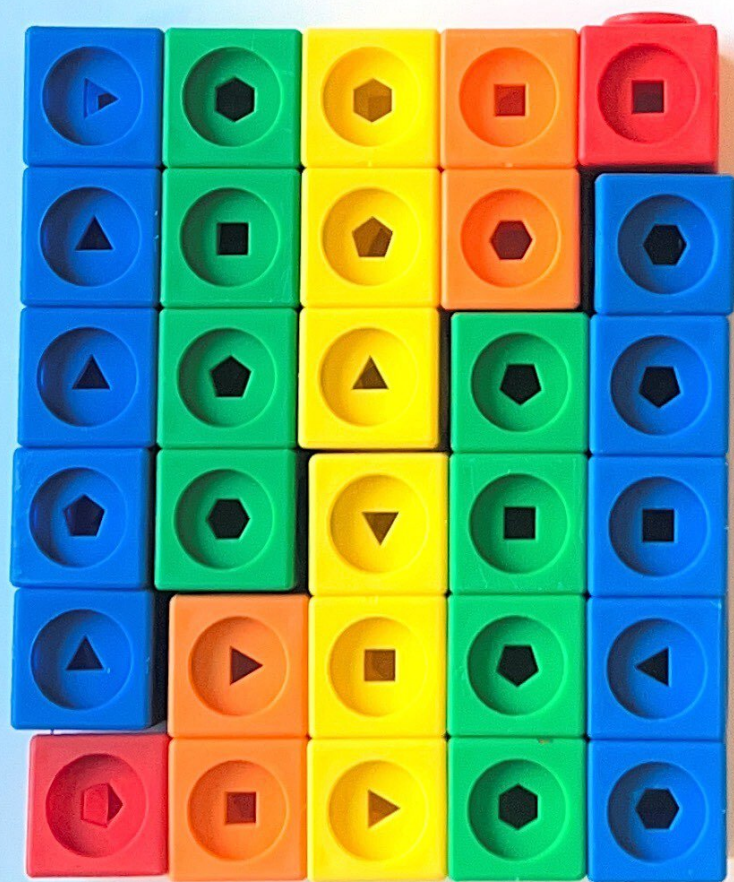
Can you find the sum of numbers from 1 to 100?

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58	57	56	55	54	53	52	51

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Formula for Sum of Numbers up to n

$$1+2+3+\dots+n = nx(n+1)/2$$

n	n+1	$nx(n+1)$	$nx(n+1)/2$
1	1	$1 \times 2 = 2$	$2/2 = 1$
2	3	6	3
3	4	12	6
4	5	20	10
5	6	30	15
10	11	110	55
17	18	306	153
20	21	420	210
100	101	10100	2070

X	11	12	13	14	15	16	17	18	19
11	11+1=12 12x10=120 1x1=1	12+1=130 13x10=130 1x2=2	13+1=14 14x10=140 1x3=3	14+1=15 15x10=150 1x4=4	15+1=16 16x10=160 1x5=5	16+1=17 17x10=170 1x6=6	17+1=18 12x10=180 1x7=7	18+1=19 19x10=190 1x8=8	19+1=20 20x10=200 1x9=9
12		12+2=14 14x10=140 2x2=4	13+2=15 15x10=150 2x3=6	14+2=16 16x10=160 2x4=8	15+2=17 17x10=170 2x5=10	16+2=18 18x10=180 2x6=12	17+2=19 19x10=190 2x7=14	18+2=20 20x10=200 2x8=16	19+2=21 21x10=210 2x9=18
13			13+3=16 16x10=160 3x3=9	14+3=17 17x10=170 3x4=12	15+3=18 18x10=180 3x5=15	16+3=19 19x10=190 3x6=18	17+3=20 20x10=200 3x7=21	18+3=21 21x10=210 3x8=24	19+3=22 22x10=220 3x9=27
14				14+4=18 18x10=180 4x4=16	15+4=19 19x10=190 4x5=20	16+4=20 20x10=200 4x6=24	17+4=21 21x10=210 4x7=28	18+4=22 22x10=220 4x8=32	19+4=23 23x10=230 4x9=36
15					15+5=20 20x10=200 5x5=25	16+5=21 21x10=210 5x6=30	17+5=22 22x10=220 5x7=35	18+5=23 23x10=230 5x8=40	19+5=24 24x10=240 5x9=45
16						16+6=22 22x10=220 6x6=36	17+6=23 23x10=230 6x7=42	18+6=24 24x10=240 6x8=48	19+6=25 25x10=250 6x9=54
17							17+7=24 23x10=230 7x7=49	18+7=25 25x10=250 7x8=56	19+7=26 26x10=260 7x9=63
18								18+8=26 26x10=260 8x8=64	19+8=27 27x10=270 8x9=72
19									19+9=28 28x10=280 9x9=81

Math Circle 3 08-30-2023 3:30 to 4:00 pm

Rooz, Adam, and Dar

We started by doing mental multiplications for numbers between 11 and 19 that they had seen last time. Adam and Rooz would take turns, but they would put their hands up once they had the answer. Adam used his own method first for the simpler examples 12×11 but switched to the one for harder examples like 17×18 after e reviewed it again. They were both doing it very well. I also showed them the rectangular model for this, and Adam said he had seen in. They will use this model this year for him a lot in school.

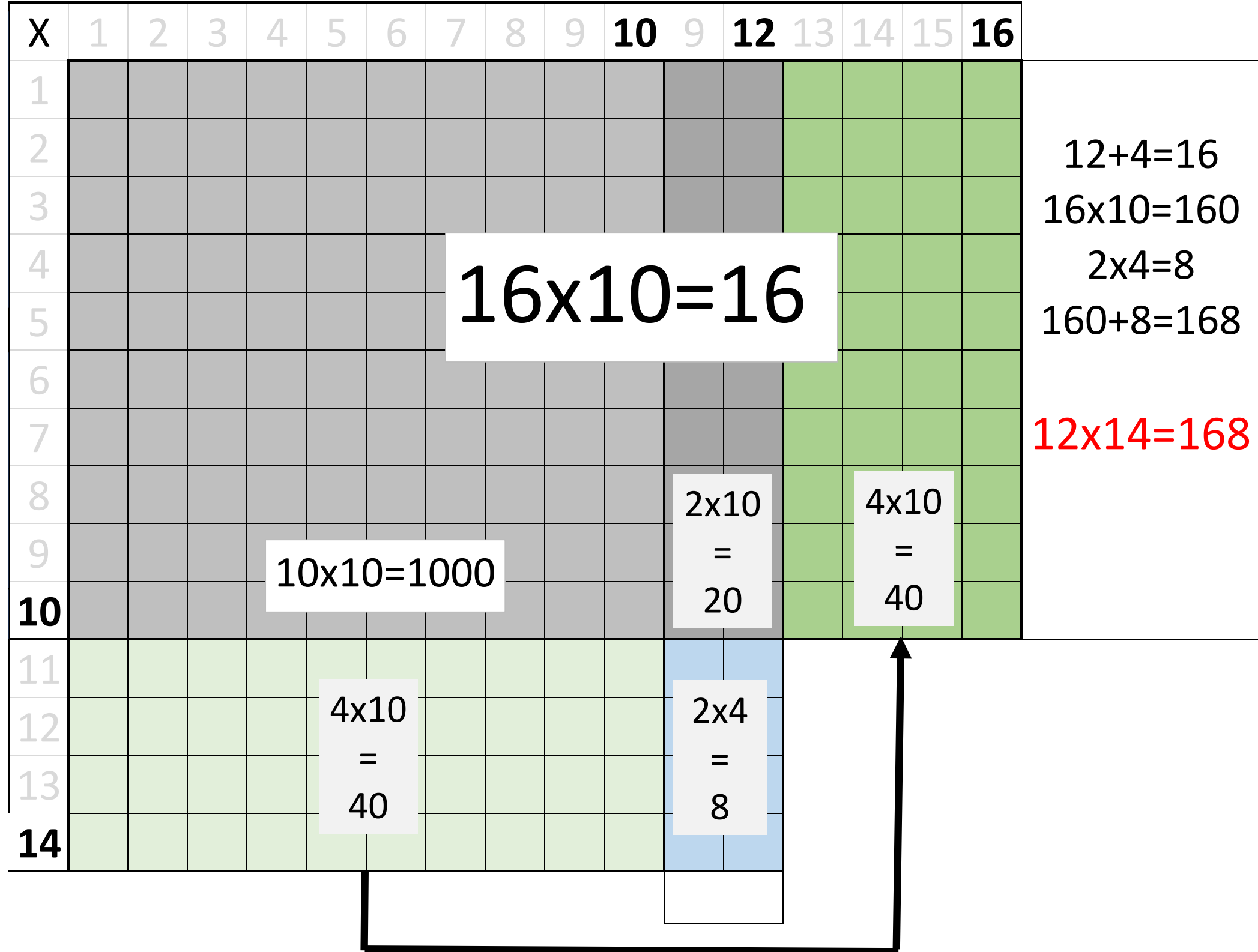
Next I showed them the trick for multiplying numbers in teens, twenties, thirties, etc whose 1-digit adds up to 10. Adam had told me that he had seen this one too. We started easily and went all the way up to nineties. They liked this but wanted to move on after we did 5 or 6 examples that got progressively harder, so we started the main activity.

I gave them the Dienes Blocks (These are the original block from [Zoltán Pál Dienes](#) that I got thru a collaborator of mine) First I had them arrange the cubes. Of course they were more interested in playing with these but I managed to get them to arrange the 1, 8, 27, 64 and 125 cube and remake them with the flat pieces. Next, I asked them to use their flat pieces for 1 and 8 cube to make a square and they said if cannot be done! But they worked with it and Adam then Rooz made it. Next, I asked them to add the pieces from the 27 cube and see if they could make a larger square. They had mixed the pieces for the first square so they had to remake it and then add the three 3×3 squares. They were amused and then I asked them to continue and make bigger squares. They did it and eventually got a 15×15 cube. I asked them if they thought that this would go on forever and they said that they were sure that it would.

I wanted them to see that the sides were $1+2+3+4+5=15$ and Adam did that but Rooz was busy playing with all the pieces but Adam figured out that the square was 15 by 15 and was 225. Next I asked him what happens if we added the pieces from the $6 \times 6 \times 6$ cube and he correctly said that it would be 21 by 21 and calculated with a little help that it would be 441. I told them about Cyrus's Reversible Square Numbers. I will have them look at it next time these are 12 and 13 that if we reverse the digits we get 21 and 31 and if we square them we get 144 and 169 and if we square the reversed one we get 441 and 691 that is the reverse of the squares! Cyrus figured this out by himself a couple of years back that was remarkable.

Next I gave them the laminated sheet that I had made for cubes going to $10 \times 10 \times 10$ and first I had them read the text on the back of it with each one of them reading one of the sentences and alternating and then they turned it over. Adam was really impressed, Rooz had seen this but was also impressed, I think.

Next Adam wanted to run to the Ping Pong Ball machine and Rooz wanted to eat so we stopped. Even though the session was for only 30 minutes, it went quite well, and they enjoyed it. I think for Rooz and Adam this may be the right amount of time. If we play math games, we may take longer a little longer.



Multiplying 2-digit numbers When
the ones-digits add up to 10

11-19	10x20	200
21-29	20x30	600
31-39	30x40	1,200
41-49	40x50	2,000
51-59	50x60	3,000
61-69	60x70	4,200
71-79	70x80	5,600
81-89	80x90	7,200
91-99	90x100	9,000

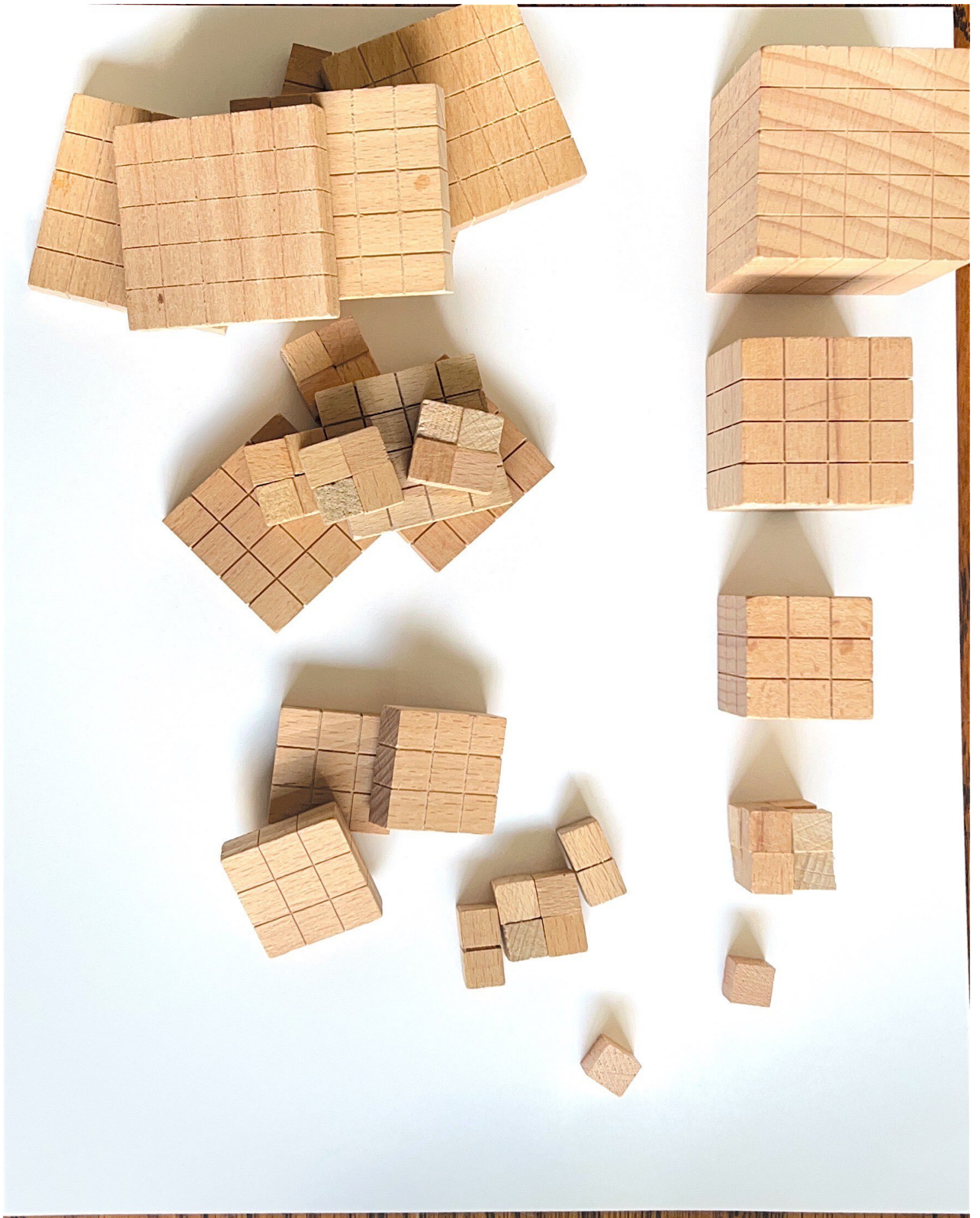
1x9	9
2x8	16
3x7	21
4x6	24
5x5	25

Examples

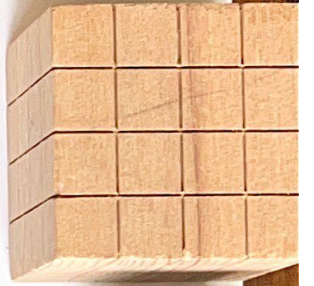
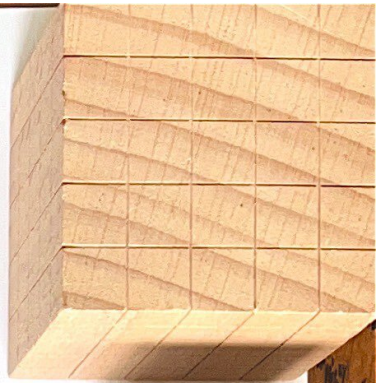
$$23 \times 17 = 200 + 21 = 221$$

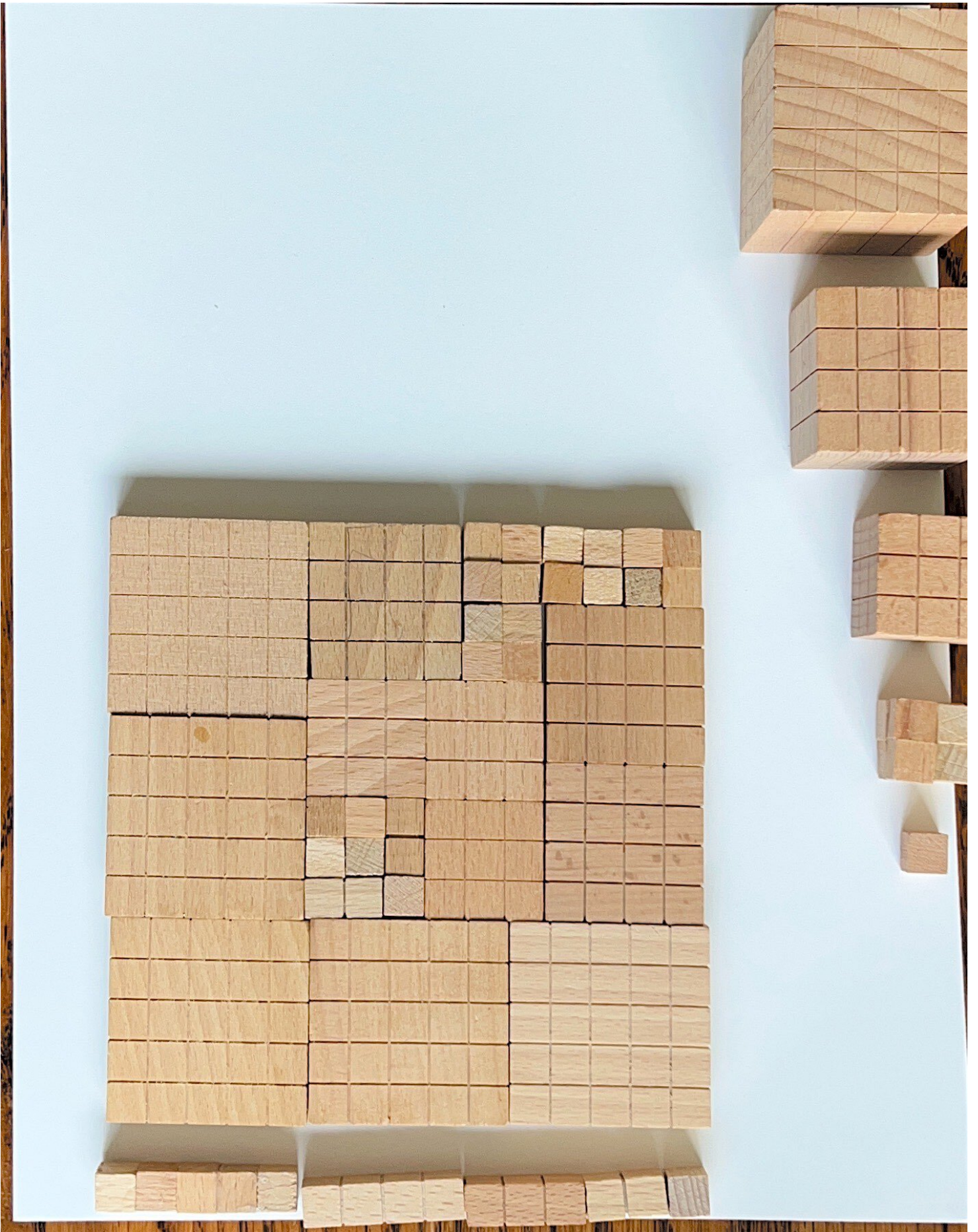
$$34 \times 36 = 1,200 + 24 = 1,224$$

$$75 \times 75 = 5,600 + 25 = 5,625$$









The Secret of the Cubes

One is both a **Cube** and a **Square**

One plus eight is nine that is **three** times **three** a **square**

Nine plus twenty seven is thirty six that is **six** times **six** a **square**

Thirty six plus sixty four is a hundred that is **ten** times **ten** a **square**

One Hundred plus one hundred twenty five is
two hundred twenty five that is **fifteen** times **fifteen** a **square**

Two hundred twenty five plus two hundred sixteen is four
hundred forty one that is **twenty one** times **twenty one** a **square**

Four Hundred forty one plus three hundred forty three is seven
hundred eighty four that is **twenty eight** times **twenty eight** a **square**

Is it always true that **sum of all consecutive
cubes is a square?**

(Turn the page over for a picture)

Sum of All Consecutive Cubes up to any Cube
is always a **SQUARE**

1	2x2x2=8		3x3x3=27			4x4x4=64				5x5x5=125					6x6x6=196						7x7x7=343							8x8x8=512								9x9x9=729									10x10x10=1,000																															
																																																																												
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55																						
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21																																																								
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Math Circle 4 9/6/2023 3:45-4:30 pm

Rooz, Adam, Papa and Dar

We started with the Magical Mind Reader Cards. I have a version of this in Scratch that you can have Adam play with it. [Magical Mind Reader on Scratch \(mit.edu\)](#). I used the big cards and had them think of a number when I left the room and then I would come back and find the number for them. Next, I showed them the trick and explained how I found the answer by adding the first number on each card that contained it. Then Adam and Rooz took turns and left the room, and they would find the answer when they came back. It took a little time at the beginning, but they picked it up quickly. Rooz had seen that a bit before. I also gave them smaller cards to take home.

Then we discussed power and in particular powers of 2 and looked at cards showing powers of 2 from 0 to 11 (see figure). I also told them about 2^{10} that is 1K and it is nearly a 1,000. I challenged them to find 2^{20} . We went back and forth and finally they came up with about 1,000,000.

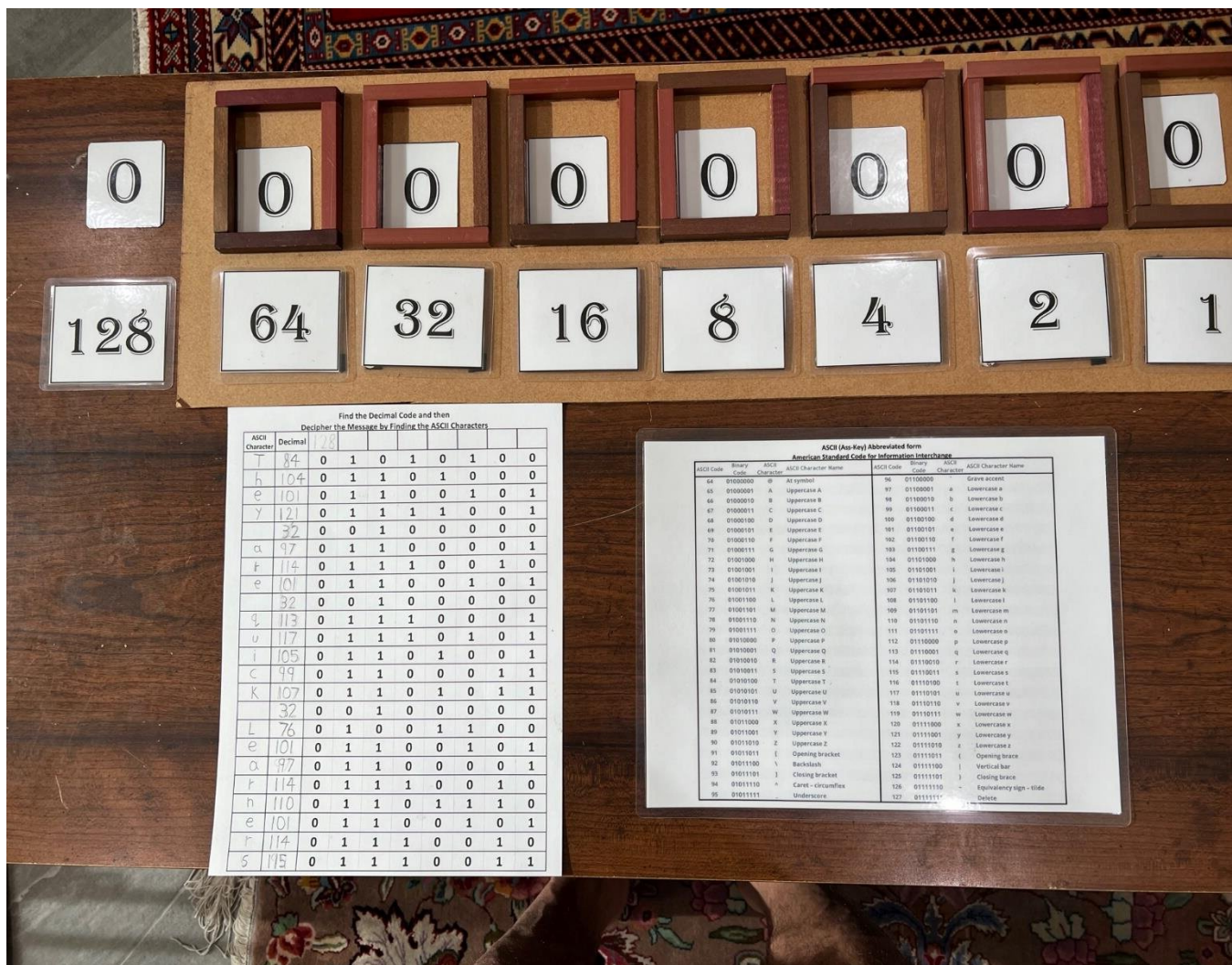
Then we examined the $2 \rightarrow 1$ machine (see figure) that can code numbers between 1 to 127 in decimal. I explained that this was called a binary number as it only had zeros and ones. They did the ones and zeros for a few numbers and then proceeded with the challenge of deciphering the code that was given to them by finding the decimal value for each row representing a number between 1 to 127 (see figure). They found all the decimal numbers. At first, I made the number on the $2 \rightarrow 1$ machine for them and they then added the relevant powers of 2 indicated by a 1 in the corresponding square. But after a few times they just used the powers of 2 on their sheet to find the decimal number. They worked well together and would even double check the decimal number that they found. Rooz had seen some of this before so he was trying to give all the answers, but Adam picked up and by then end Rooz was coping some of the numbers from Adam. Papa was also encouraging Adam and it worked well.

Then I gave them the ASCII table and using it they collaborated to decipher the message; “ROOZ&ADAM ARE SMART” They enjoyed the process very much. I explained that this was how computers kept the information and before they literally used light bulbs as 1 when it was on and 0 when it was off and now, they used transistors that are very small. We discussed this for a little and I told them that they had another message on the back of the card that they could do later at home if they wanted to. Cyrus had come back from his Piano lesson and wanted to do get in the action. This was one of their favorite activities last year in the Math Circle. So I gave him a sheet and he started on it.

I wanted to discuss the Magic Cards and their secret and how it related to what they had done but Adam was ready for the Ping Pong Ball machine so I let them go and play and will discuss it next time, so they make the connection fully.

It was a very good session and Papa was a good addition and he is welcome to participate anytime now. When they left I discussed with Jerry the $2 \rightarrow 1$ machine and the how it also worked as $10 \rightarrow 1$ or even as an $X \rightarrow 1$ machine briefly. And should a quick example of how division worked on it even for polynomials. I also mentioned the [The Power of mathematical Visualization](#) in YouTube where I have gotten some of the material from..





Find the Decimal Code and then
Decipher the Message by Finding the ASCII Characters

ASCII Character	Decimal								
		0	1	0	0	0	0	1	1
		0	1	1	1	1	0	0	1
		0	1	1	1	0	0	1	0
		0	1	1	1	0	1	0	1
		0	1	1	1	0	0	1	1
		0	0	1	0	0	1	1	0
		0	1	0	0	1	1	0	1
		0	1	1	0	0	0	0	1
		0	1	1	1	0	0	1	0
		0	1	1	0	0	0	1	1
		0	0	1	0	0	0	0	0
		0	1	1	0	0	0	0	1
		0	1	1	1	0	0	1	0
		0	1	1	0	0	1	0	1
		0	0	1	0	0	0	0	0
		0	1	0	0	0	1	1	1
		0	1	1	1	0	0	1	0
		0	1	1	0	0	1	0	1
		0	1	1	0	0	0	0	1
		0	1	1	1	0	1	0	0

Find the Decimal Code and then
Decipher the Message
by Finding the ASCII Characters from your Cards

ASCII Character	Decimal	128	64	32	16	8	4	2	1
R	82	0	1	0	1	0	0	1	0
O	79	0	1	0	0	1	1	1	1
O	79	0	1	0	0	1	1	1	1
Z	90	0	1	0	1	1	0	1	0
&	38	0	0	1	0	0	1	1	0
A	65	0	1	0	0	0	0	0	1
D	68	0	1	0	0	0	1	0	0
A	65	0	1	0	0	0	0	0	1
M	77	0	1	0	0	1	1	0	1
	32	0	0	1	0	0	0	0	0
A	65	0	1	0	0	0	0	0	1
R	82	0	1	0	1	0	0	1	0
E	69	0	1	0	0	0	1	0	1
	32	0	0	1	0	0	0	0	0
S	83	0	1	0	1	0	0	1	1
M	77	0	1	0	0	1	1	0	1
F	67	0	1	0	0	0	0	0	1
R	82	0	1	0	1	0	0	1	0
T	84	0	1	0	1	0	1	0	0

Math Circle 4 9/13/2023 3:30-4:20 pm

Roos, Adam, Papa and Dar

I asked them if they knew the trick for multiplying by 11. Adam said that he did. So I gave him one multiplication which he did correctly and I asked him how he did it and he explained it well. Then I asked Roos to do one and Adam to help him with the method. After a couple of examples Roos could also do it. For example, $26 \times 11 = 2(2+6)1 = 281$. Then I gave them a harder one something like 74×11 and Adam did it correctly and we discussed that now they had to carry a 10 as $7+4=11$ so the answer will be $(7+1)14 = 814$.

Next we started with our main activity with the $10 \rightarrow 1$ machine that is a great way to master addition and subtraction and carrying and borrowing as well as many other topics in arithmetic and even algebra (see figure) based on [The Power of Mathematical Visualizations lecture 6](#). I discussed how now instead of 2 cubes exploding 10 cubes exploded and gave rise to one in the next space (the higher power) Roos had seen this before, so he actually came up with 10 cubes exploding. Both were not quite sure how this was very similar to the $2 \rightarrow 1$ machine that they had seen last week. But I think by the end they may have grasped it.

Now I gave them some numbers to each of them in turn. The first number was in 1002 so to show mastery Adam wanted to dump a lot of cubes (1002) of them in 1s place and then explode them to get to the final answer. I asked him to cut to the chase and just show the answer that he did. We did a few more numbers like this and we started the main game.

Originally the game was for them to select two number cards from 0 to 9 and a sign card from 4 cards and make a number that they would add to the board. I explained that adding a negative is just like subtracting that number. They started at 100 and took turns (see figure). One of them did the sum on the board (I also did the number with the Super Cards) and the other did the addition on the paper. They were very engaged and after a few more numbers I decided to ask them if they wanted to do 3-digit numbers. At first, they did not want it then they OKed it. So we continued and by the end we even did a 4 digit number. They always dreaded to get the negative sign and most of the time they got a positive sign.

I asked them how many of the 4 sign cards were positive and how many were negative. First they they were not sure and wanted me to show them. Finally they both said that 3 were negative and one was positive no matter how much Jerry and I tried to tell them that they had gotten more plus signs so how could there be more negative cards they would not change their mind. Eventually Adam said that there must be 3 negative signs and one positive but Roos was still saying 2 and 2. I finally showed them the cards and there were 3 plus and one minus sign. Jerry then asked them if they had 2 negative signs and 6 positives would it be the same. Roos said yes as it is just twice as many for each card now. I asked what if there were 3 negative and

9 positives. He answered the same as they are now 3 times as many. Finally, I asked what if there are 4 negatives how many positives should we have to have the same chances. He said 12 correctly. Adam was listening and on board. This concept is a hard one at this point. The concept of probability that if you have more of something you get it more often is not an easy one. The number part that Rooz was doing is just because we have done a lot of fraction and ratio games before with him.

The session went 45 minutes and broke up when Cyrus came. They were both quite engaged and enjoyed it and could have gone on more.



Math Circle RA 5 11/01/2023 2:30 to 3:20 pm

Rooz, Adam, and Dar

I had planned to start the Factor City activity with Factor Blocks this week, but Rooz crashed the MC session for Cyrus and Marc and wanted to also do the same activity. I explained to him what he was to do and he got a piece of paper and started arranging the triangles on the rectangles and writing down the number of different triangles that made up each rectangle. Adam came in after a few minutes and wanted to do the same. So I printed the same worksheet for them and they did the activity Hratio F.D. Rectangles.

First, they covered Rectangles A to G with the Small, Medium and Large triangles. While doing this they completed the top part of the hand out recording how many triangles of what kind covered each rectangle. Some of the rectangles could be covered with different triangles. For example: Rectangle C can be covered with 6 small or 2 large triangles and most importantly Rectangle G can be covered with 8 Large, or 18 Medium or 24 Small Triangles and the relation between all the rectangles and all the Triangles can be derived from this fact.

Next, they completed the table of the relation between the Rectangles and the triangles when the ratio was a whole number. For example, $A=2S$ or $G=24s$, $G=18M$ or $G=8L$.

Rooz had several minutes of advance, so he completed the table with round numbers and wanted to help Adam. But Adam wanted to cover the rectangles himself and complete the table and he had a few cells left before he got tired and Ping Pong was calling. I gave the Triangles and the worksheet to Jerry so he could complete it if he got a chance.

Rooz started to try to find the fraction relations but this turned out to be too difficult and he gave it up. I think the Fractional reasoning may be too much for them at this point

This was a great activity that they enjoyed. It was designed with a lot more writing for 6th graders to do in a week with an evaluation, but they did it in less than an hour. I had made some short cuts for them, and we skipped more than half of it with Fractional reasoning!

A

C

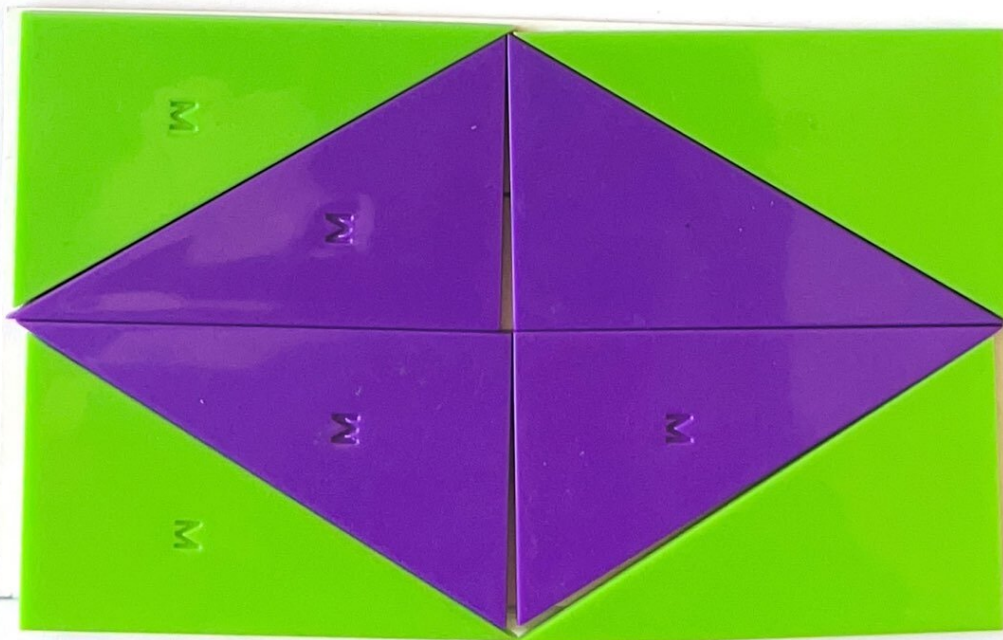
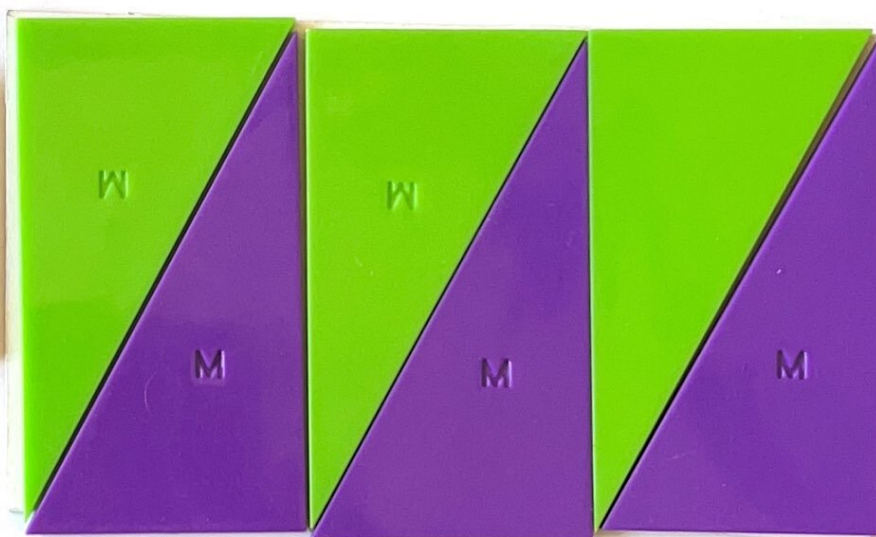
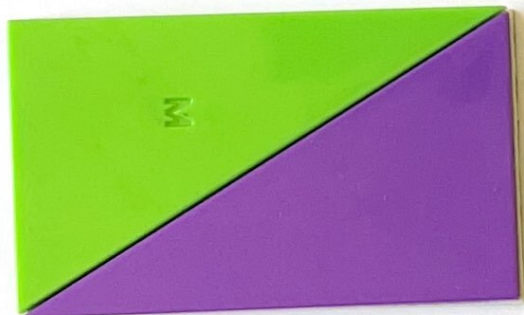
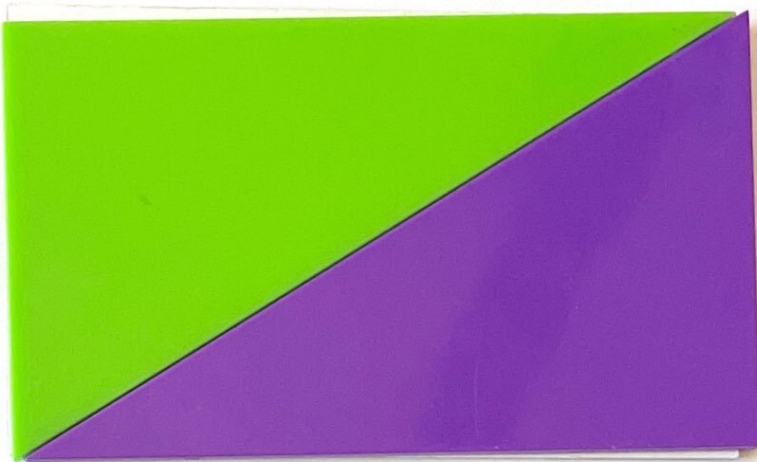
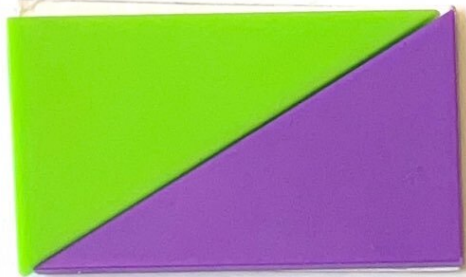
B

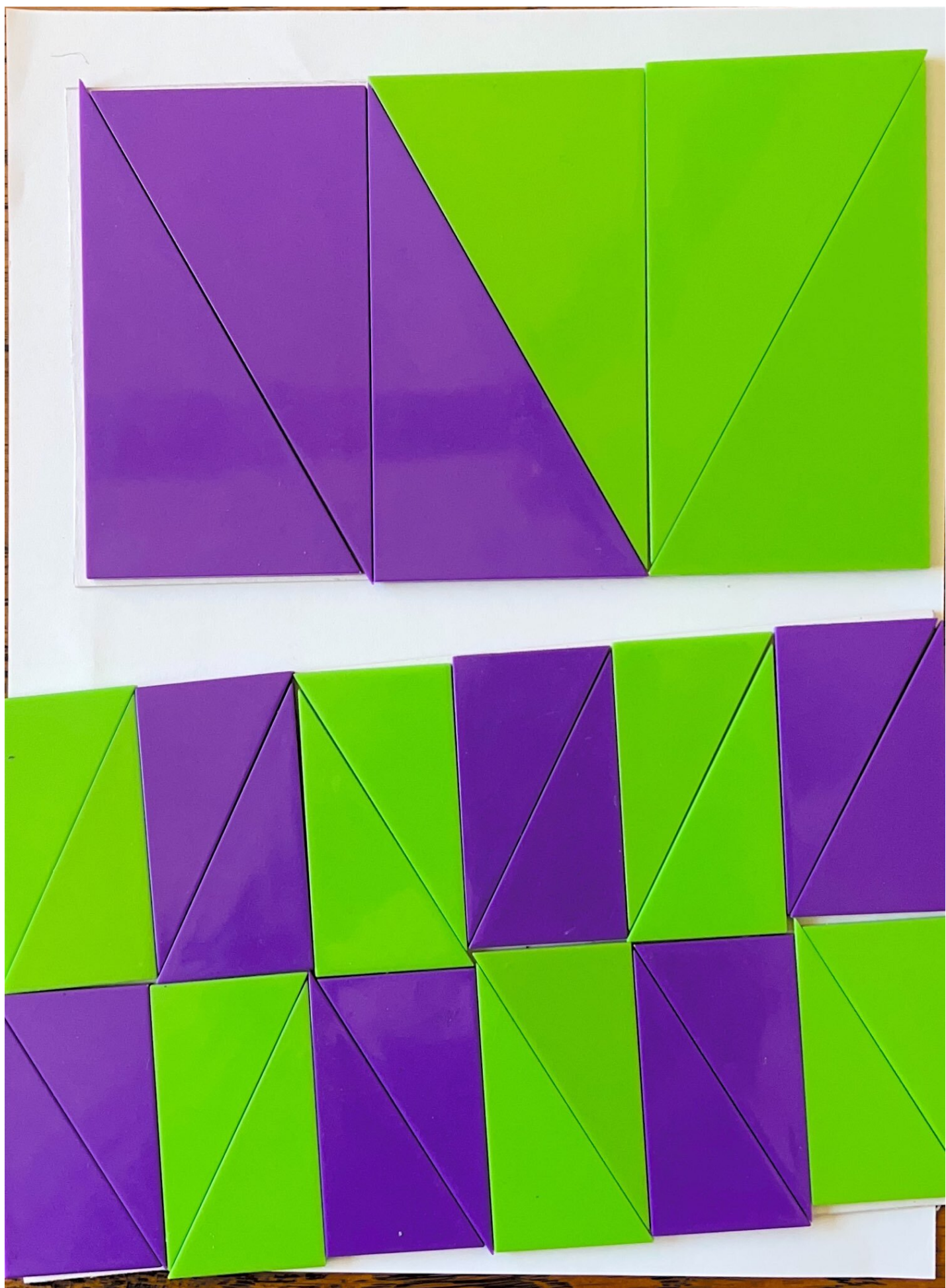
D

E

F

G





Horatio F. D. Rectangles

2	Small triangles cover rectangle A
2	Medium triangles cover rectangle B
6	Small triangles cover rectangle C
2	Large triangles cover rectangle C
8	Small triangles cover rectangle D
6	Medium triangles cover rectangle D
8	Medium triangles cover rectangle E
18	Small triangles cover rectangle F
6	Large triangles cover rectangle F
24	Small triangles cover rectangle G
18	Large triangles cover rectangle G
18	Medium triangles cover rectangle G

	S	M	L
A	$A=2S$		
B		$B=2M$	
C	$C=6S$		$C=2L$
D	$D=8S$	$D=6M$	
E		$E=8M$	
F	$F=18S$		$F=6L$
G	$G=24S$	$G=18M$	$G=8L$

G	=	24	S
G	=	8	L
G	=	24	S
G	=	18	M
G	=	8	L
G	=	18	M

L	=	3	S
S	=	$\frac{1}{3}$	L
M	=		S
S	=		M
L	=		M
M	=		L

Math Circle 7 11/28/2023 2:30 to 3:30 pm

Roosz, Adam and Dar

We worked on Factor Blocks. The activities are from 3 teachers guides that I have co-authored for the use of Factor Blocks. This manipulative was the most successful of my 3 major manipulatives and it had a 10+ year run in schools and it is a shame that was discontinued by ETA. It helps students in grades 4-6 visualize factors and multiplication and eventually Greatest Common Factor GCF and Least Common Multiple LCM. We will spend the next 2 weeks also on these activities and may come back to it for GCF and LCM later on after more work on Fractions.

I told them about Factor Blocks and the fact that I had invented the Manipulatives and showed them the Patent and discussed what it meant. They seemed interested and also I gave them their sets to open and explore.



Roosz with my help read and translated the scroll that describes the definition of the Factor Blocks in French after they had both manipulated and ordered and reordered the Factor Block finding how many different blocks were in it and how they were called. We discussed the story and also what it meant by looking at the different Factor Blocks.

Next they organized together the Factor Blocks and grouped them in different ways and eventually using one of each kind ordered them from the smallest to largest and explored what their dimensions were and how they should be named and how the Scroll had defined them. The Scroll defined them as “3 Dimensional blocks that have 1 or a one-digit prime number for each dimension”

Next, they completed Activity Sheet 1 and they discussed the naming of the dimensions and went back and forth to get the correct names. They did this much faster than I thought would take so I also discussed the fact that not all Factor Blocks were in the set and gave them the challenge that I had initially planned to send home with them.

They took more time with this, but they seemed to enjoy it and both completed it checking from time to time with each other to make sure they had the right name and the number of Units. They also came up with the fact that since there were 20 more than the original 15 Factor Blocks in this set if they were all included the set would have become way too large for students to use.

Next I told them about the Factor City that they were going to built next time and the fact that the building have footprints and they used the Bird’s Eye View Square Paper and the Factor Blocks to draw the foot prints of all the possible Factor Blocks and found that there were 11 different footprints. They colored these footprints and played with placing the Factor Blocks on them.

I was going to let them use the engineering paper to draw 3-D pictures of some of the Blocks (which is a challenging task initially) but since we had spent an hour already I decided to leave that for next time (maybe).

It was quite impressive that they worked for that long and Adam asked for a break in the middle and they got 5 minutes. A very good session all in all.

Grades 4-8

factor city™



Teacher's Resource Book
for Dr. Super's™ Factor Blocks™



Authors
Behrouz Aghevli
Noel Klimenko
Mark Spikell

ETA 
Cuisenaire

The
United
States
of
America



The Commissioner of
Patents and Trademarks

Has received an application for a patent for a new and useful invention. The title and description of the invention are enclosed. The requirements of law have been complied with, and it has been determined that a patent on the invention shall be granted under the law.

Therefore, this

United States Patent

Grants to the person(s) having title to this patent the right to exclude others from making, using, offering for sale, or selling the invention throughout the United States of America or importing the invention into the United States of America for the term set forth below, subject to the payment of maintenance fees as provided by law.

If this application was filed prior to June 8, 1995, the term of this patent is the longer of seventeen years from the date of grant of this patent or twenty years from the earliest effective U.S. filing date of the application, subject to any statutory extension.

If this application was filed on or after June 8, 1995, the term of this patent is twenty years from the U.S. filing date, subject to any statutory extension. If the application contains a specific reference to an earlier filed application or applications under 35 U.S.C. 120, 121 or 365(c), the term of the patent is twenty years from the date on which the earliest application was filed, subject to any statutory extension.

J. Todd Johnson

Acting Commissioner of Patents and Trademarks

Pamela L. Morton

Attest

L'Annee 1235, Le 7 Janvier

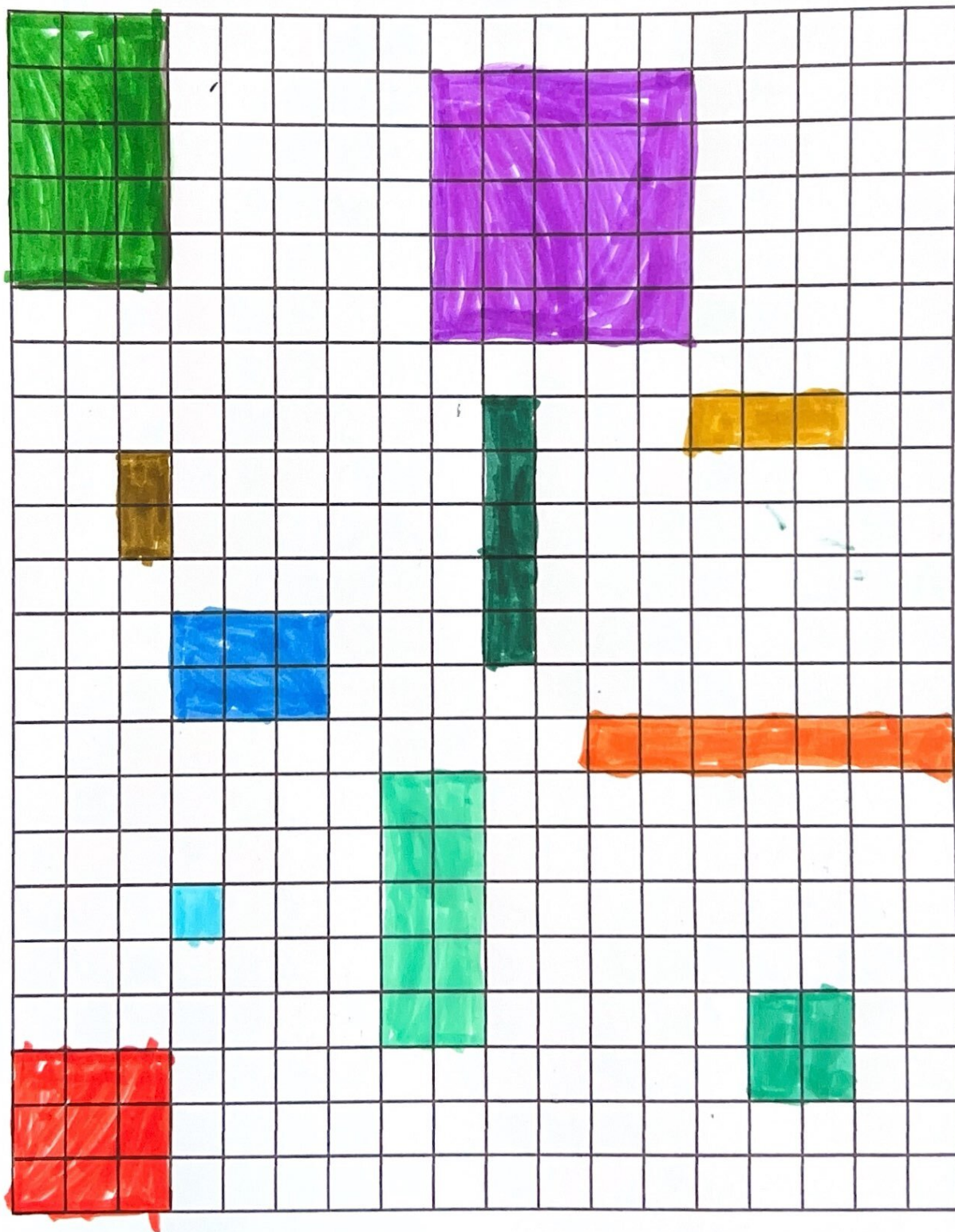
**Les Blocs ont
trois dimensions.**

**Chacun des dimensions
doit être un ou
un numero premier d'un chiffre.**

Translation Box

French	English
chacun	each
chiffre	digit
doit	must
des	these
être	are
les	the
numero	number
ou	or
premier	prime
trois	three
un	one

Bird's-Eye View

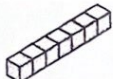
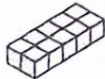


Activity Sheet 1

Name Rooz

Date _____

Meet the Factor Blocks™

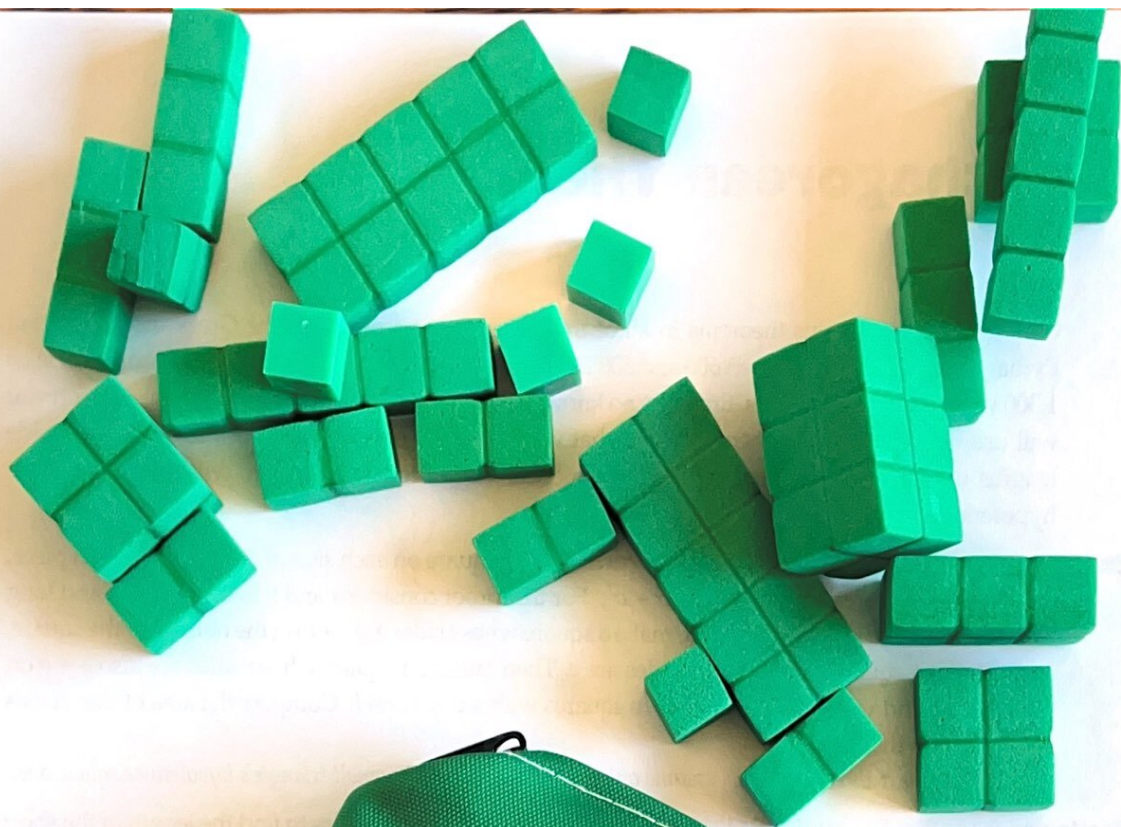
Factor Blocks	Type	Height	Width	Length	Name	Number of Units
	Unit	1	1	1	1	1
	Long	1	1	2	2	2
	Long	1	1	3	3	3
	Long	1	1	5	5	5
	Long	1	1	7	7	7
	Flat	1	2	2	4	4
	Flat	1	2	3	⁶ 2x3	6
	Flat	1	3	3	9	9
	Flat	1	2 5	5	10	10
	Flat	1	3	5	15	15
	Flat	1	5	5	25	25
	Block	2	2 2	2	8	8
	Block	2	2	3	12	12
	Block	2	2 5	5	20	20
	Block	2	3	5	2x3x5	30

©1998 EIA

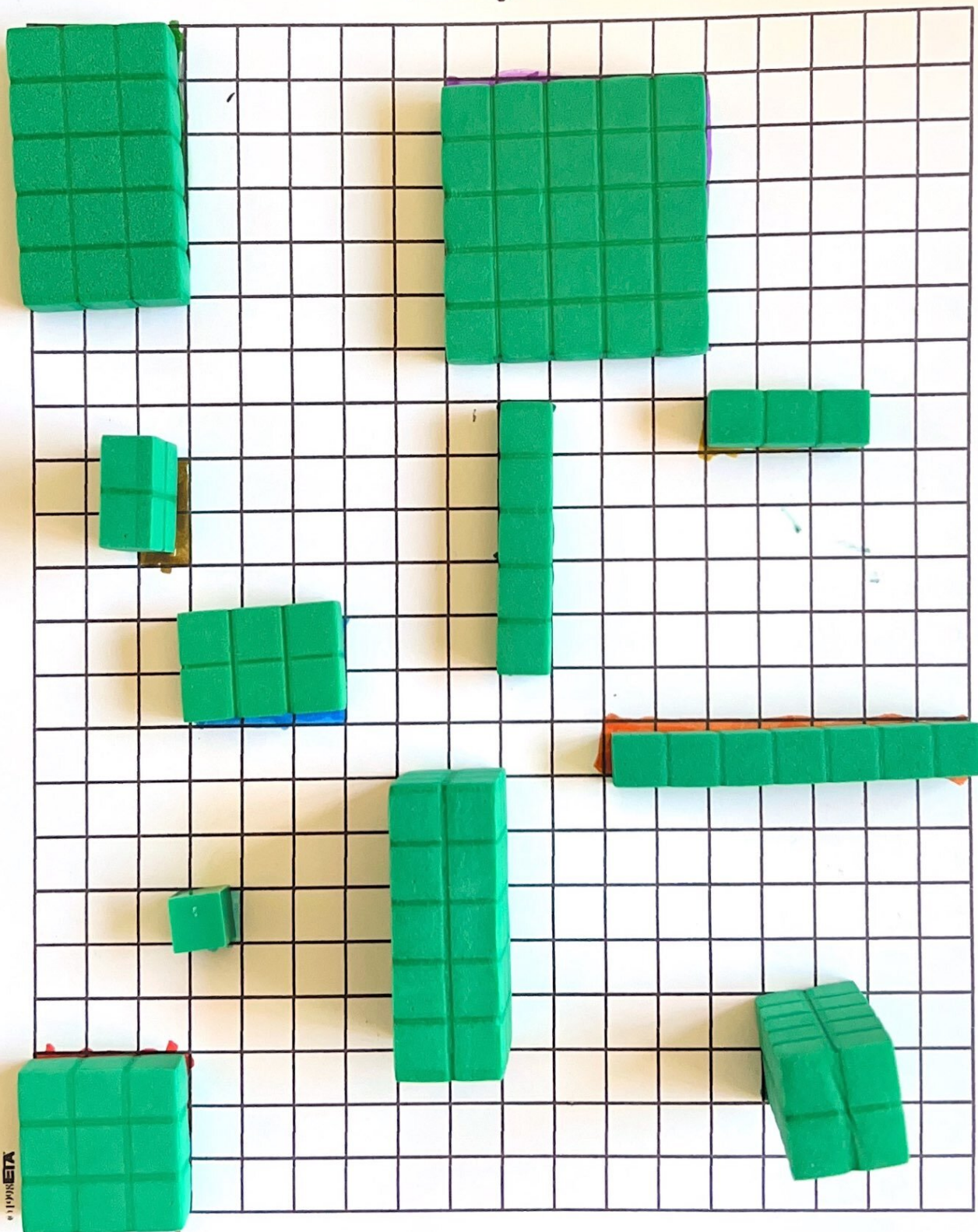
Complete the Height and find the Name and the Number of Units that are missing from Dr. Super's Factor Blocks set.
Why do you think these were not included in the set?

Type	Height	Width	Length	Name	Number of Units
Block	2	3	3	2x3x3	18
Block	3	3	3	3x3x3	27
Block	5	3	5	5x3x5	75
Block	7	5	5	7x5x5	175
Block	8	5	5	8x5x5	200
Block	5	5	5	5x5x5	125
Block	1	2	7	1x2x7	14
Block	1	3	7	1x3x7	21
Block	2	5	7	2x5x7	70
Block	3	3	7	3x3x7	63
Block	1	5	7	1x5x7	35
Block	3	5	7	3x5x7	105
Block	1	2	7	1x2x7	14
Block	2	3	7	2x3x7	42
Block	3	5	7	3x5x7	105
Block	2	7	7	2x7x7	98
Block	1	7	7	1x7x7	49
Block	2	7	7	2x7x7	98
Block	3	7	7	3x7x7	147
Block	7	7	7	7x7x7	343

Factor Blocks are rectangular prisms that have 1 or a prime digit for the length of their 3 dimensions.



Bird's-Eye View



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Math Circle 8 12/06/2023 3:30 to 4:30 pm

Rooz, Adam, and Dar

In this session we worked on examining how a Factor Blocks can be rotated to fit on different plot using the plots that they had from last session and also how many factor blocks can be placed on the same plot.. Then they completed Activity Sheet 4, Table 1. They did that fast but did not spend enough time to place the Factor Blocks on the birds eye view work sheet. Roozs did more of the placing of the Blocks than Adam who rushes a bit more but they wanted to fill up the table and they did a good job.

Next, we moved on to Activity Sheet 4, Table 2 where they find on how many different plots each Factor Block can be place. Here they used the blocks more, but I had to keep encouraging them to use the blocks before they filled the table. We did have a good discussion about the number of faces that each block has and how this related to the table they had filled up.

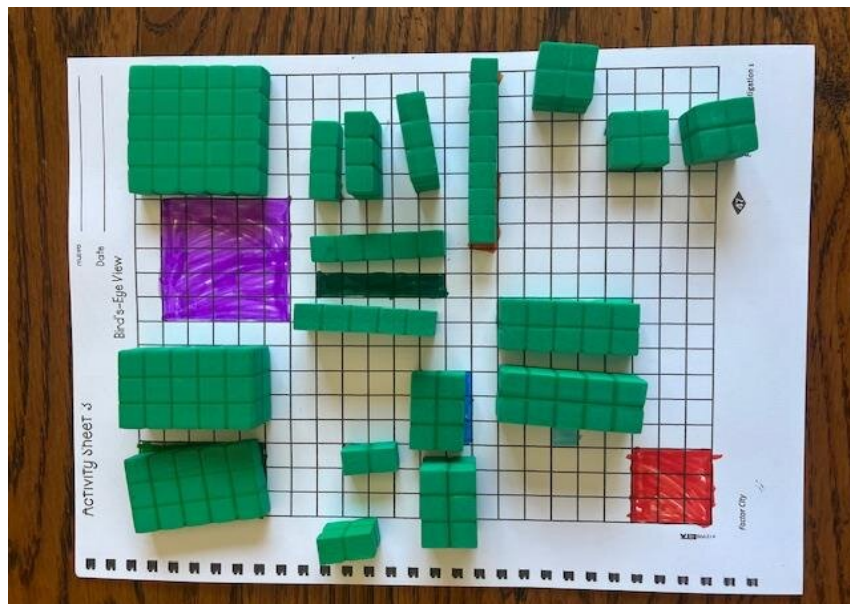
They wanted to stop (Adam is a big fan of Breaks) So I give them 8 minutes.

Then they did the activity of relating floors to a building to the area of the plot and finding multiple to be done before the next session at home.

They also used the 3-D paper to color a few of the Factor Block and Other Blocks and they enjoyed it.

They also got the nets for the building for Factor City that they will color and cut and make into rectangular prisms with tape in the next session.

I had planned to show them the Powers of Ten Videos but I forgot about it so I will do it in the next session.



Activity Sheet 4

Name: _____ Date: _____

Plot Area

Plot Drawing	Plot Dimension	Plot Area	Factor Blocks Buildings that Fit on this Plot
	1x1	1	1, 2, 3, 5, 7
	2x1	2	2, 4, 6, 10
	3x1	3	3, 6, 9, 15
	4x1	4	4, 8, 12, 20
	5x1	5	5, 10, 15, 25
	2x2	4	4, 8, 20
	3x2	6	6, 12, 24, 30
	3x3	9	9
	2x5	10	10, 20, 30
	3x5	15	15, 30
	5x5	25	25

Factor Blocks that only Fit on 1 Plot (all faces the same)	Factor Blocks that Fit on Exactly 2 Plots (2 different faces)	Factor Blocks that Fit on Exactly 3 Plots (3 different faces)	Factor Blocks that Fit on Exactly 4 Plots (4 different faces)
Unit	2x2 Plot		
1	2, 3	15	
8	2	30	
	3	10	
	5		
	7		
	9		
	20		
	12		
	4		
	6		

Factor City Investigation 1

Factor City

Name _____

Date _____

Factors and Multiples

Tables 1A - 1E

A Building with a Value of 6

Factor Block	Number of Rooms
1	6
2	3
3	2
6	1

B Building with a Value of 12

Factor Block	Number of Rooms
1	12
2	6
3	4
4	3
6	2
12	1

C Building with a Value of 15

Factor Block	Number of Rooms
1	15
3	5
5	3
15	1

D Building with a Value of 30

Factor Block	Number of Rooms
1	30
2	15
3	10
5	6
6	5
10	3
15	2
30	1

E Building with a Value of Your Choice

Factor Block	Number of Rooms
1	50
2	30
3	20
4	15
5	12
6	10
10	6
15	5
20	4
30	3
50	1

Table 2

Factor Blocks	Number of Rooms					
	1 Story	2 Stories	3 Stories	4 Stories	7 Stories	10 Stories
2 Long	2	24	6	8	10	20
5 Long	5	10	15	20	25	50
2x3 Flat	6	12	18	24	30	60
2x5 Flat	10	20	30	40	70	100

Math Circle 9&108 12/13&20/2023 1:50 to 3:45 pm

Rooz, Adam and Dar (with help from Cyrus)

Rooz and Adam completed the Factor City Activity, and it took them two long session with no breaks. They colored taped to make their cities. good for them. (This activity is really for 4-6 graders, so it requires more dexterity for cutting and making the buildings and it was done in five sessions.)

Before continuing the activities with Factor Blocks we watched the video on YouTube called [**Powers of Ten™ \(1977\)**](#) made by Eames husband and wife that is amazing as it was made in 1977, it not only to give a great introduction to the positive and negative power of 10 but also to give a sense of where we as humans fit into the universe at large and the microscopic world also. There is also a video called [**Scales of the Universe in Powers of Ten**](#) which is very similar with some better images when we travel out into the cosmos and also to the microscopic world. They enjoyed it and we did talk about the powers of ten some. These videos are well worth seeing again and discussing them afterwards.

Then, they initialed and colored the different buildings on Activity Sheet 12 using the buildings they had cut out and glued together. They had both cut out the building before and with help from Julia and Aryan. This was a slow process, but they enjoyed it. They enjoyed this and made a lot of noise doing it. Their interaction is very nice to see. It is a great mixture of cooperation and competition and copying from each other.

Next in the second session they completed Activity Sheet 12 and chose and colored the plots where they would place each building and used the same color as the corresponding buildings and initialed them. Then they built the houses with the Factor Blocks that were designated on the designated plots.

Next, I worked with both of them to put in the buildings that they had built on their corresponding plots with double-sided glue. Adam actually took the initiative and glued some of his houses himself. They enjoyed this process. This was not as slow as the other activities but proceeded at a good rate for both.

They also completed the Activity Sheet 13 to find the total cost for the city. Both really wanted to complete their work. Figuring out the Total Cost was not easy, but I pointed out that they could look at Activity Sheet 12 and

there was a clue there. They did figure out the Total Cost when they realized that the answer was the total value multiplied by the \$10,000 which is the Price of a Unit volume.

These guys are amazing! You should all be proud of them and let them know that Dar (Dr. Super) said so;=>

Activity Sheet 12

Name: Just Z
Date: 11-13-13

Factor City Building Chart (City Planner)

Building	Factor Blocks Used	Number of Factor Blocks Used	Volume (cm ³)
One	One	6	6
Two	2 Long	6	12
Three	3 Long	4	12
Four	4 Long	2	10
Five	5 Long	1	7
Grocery Store	2x2 Flat	3	12
Movie Theater	2x2 Flat	2	12
Shed	2x5 Flat	2	20
Restaurant	3x3 Flat	1	9
Post Office	3x5 Flat	1	15
Supermarket	5x5 Flat	1	25
Warehouse	2x2x2 Block	1	8
Hospital	2x2x3 Block	1	12
School	2x2x5 Block	1	20
Office Building	2x3x5 Block	1	30
Totals		33	240

Activity Sheet 13

Name: Rose
Date: 11-13-13

Factor City Budget Chart (Accountant)

Building	Factor Blocks Used	Number of Factor Blocks Used	Value of Each Factor Block Used	Building Cost
One	One	6	\$10,000	\$60,000
Two	2 Long	6	20,000	120,000
Three	3 Long	4	30,000	120,000
Four	4 Long	2	50,000	100,000
Five	5 Long	1	70,000	70,000
Grocery Store	2x2 Flat	3	40,000	120,000
Movie Theater	2x2 Flat	2	60,000	120,000
Shed	2x5 Flat	2	100,000	200,000
Restaurant	3x3 Flat	1	90,000	90,000
Post Office	3x5 Flat	1	15,000	15,000
Supermarket	5x5 Flat	1	25,000	25,000
Warehouse	2x2x2 Block	1	80,000	80,000
Hospital	2x2x3 Block	1	120,000	120,000
School	2x2x5 Block	1	200,000	200,000
Office Building	2x3x5 Block	1	300,000	300,000
Totals		33	330,000	2,400,000

Activity Sheet 12

Name: _____
Date: _____

Factor City Building Chart (City Planner)

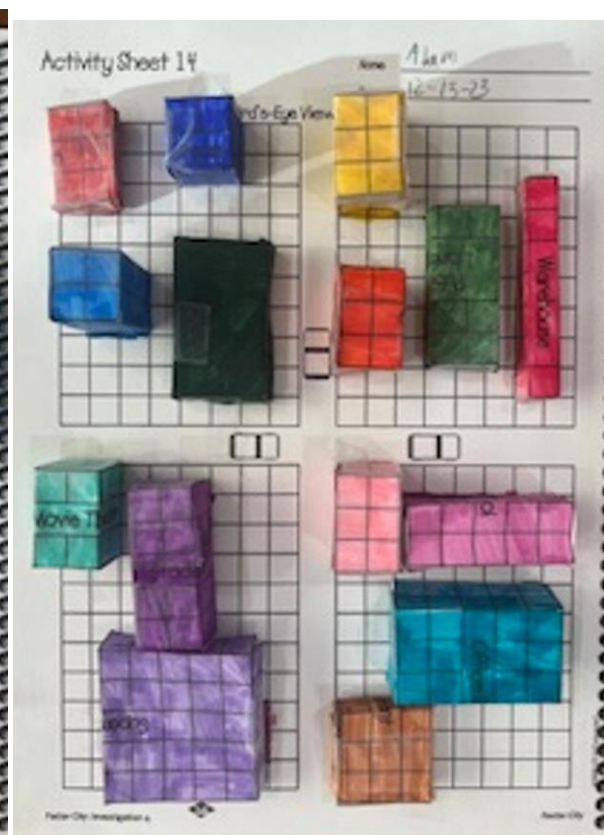
Building	Factor Blocks Used	Number of Factor Blocks Used	Volume (cm ³)
One	One	6	6
Two	2 Long	6	12
Three	3 Long	4	12
Four	4 Long	2	10
Five	5 Long	1	7
Grocery Store	2x2 Flat	3	12
Movie Theater	2x2 Flat	2	12
Shed	2x5 Flat	2	20
Restaurant	3x3 Flat	1	9
Post Office	3x5 Flat	1	15
Supermarket	5x5 Flat	1	25
Warehouse	2x2x2 Block	1	8
Hospital	2x2x3 Block	1	12
School	2x2x5 Block	1	20
Office Building	2x3x5 Block	1	30
Totals		33	240

Activity Sheet 13

Name: _____
Date: _____

Factor City Budget Chart (Accountant)

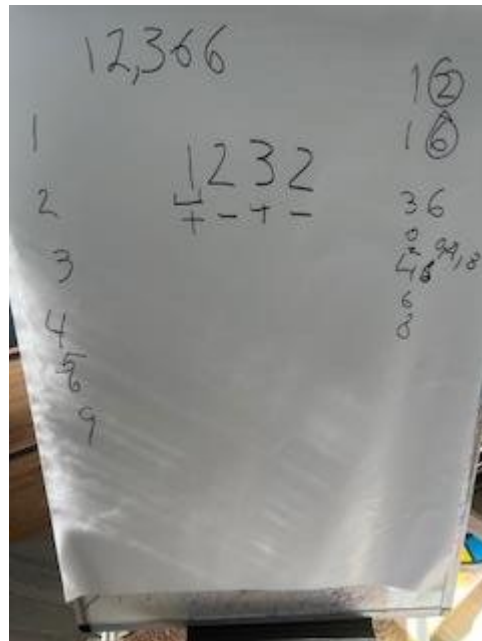
Building	Factor Blocks Used	Number of Factor Blocks Used	Value of Each Factor Block Used	Building Cost
One	One	6	\$10,000	\$60,000
Two	2 Long	6	20,000	120,000
Three	3 Long	4	30,000	120,000
Four	4 Long	2	50,000	100,000
Five	5 Long	1	70,000	70,000
Grocery Store	2x2 Flat	3	40,000	120,000
Movie Theater	2x2 Flat	2	60,000	120,000
Shed	2x5 Flat	2	100,000	200,000
Restaurant	3x3 Flat	1	90,000	90,000
Post Office	3x5 Flat	1	15,000	15,000
Supermarket	5x5 Flat	1	25,000	25,000
Warehouse	2x2x2 Block	1	80,000	80,000
Hospital	2x2x3 Block	1	120,000	120,000
School	2x2x5 Block	1	200,000	200,000
Office Building	2x3x5 Block	1	300,000	300,000
Totals		33	330,000	2,400,000



Math Circle 11 01/10/2024 3:30 to 4:30 pm

Rooz, Adam, and Dar (Big Cyrus Observed and took Photo) (Little Cyrus Joined at the end)

We played the Divisibility Game that I had adopted from a more complete version that I had done for Cyrus and Marc. In this game the divisors went up to 12 and they needed the divisibility rules for 1 to 12. I started explaining the rules and we did 2, 3, 9 and 5 (that they mostly knew) and discussed 4 and 6 and 11 and I was going to talk about the rules for 7 and 8 that are more difficulty but they wanted to start playing so we started.



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-98dc-76c9d4d51daf

The rules for the game are somewhat like Monopoly, and we used the money and tokens from Monopoly. Each person has a token and gets \$1,500 to start the game. The board has 20 boxes numbered from 1 to 12 with some of the numbers repeated. The board also has 3 corner boxes Called Win the Bank where if a player lands on it he will win the money in the Bank (in the middle). Each player receives an extra \$200 for going across the Big Divide (Starting Block), also \$50 is placed in the Bank in the middle. Players take turns and roll 2 dice and will land on a number or win the bank. They will get 3 small number cards that have 0-9 on them where they can make a 3-digit number with it (generally there are 6 possible ways to make different numbers). If they can make a number that is divisible by the number they have landed on they will get 10 time that number. If not, they can move their token to another number that is a divisor for their chosen 3-digit number and get 5 times the mount of that number. If they must pass the Big Divide to do this, they will get another \$200. They can also declare their number as Prime and get \$100 for it. Each player gets 4 special cards called Calculator Check, Calculator Challenge, Prime Check and Prime Challenge. They can use these cards to either check for divisibility or to be prime or challenge another player. If a challenge occurs then if the challenger wins he will receive the amount from the Bank. If the challenger is wrong the player gets the money from the Bank and the challenger must put an equal amount in the Bank in the middle.

They were both very quick to pick up on the rules and played well. They really enjoyed the game and there was not that much competitive spirit. When the number cards ran out, I asked them if they wanted to play another round or stop and they both said that they wanted to continue. In the end everyone counted their money, Dar lost badly and because of a last move Rooz got a little ahead of Adam and won.

They really enjoyed themselves very much and would help each other about what number to make or the divisibility.

I need to modify some of the rules a bit in particular on the primes. We will cover the divisibility rules up to 12 once again with emphasis on 4, 7, 8 and 11 and play one more time.

They were both all over the place and Rooz was mostly on the table and Adam was rolling around on the coach. but they were both very focused. I helped them some times with pointing out bigger numbers that they could use as a divisor, Little Cyrus also joint in the last 15 minutes and was helping both of them.



Math Circle 12 & 13 Divisibility Games 01/24&31/2024 3:30 to 4:20 pm

Rooz, Adam, and Dar

In Session 12 we continued to play the Divisibility Game that was described and played the week before. To start with I went over the divisibility rules by numbers 1 to 12 and for 7 and 11 again. They had an easy time recalling these rules, but they will need more practice to keep them all. We also talked about divisibility by 4 and 8 that are also a bit more difficult.

They enjoyed playing the game and finding primes continued to be a favorite search as they got more money for it even though they were getting less than the last time. I would help them with some of the numbers, but they mostly would find the answers themselves. They continued to enjoy the game and there was not that much competitive spirit this time either and they would help each other. At the end they counted their money but even though it was clear that Dar was last it did not become clear who had won, and they did not insist on it.

List of Prime Numbers Up to 1,000									
2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541
547	557	563	569	571	577	587	593	599	601
607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733
739	743	751	757	761	769	773	787	797	809
811	821	823	827	829	839	853	857	859	863
877	881	883	887	907	911	919	929	937	941
947	953	967	971	977	983	991	997		

They really enjoyed themselves very much again and would help each other with what number to make with the 3 cards that they drew or the divisibility rule for each number. Also, they were both all over the place again, Rooz was mostly on the table and Adam was rolling around on the Couch. I am attaching the picture from last session that got repeated many times. We enjoy ourselves as we learn; =>

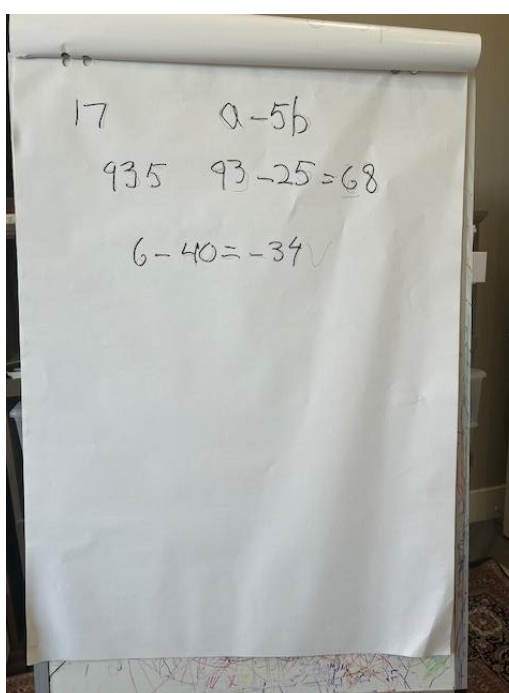
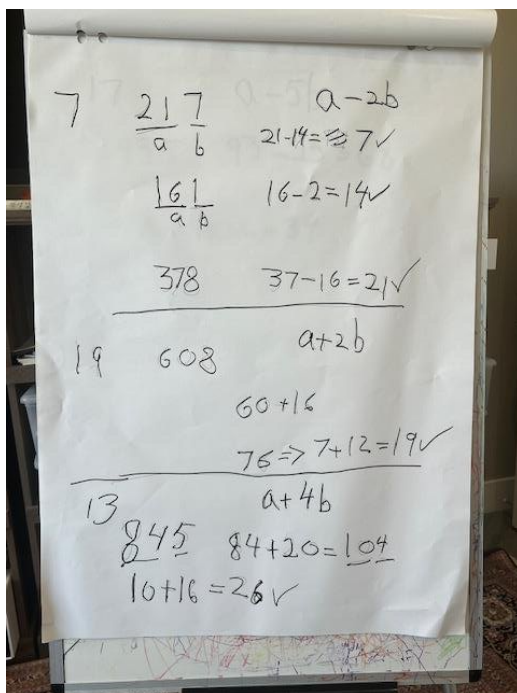


In the 31 session we played another divisibility game that I had also invented. It was based on a video that I saw on you tube on how to check [Divisibility for 7 with a graph](#).

Before starting I went over the divisibility rules up to 20 in particular for 7, 13, 17, and 19. They were interested but wanted to get to play the game as the board looks kind of cool and strange.

Rules for Divisibility by Numbers from 1 to 20

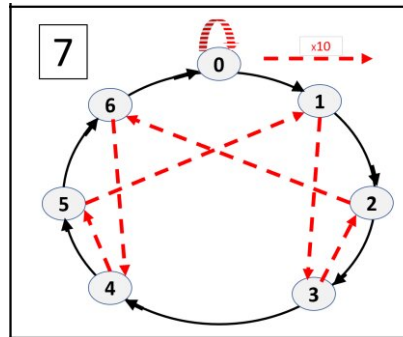
Everything is divisible by 1	If the number is even 1s-digit is: 0, 2, 4, or 8 then it is divisible by 2	If the sum of the digits is divisible by 3 then the number is divisible by 3	Write your number ab as $ax10 + b$ ab is divisible by 4 iff $2a-3b$ is divisible by 4 e.g. 388 is divisible by 4 because $76 - 24 = 52$ that is divisible by 4	If the number ends in 5 or 0 it is divisible by 5
If the number is even and divisible by 3 it then is divisible by 6	Write your number ab as $ax10 + b$ ab is divisible by 7 iff $a-2b$ is divisible by 7 e.g. 203 is divisible by 7 because $20 - 6 = 14$ that is divisible by 7	Write your number ab as $ax10 + b$ ab is divisible by 8 iff $2a-3b$ is divisible by 8 e.g. 336 is divisible by 8 because $66 - 18 = 48$ that is divisible by 8	If the sum of the digits is divisible by 9 then the number is divisible by 9	If the number ends in 0 it is divisible by 10
If the alternating + - sum of the digits is divisible by 11 then number is divisible by 11	If the number is divisible by 3 and 4 then it is divisible by 12	Write your number ab as $ax10 + b$ ab is divisible by 13 iff $a+4b$ is divisible by 13 e.g. 351 is divisible by 13 because $35+4=39$ that is divisible by 13	If the number is even and divisible by 7 then it is divisible by 14	If the number is divisible by 3 and 5 then it is divisible by 15
Write your number ab as $ax10 + b$ ab is divisible by 16 iff $2a-3b$ is divisible by 16 e.g. 192 is divisible by 16 because $38 - 6 = 32$ that is divisible by 16	Write your number ab as $ax10 + b$ ab is divisible by 17 iff $a-5b$ is divisible by 17 e.g. 272 is divisible by 17 because $27 - 10 = 17$ that is divisible by 17	If the number is even and divisible by 9 then it is divisible by 18	Write your number ab as $ax10 + b$ ab is divisible by 19 iff $a+2b$ is divisible by 19 e.g. 399 is divisible by 19 because $39+18=57$ that is divisible by 19	If the number ends in 0 and the 10s-digit is even then it is divisible by 20



They got some money as before but to check for divisibility they would need to see if the started with the 100s-digit and went around the graph and jumped at the end. Then did the same for 10s- digit and finally for the Ones-digit. The game went well but we ran into a little

trouble for the graph for numbers 2, 5 and 10 that seem to indicate that all numbers were divisible by them. (I had forgotten that there was no jump on the ones-digit). But it was a good exercise for them. I know this sounds kind of complicated, but it really is not really complicated when they are playing and moving their pieces on a graph to see if they end up at 0 or not.

For example, to check that 427 is divisible by 7 using the graph for 7 first they move 4 to get to number 4 on the graph and then jump to 5 as indicated on the graph, next they go 2 steps from 5 to get to 0 and remain there as the loop shows. Final they go 7 steps to get back to 0. Since they ended up at 0 this means 427 is divisible by 7.



The jumps are the remainders after dividing by 7 which is indicated by mod:

$10 \bmod(7)$ is 3 → $10:7=1$ rem 3

$20 \bmod(7)$ is 6 → $20:7=2$ rem 6

$30 \bmod(7)$ is 2 → $30:7=4$ rem 2

$40 \bmod(7)$ is 5 → $40:7=5$ rem 5

$50 \bmod(7)$ is 1 → $50:7=7$ rem 1

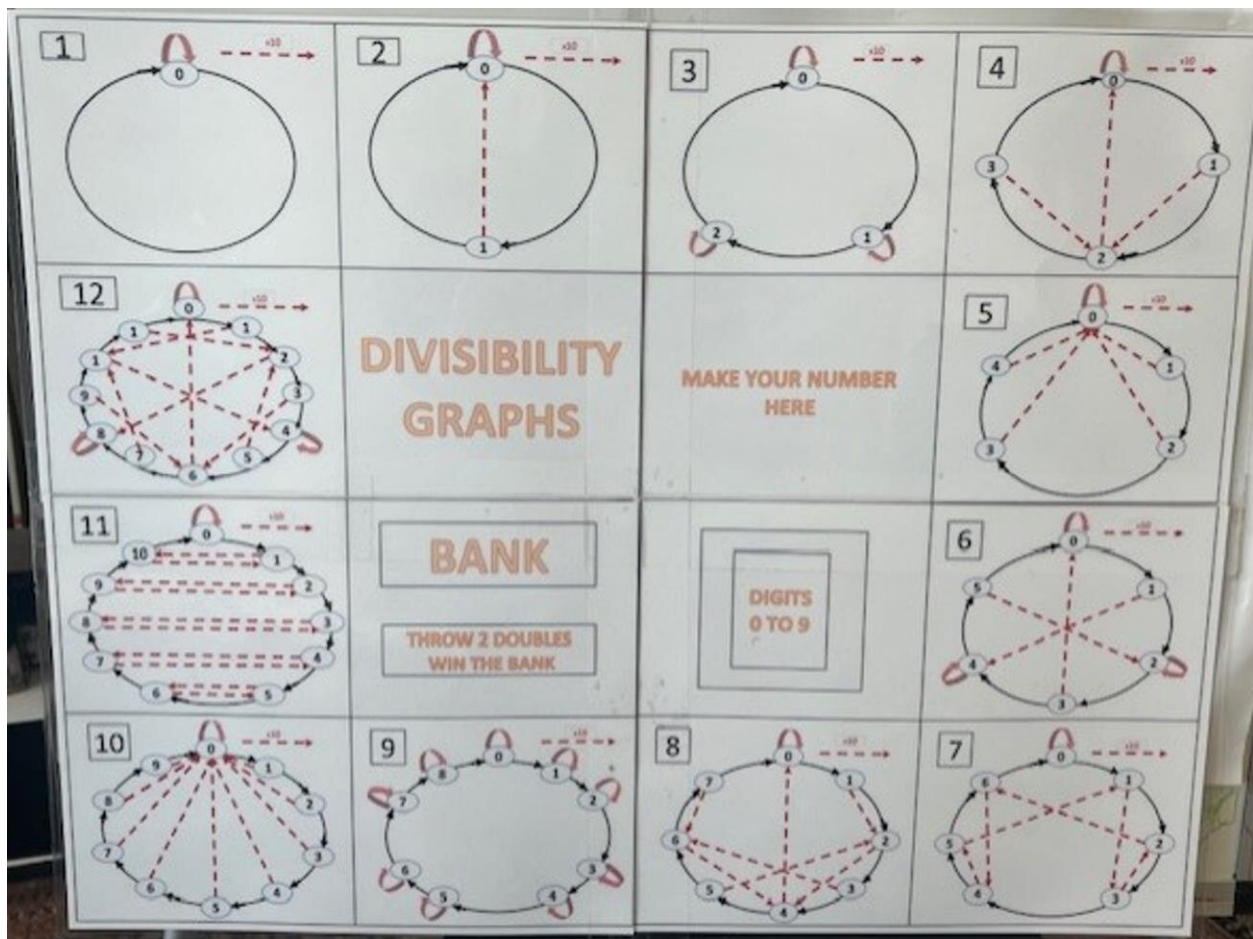
$60 \bmod(7)$ is 4 → $60:7=8$ rem 4

$70 \bmod(7)$ is 0

$80 \bmod(7)=10 \bmod(7)$ is 3

$90 \bmod(7)=20 \bmod(7)$ is 6

Now to check for divisibility of 427 we write as $10(10(4)+2)+4$ and find the remainders from left to right first inside the parenthesis for 4 with a jump to 5 for 10×4 then we add the 2 to get to 0 from 5 and finally 7 to get back to 0 that indicates that the remainder is 0.



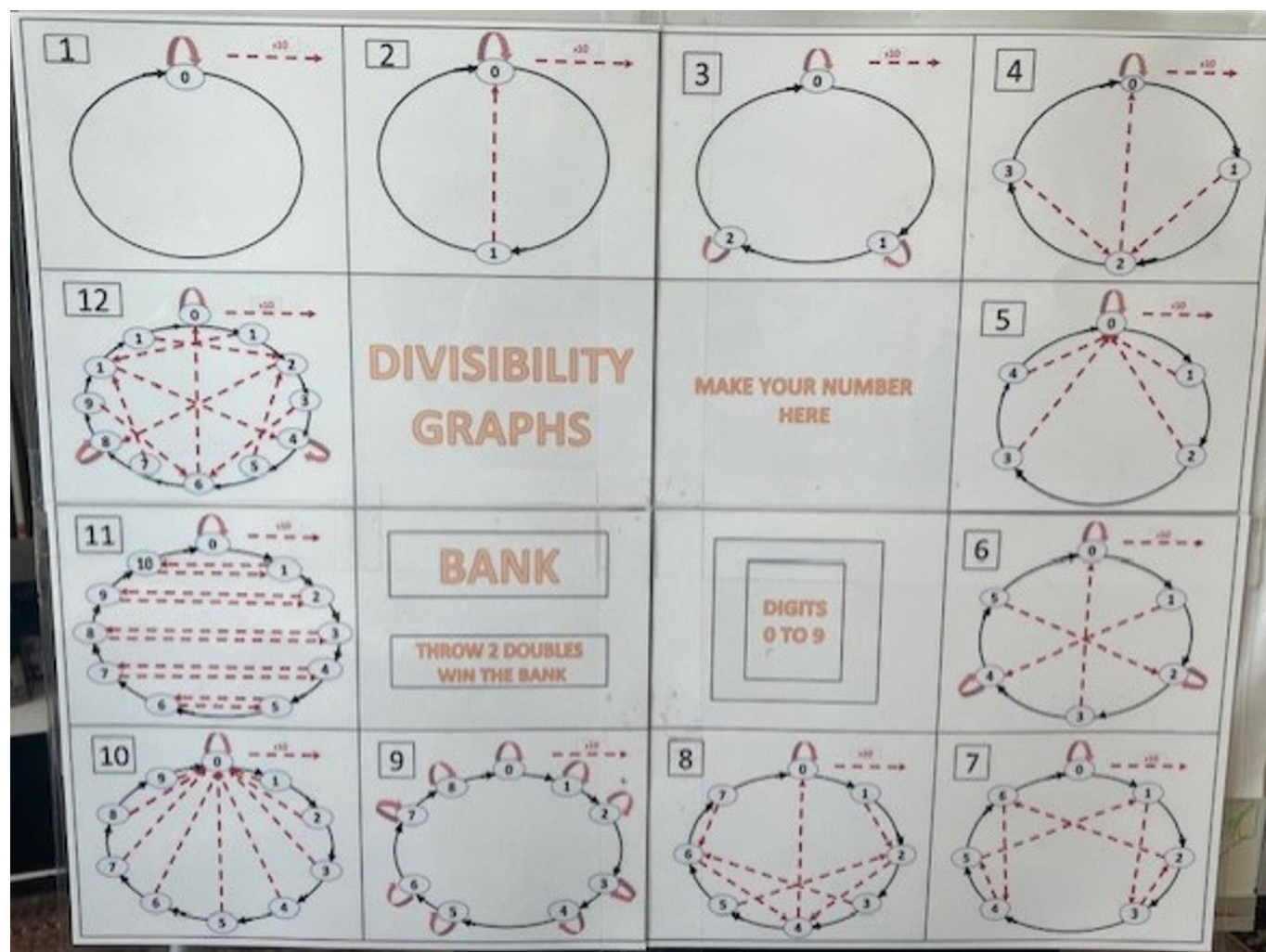
The game went well and as I said they really enjoyed finding a problem with it. We did not get into why the moving around on the graph could determine whether a number is divisible by the number on that graph but they were fascinated by it and also used the other technique to test for divisibility also.

They wanted to run off to the cave to play with video games in my car as Cyrus and Marc were also there so we ended a little early.

Roosz, Adam, and Dar

We initially talked about the error that we had found for checking divisibility by 5, 8 and 10 and it turns out that there is no error and I had forgotten to tell them that when they make the moves for the ones-digit they should not follow the red arrow that is only when that digit is multiplied by 10. So, for example for checking divisibility by 5 for 342 they will end up at 0 before taking the 2 steps for the ones-digit so they will end up at 2 and this says that 342 is not divisible by 5. Note that if a number ends in 5 we will always come back to 0 after five steps as we will start from 0. For all the graphs if you end up at 0 after you have taken all the steps for all the digits in the number then the number is divisible by the number indicated on that graph. For example to check if 224 is divisible by 7 we first go 2 steps for the 100s-digit and jump to 6 as indicated by the red arrow, next we go another 2 steps and end at 1 and jump to 3 as indicated by the red arrow last we go 4 steps for the one-digit and end at 0 so 224 is divisible by 7

(Julia, Dan and Jerry) I will send you instructions for building the game and the rules so you can play with Adam. This game with the rules that I sent home will give Adam a solid background on divisibility that most kids never get.



Next, I had them play with the following table and told them that: In this table if you covered any 3 consecutive numbers horizontally or vertically, I can tell you what the sum of the 3 covered numbers is equal to. I told them that I had memorized the table completely. We did this a few times and they were impressed but I also made some mistakes as I had forgotten how to find the sum if the 3 squares touched one of the edges. Next, I taught them the trick. It is simply that any five adjacent numbers in this table add up to 20. So, all they must do is find the sum of the two numbers around the covered numbers and subtract it from 20 to get the sum. If the numbers touch a side, then the two numbers preceding or following them should be added and then subtracted from 20. This is what I had forgotten. We did this several times and they became proficient in it in particular Adam. Rooz was a little hesitant still. I also was a bit hazy about the vertical numbers but the trick works for them also. This is what they can do for friends and family, and it looks quite impressive at their age or any age if you do not know the trick.

0	3	9	1	7	0	3	9	1	7	0	3
4	5	2	8	1	4	5	2	8	1	4	5
7	6	0	4	3	7	6	0	4	3	7	6
8	4	6	2	0	8	4	6	2	0	8	4
1	2	3	5	9	1	2	3	5	9	1	2
0	3	9	1	7				1	7	0	3
4	5	2	8	1	4	5	2	8	1	4	5
7	6	0	4	3	7	6	0	4	3	7	6
8	4	6	2	0	8	4	6	2	0	8	4
1	2	3	5	9	1	2	3	5	9	1	2
0	3	9	1	7	0	3	9	1	7	0	3
4	5	2	8	1	4	5	2	8	1	4	5

Next I gave them a table that had most of the cubes for two digit numbers and told them that I could find the two digit number if they gave me the cube. I did this a few times and they were impressed. Then I asked them if they believed that they could do the same in few minutes. They both said no way.

X	X ³	X	X ³	X	X ³	X	X ³	X	X ³
81	531,441	79	493,039	43	79,507	58	195,112	52	140,608
100	1,000,000	38	54,872	56	175,616	24	13,824	90	729,000
44	85,184	61	226,981	43	79,507	94	830,584	92	778,688
40	64,000	91	753,571	41	68,921	51	132,651	73	389,017
38	54,872	34	39,304	93	804,357	81	531,441	14	2,744
82	551,368	80	512,000	34	39,304	67	300,763	19	6,859
19	6,859	40	64,000	50	125,000	71	357,911	69	328,509
81	531,441	46	97,336	41	68,921	26	17,576	61	226,981
76	438,976	99	970,299	44	85,184	42	74,088	14	2,744
38	54,872	26	17,576	54	157,464	57	185,193	59	205,379
88	681,472	58	195,112	18	5,832	72	373,248	33	35,937
30	27,000	61	226,981	95	857,375	14	2,744	24	13,824
26	17,576	52	140,608	82	551,368	69	328,509	58	195,112
22	10,648	26	17,576	18	5,832	18	5,832	22	10,648
38	54,872	70	343,000	93	804,357	77	456,533	82	551,368
49	117,649	15	4,096	15	3,375	64	262,144	73	389,017
26	17,576	15	3,375	25	15,625	19	6,859	82	551,368
22	10,648	55	287,496	88	681,472	80	512,000	30	27,000
59	205,379	48	110,592	17	4,913	62	238,328	100	1,000,000
21	9,261	71	357,911	20	8,000	14	2,744	43	79,507

So I gave them the following table and asked them to find the cube for numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 and write it in front of them. They did this. For 6 onwards they went slower and Rooz knew a few of them like 8 cubed that is 512 because it is 2 to the power nine but I helped them find these cubes. They both completed the table. Then I explained the trick for finding the cube root to them: When someone gives you a cube like 438,976 all you have to remember is the thousands part of the number and the ones-digit. First, you look at the thousands part of the number and see where in the table that falls. In this case it falls between 343 and 512 and then you need to pick the smaller number and look up the tens par for your number, in this case 343 and the tens-digit for the answer will be 7. For the ones- digit of the answer if the cube ends in 1, 4,5,6,and 9 the number will also end with the same digit so here the answer is 76. For numbers ending in 2 or 8 we must switch them, and we have to do the same for 3 and 7 as indicated by the colors on their charts. We took turns and found the cube root when another person gave the cube from the original table. They were getting confused about what to look up so I had them add the zeros both to cube and the cube root numbers did this helped some but still they missed the ones digit some times. With a little practice they can memorize the cubes of the numbers 1 to 9 and the rules for looking up the numbers this really impresses grown up who can not do something like taking cube roots in their head.

We did end a little early and since I had shut down the car they went and hit some ping pong balls and then ran around a bit. It was a good session, and we will practice both tricks a little more next time.

Lookup Table for Third Power of 2-Digit Numbers

X	$X^3(000)$	
1	1	
2	8	
3	27	
4	64	
5	125	
6	216	
7	343	
8	512	
9	729	

Lookup Table for Third Power of 2-Digit Numbers

X	$X^3(000)$	
10	1,000	
20	8,000	
30	27,000	
40	64,000	
50	125,000	
60	216,000	
70	343,000	
80	512,000	
90	729,000	

2 $\longleftrightarrow$ 8

3 $\longleftrightarrow$ 7

Math Circle Mathemagic 2/14/2024 3:00-3:45

Roos, Adam and Dar.

We started with a challenge activity “The Sieve of Eratosthenes” in which they crossed out multiples of primes starting with 2 to find all the prime numbers on a table of numbers up to 100.

They both completed the task and found all the 25 primes after a few rounds counting and checking. Next, I asked them what prime number they had to go up to and cross out it’s multiples to find the rest of the primes and they both said 7. I asked them why they did not have to go to 11 and after some discussion they end up with the fact that 11×11 was 121 and so it was not in the table (and all the other multiples of 11 were already crossed out – I am pretty sure they made this conclusion even though they did not say it).

We did an advanced version of this with Cyrus and Marc. Adam and Roos did the first part of this challenge as well as the older boys.

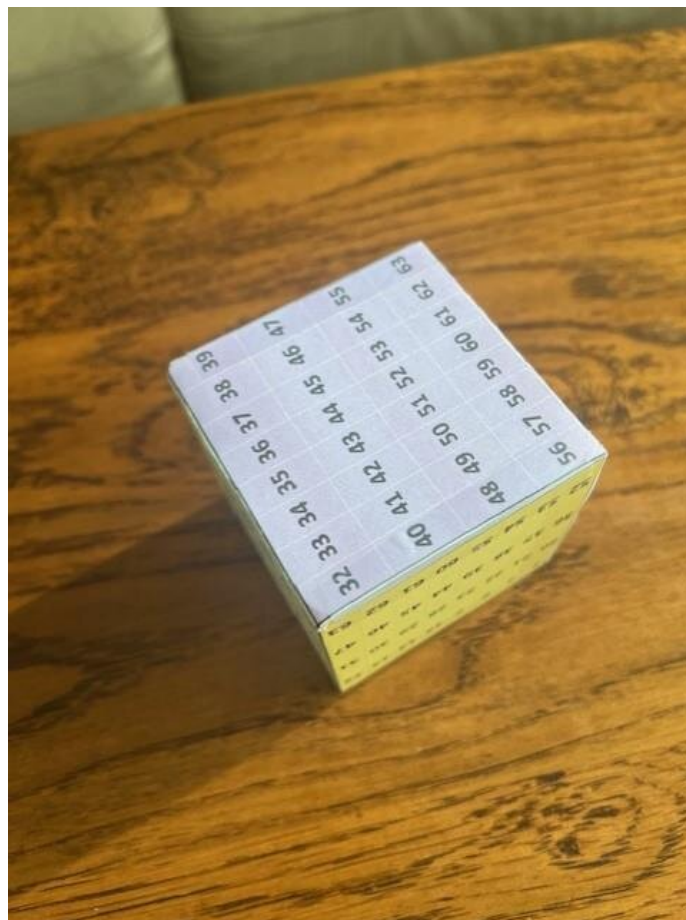


Next, we did the two activities from last time the table with the 5 numbers that add up to 20. This time we saw that we could do horizontal and well as vertical series and numbers at the edges also. They really liked this, and we did several tries each.

After that we reviewed the cube root technique and we each had to do several cube roots. They also liked this very much. They had to write down the numbers still and consult the lookup table but are getting close to doing it without pencil and paper.

Finally, we did the Magic Color Cube that I have created. First, I had them think of a number between 1 and 63 and look at the cube and tell me the colors and I would tell them what number they had selected. Next, I told them that they could do this in a minute and Adam said no way. I showed them the key and that for each color that is mentioned all they had to do was to add up the top right number and the sum was the number that the other person had selected. We had done this before with cards, but the cube is more interesting with it's colors. Then they did several tries each. I have a version of this on Scratch called the [Magical Mind Reader](#) that they can try. Scratch is good programing tool for them to work with. We may do a session playing with it once I get my new computer (this one is on the brink)

The session was good, and I let them run to the car and play with the video game in the car cave. It was fun!



Math Circle 16 Strategy Games etc. 3/06/2024 3:30-4:30

Roosz, Adam, and Dar

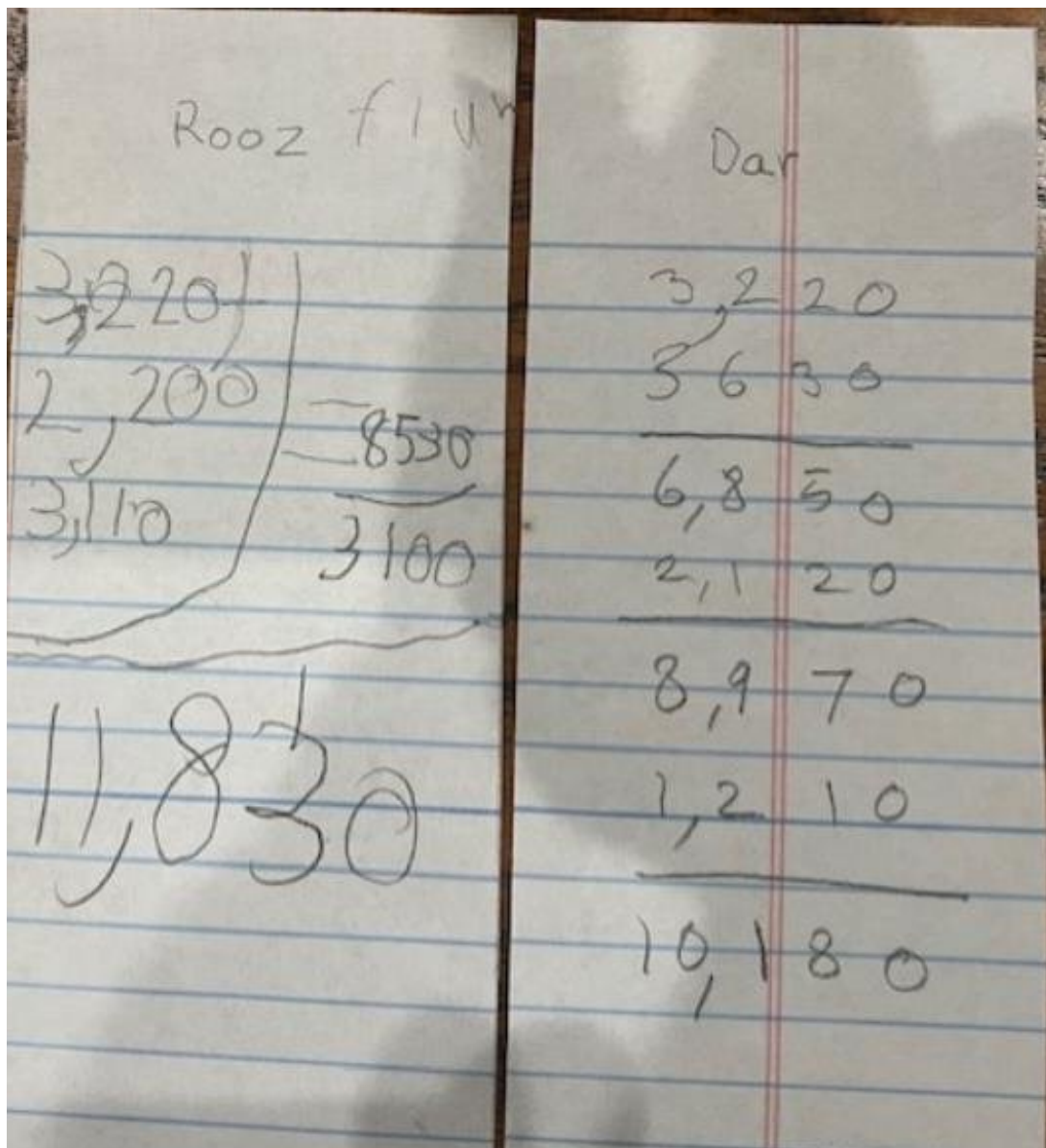
We started by playing Traingo!. I had made the smaller version of the losing positions that you had to put your partner in to win and gave it to them before we started playing. First, I played with them one by one. They both started the right move and then consulted their cards and would take time to make their second and subsequent moves. When I went first, I did not use the winning opening and also, I would make mistakes so they could search and find a position to beat me, and they did most of the time. And they both really liked beating Dar!. This is a good strategy game where they must use both spatial and logical reasoning to win. Next, they wanted to play against each other, and they did, and it was fun for both. As usual they were not so concerned about who won even though they both obviously wanted to win.



[illegible]

Decimal	128	64	32	16	8	4	2	1
17	0	0	0	16	0	0	0	1
29	0	0	0	16	8	0	0	1
31	0	0	0	16	8	4	2	1
107	0	64	32	0	8	0	2	1
111	0	64	0	16	8	0	2	1
127	0	64	32	16	8	4	2	1
135	0	64	32	0	0	0	1	1

Finally, we started playing our main game "Super Match" with Supe Cards. I am attaching the instructions for this game and several other games that can be played with these cards. This is a very good card game where you get tricks by matching cards that add up to 10, 100 or 1,000. I gave Adam a deck to play with at home and recommend it highly in particular keeping track of the score that can help them add numbers in thousands. They both need practice there. For now, I let them do it any way they wanted to but did show them how I was doing it. We played several hands and they got used to dealing the cards also. I had played this game with Rooz before, and he always liked it. Below I do not have Adam's scores as I forget to take a picture of it before I let him take it home.

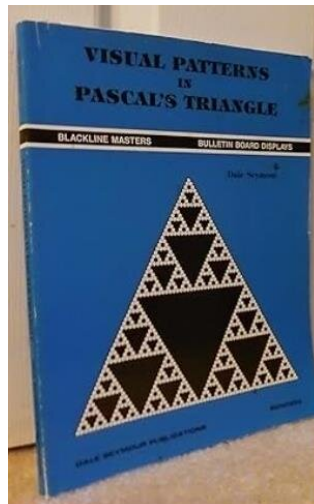


It was a good session and lasted about an hour and we could have gone on as they both liked Super Match and were not so concerned about who won or who last. In general, they both get along well in the games we play and are not super competitive that I think is a very good thing.

Math Circle 17-18 Pascal's Triangle 3/13 and 3/20 2024 3:30-4:30

Roos, Adam, and Dar

In these sessions I used the book "[Visual Patterns in pascal's Triangle](#)" by Dale Seymore that is one of my favorites.



They started by completing Pascal's Triangle and they figured out that any number is the sum of the two numbers above it except the two outside diagonals that are equal to 1. I had them stop around the 10th row and they completed the table at home. Adam completed it in the second session. Next, we talked about finding numbers in the Pascal's Triangle that are divisible by 2 and by 3

PASCAL'S TRIANGLE

Use the pattern to fill in missing numbers in Pascal's Triangle.



I Gave them copies of the completed table and they colored the numbers that were divisible by 2 and they saw the pattern we discussed how this pattern may continue.

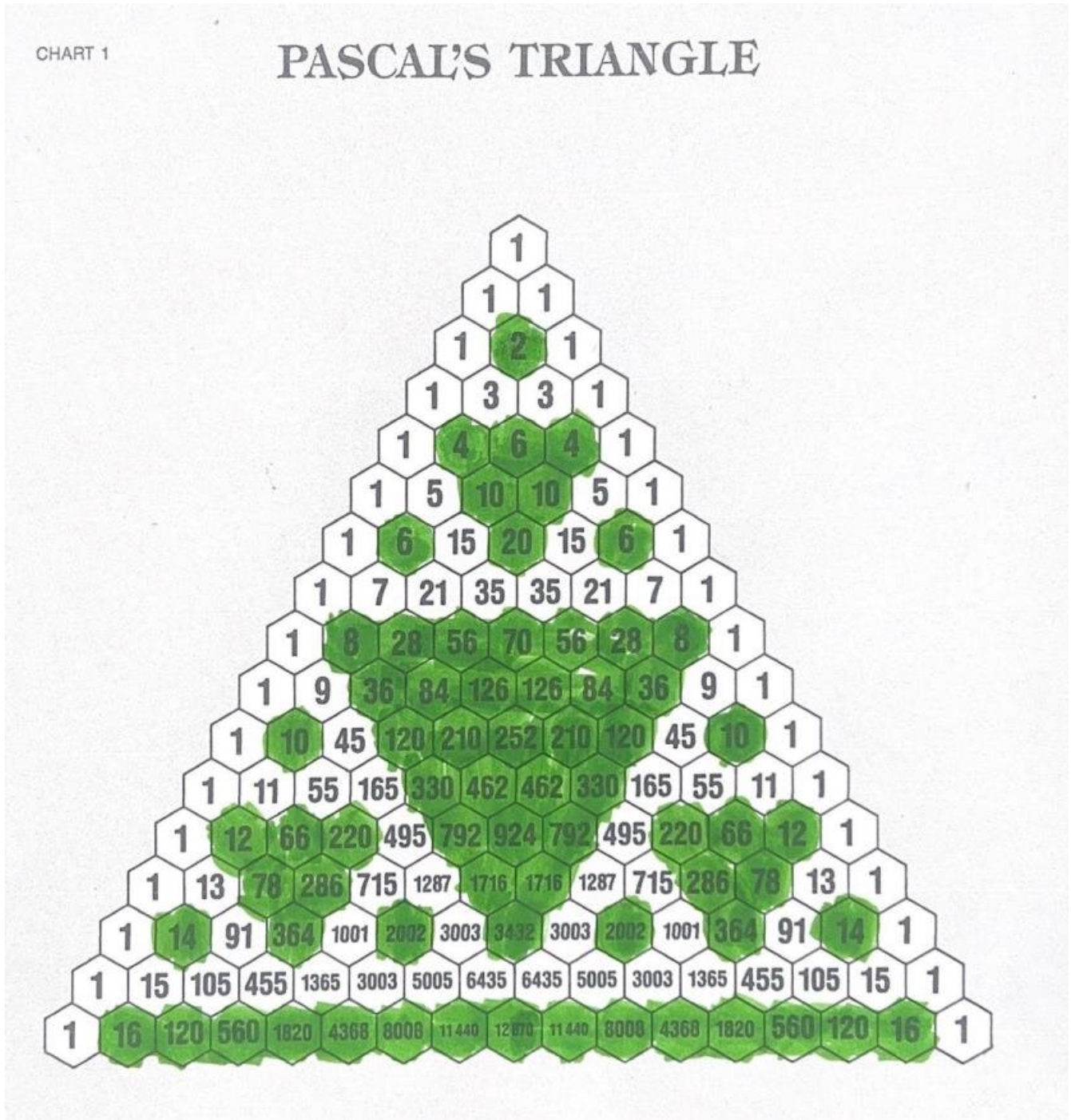
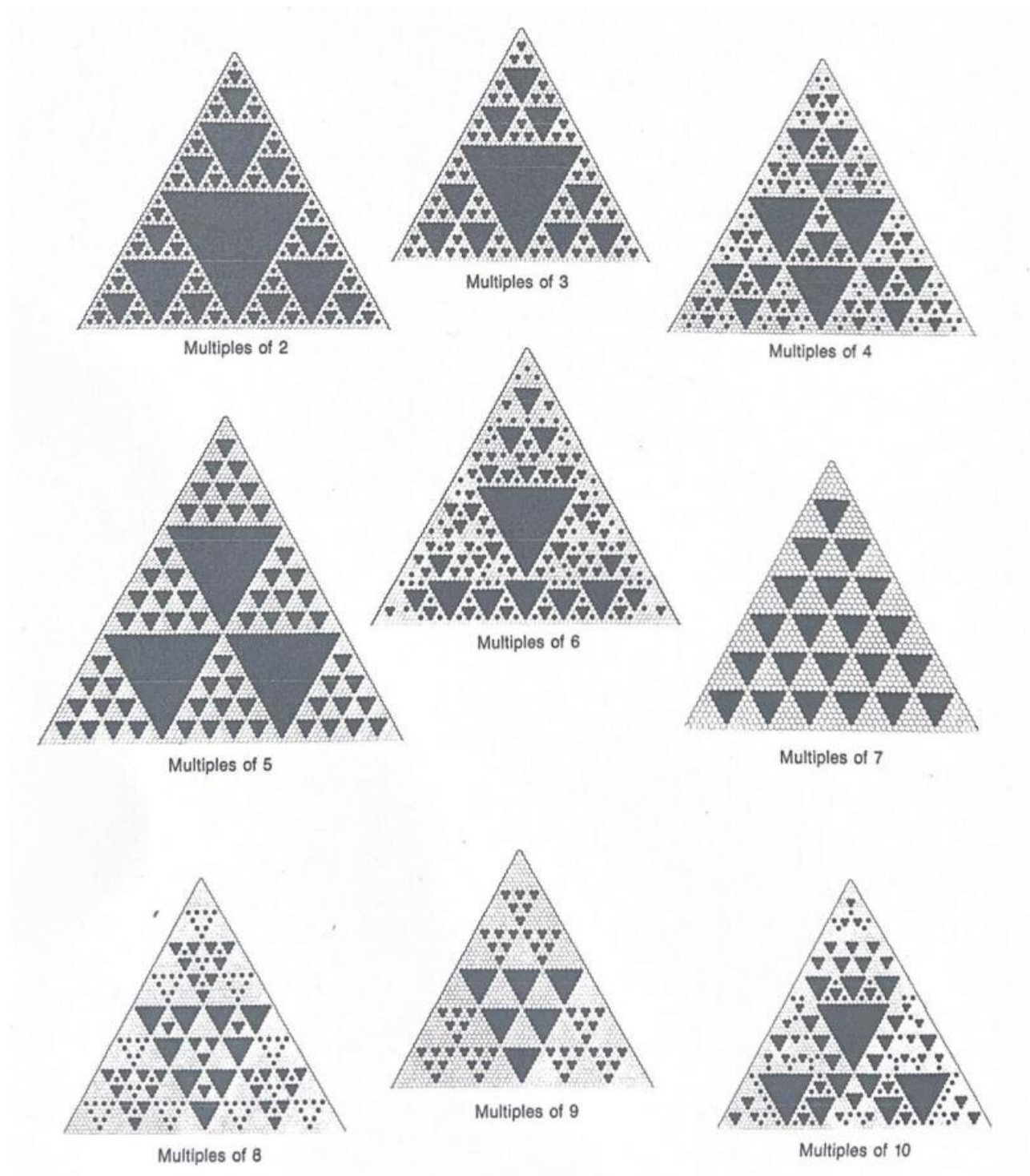


CHART 1

Next, they looked at the patterns that will emerge for other primes and non-primes less than 10 shown below and we briefly talked about these patterns.



In the second session they initially colored the numbers divisible by 7 shown on the first page of this note. We discussed the rule for divisibility by 7 (a number ab where b is the 1s digit is divisible by 7 if $a-2b$ is divisible by 7 for example 231 is divisible by 7 since $23-2\times1=21$ is divisible by 7) I still had to help them some, but they could see the pattern that was emerging. Next, we talked about the Figurate numbers: Triangular, Square, Pentagonal, and Hexagonal numbers and I helped them see that if you calculated the differences they could continue each series easily.

They also figured out that:

The square numbers are the sum of consecutive Triangular numbers.

The Pentagonal numbers are twice the previous Triangular number and the Triangular number at the same level. For example: $5=2\times1+3$ and $12=2\times3+6$ and $22=2\times6+10$ etc.

Hexagonal numbers are the same as the Triangular numbers when we skip one element each time.

INTRODUCTION TO FIGURATE NUMBERS IN PASCAL'S TRIANGLE

The ancient Greeks were very interested in numbers that are connected by geometric forms. *Figurate numbers* (also known as polygonal numbers) are those numbers that can be arranged in a polygonal array.

Triangular numbers can be represented by dots in a triangular array.

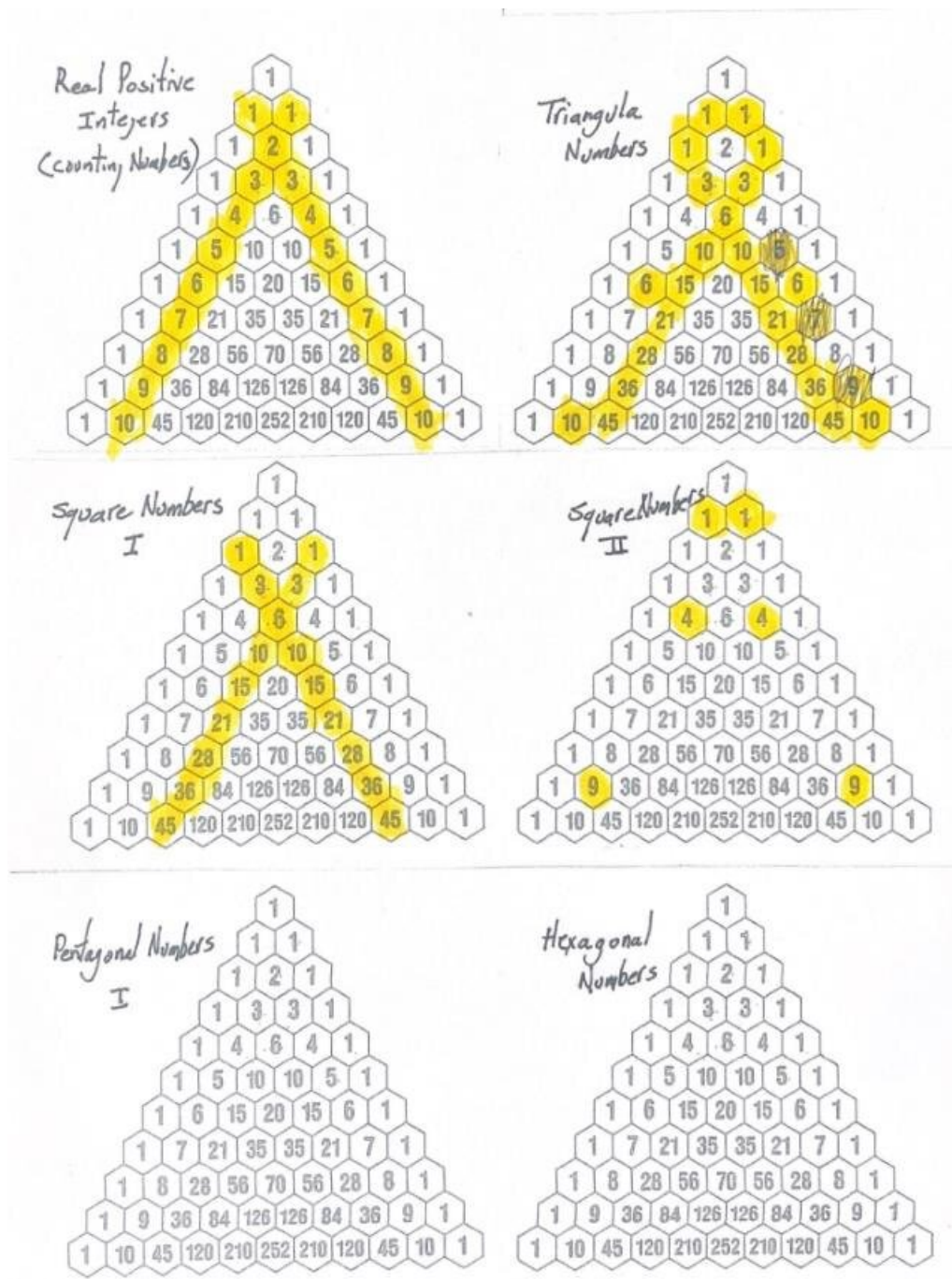
Square numbers can be arranged in a square array and are of the form n^2 where n equals the number of dots on a side.

Pentagonal numbers can be arranged in a pentagonal array.

Hexagonal numbers can be shown as dots in a hexagonal array.

1. (a) What is the relationship between triangular numbers and square numbers?
 (b) Triangular and pentagonal numbers?
 (c) Triangular and hexagonal numbers?

Next they found and colored the real positive integers and Triangular and Square numbers of the sheet that I had prepared for them.



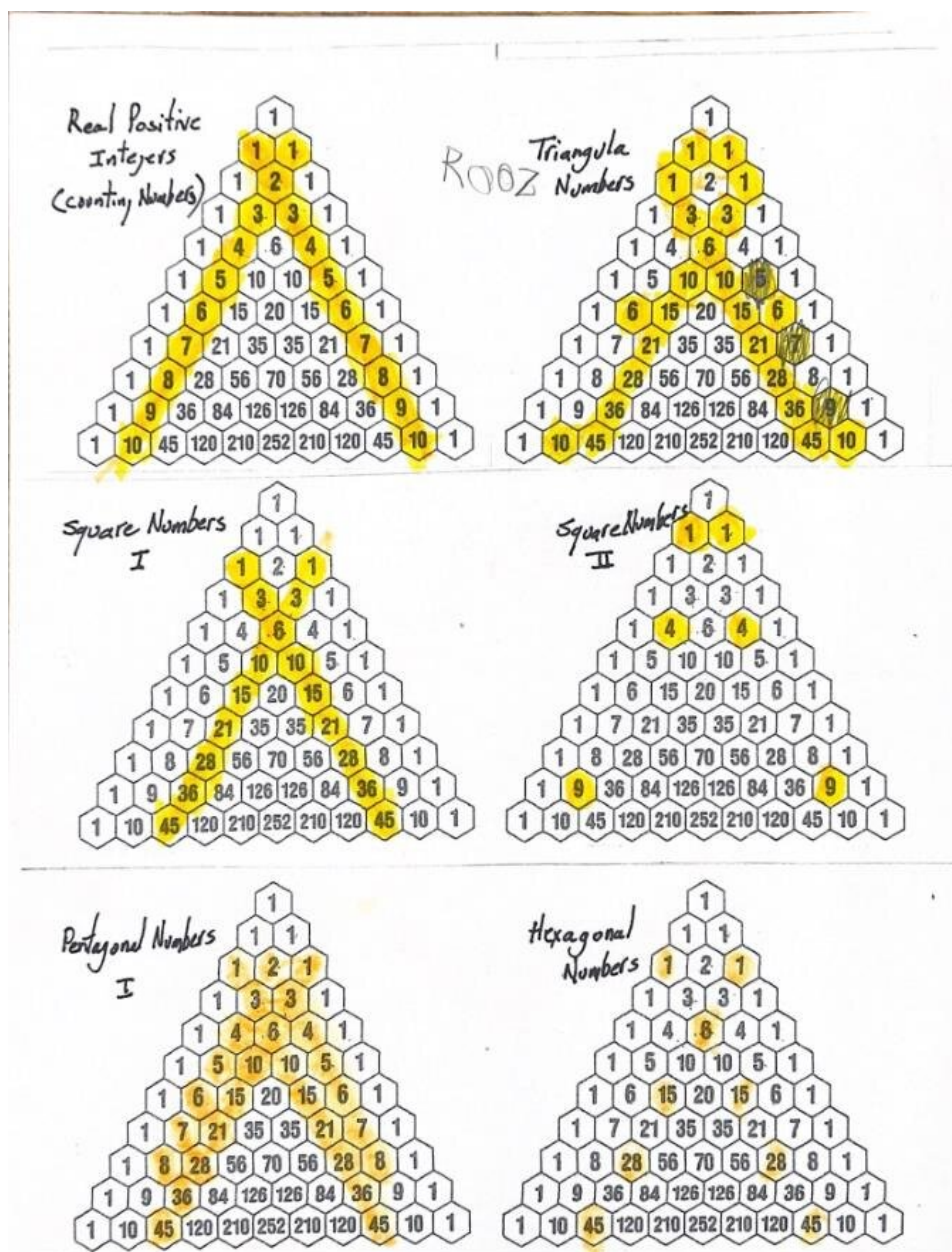
These were good sessions and helped them see that math is not just about numbers and arithmetic but has to do with patterns and symmetry. It is also good for them to see such a simple pattern used for Pascal's Triangle may give such interesting divisibility patterns.

Math Circle 19 Pascal's Triangle 4/3/2024 3:30-4:30

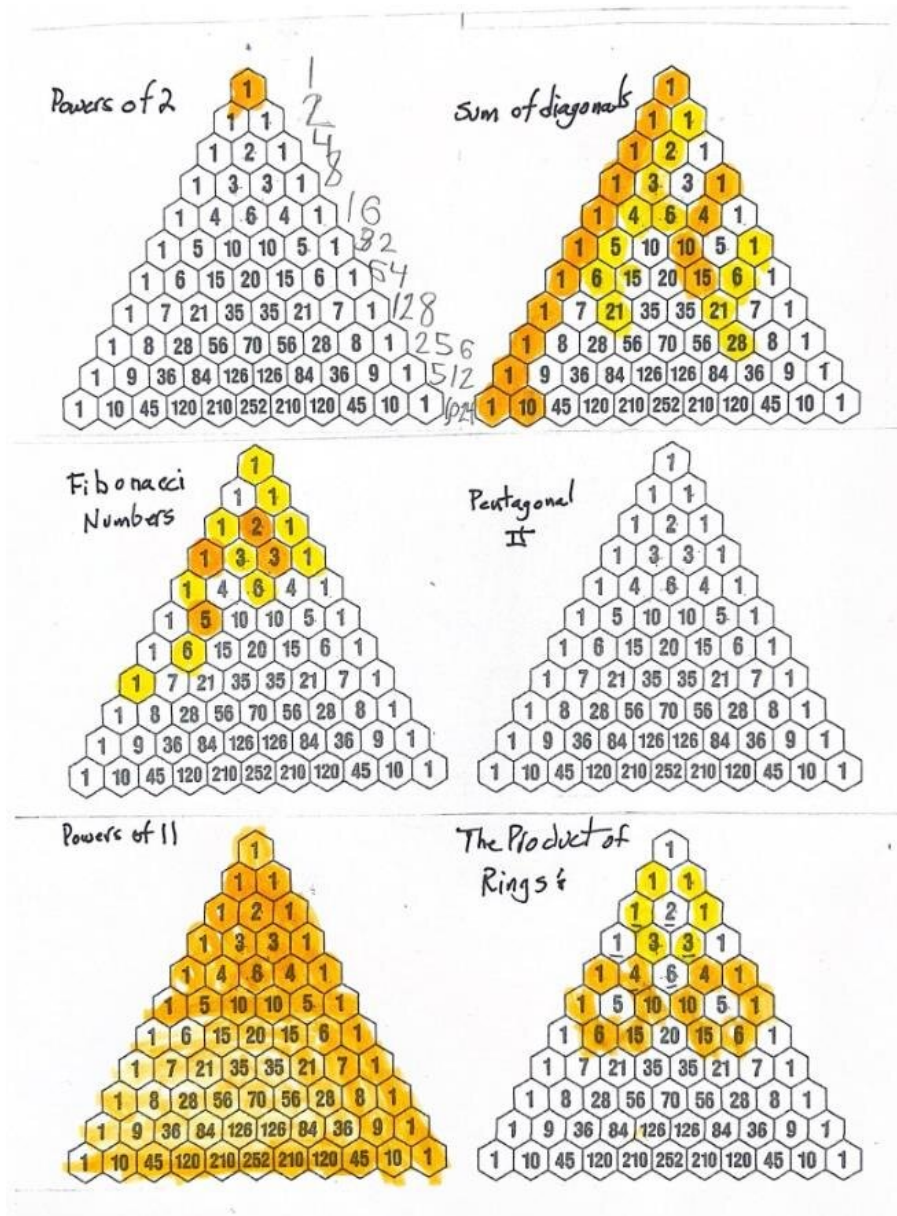
Roos, Adam, and Dar

In this session we continued with activities from "[Visual Patterns in pascal's Triangle](#)" by Dale Seymore.

We reviewed Pascal's Triangle and the patterns that Roos and Adam had found in it in the last two sessions (Real Positive Integers, and Triangular and Square Numbers). We also reviewed what Triangular, Pentagonal and Hexagonal numbers were again. They continued by finding patterns for Pentagonal and Hexagonal Numbers. We also discussed again the relation of pentagonal and hexagonal numbers to the Triangular Numbers that they had discovered last time.



Next, they discovered that the sum of the rows in the Pascal's Triangle gave the powers of 2 and wrote these numbers next to each row. They then found that dog legs starting from any of the sides with number 1 would sum up to the last number. For example, below $1+4+10$ adds up to 15. They did a few of these dog legs and I think they liked these patterns the best. We also discussed how they can find Fibonacci numbers and products of Rings that always gave square numbers. These were more difficult for them to find and I had to help them. Finally, they saw that powers of 11 could be found in the rows of the Pascal's Triangle. So $11^1=11$ in the second row and $11^2=121$ in the third row etc. I also told them that when the entry in the Pascal's Triangle were larger than 9 there was a carrying to be done and should them using the calculator that $11^5=161051$ that is shown as 1 5 10 10 5 1



This was getting a bit too much, so I stopped, and we played a round of Super match with the special paper I had prepared for them.

The game went well, and the table helped them to see the place value better and do their additions easily. Rooz and Adam are not very competitive, so the game went smoothly. Cyrus walked in at the end and played the last hand for me (as we did for almost an hour instead of 45 minutes) even though Adam had the highest score and won it created a bit of tension for him. As I discussed with Julia and Aryan, I will make sure that Cyrus comes in after the Circle for Rooz and Adam is finished.

Rooz						Adam					
100,000	10,000	1,000	100	10	1	100,000	10,000	1,000	100	10	1
		2	2	4	0			3	1	2	0
		2	1	1	0			1	2	1	0
		4	3	5	0			4	3	3	0
		1	3	2	0			2	0	2	0
		5	6	7	0			6	3	5	0

This was a good session they seemed a bit more restless than usual but they loved the Super match that is really a good game for them at this stage of their math development.

Math Circle 20 Supers and Babylonian Fraction Game 04-10-2024 3:30-4:15

Rooz, Adam, and Dar

I discussed with Cyrus and Rooz that I wanted to come to their class and asked for their advice. We first read the Story of the first Census. Adam Started then Dar continued and Rooz read the third paragraph. They got their Rice and Pomegranate at this point, so Dar continued reading. Then they completed the work sheet for the first Census. Then I told them what I had in mind: to have the class assemble (or maybe all the multigrade 1-2 students) and we do a census (I told them what a census was and how often it was done in US), Adam made the good point that it can never be exact as so many babies are born every minute! Dar agreed but said this is really an estimate that will help decision making and does not have to be precise and gets adjusted every year. They both said it may work but Adam thought that some first graders will not get it as they could not even count to a hundred.

First Census – A Sumerian Folk Legend

Week 2, Day 1

The First Census-A Sumerian Folk Legend

Aryan had just finished brushing her teeth and had climbed into bed when she heard her father walking down the hallway. Even though Aryan was now in second grade, she still loved having her daddy tell her bedtime stories that she had listened to since she could remember. "Daddy, Daddy tell me the story about our relative, the king. That's one of my favorites." So her father sat down next to her and began telling the story about many, many, many, many, many hundreds, even thousands of years ago, when her great, great, ..., great grandfather, the king, Supra, ruled a far away land called Sumer.

King Supra let out a sigh as he stood on the steps in front of the great temple of Sumer. He looked out at all the people of the town scurrying about to prepare for the spring celebration. This was a very special time of year because it's when everyone would be counted in the census to find out how much the tribe had grown since the spring census the year before. After the official count took place, the people would celebrate by eating, playing games, and dancing.

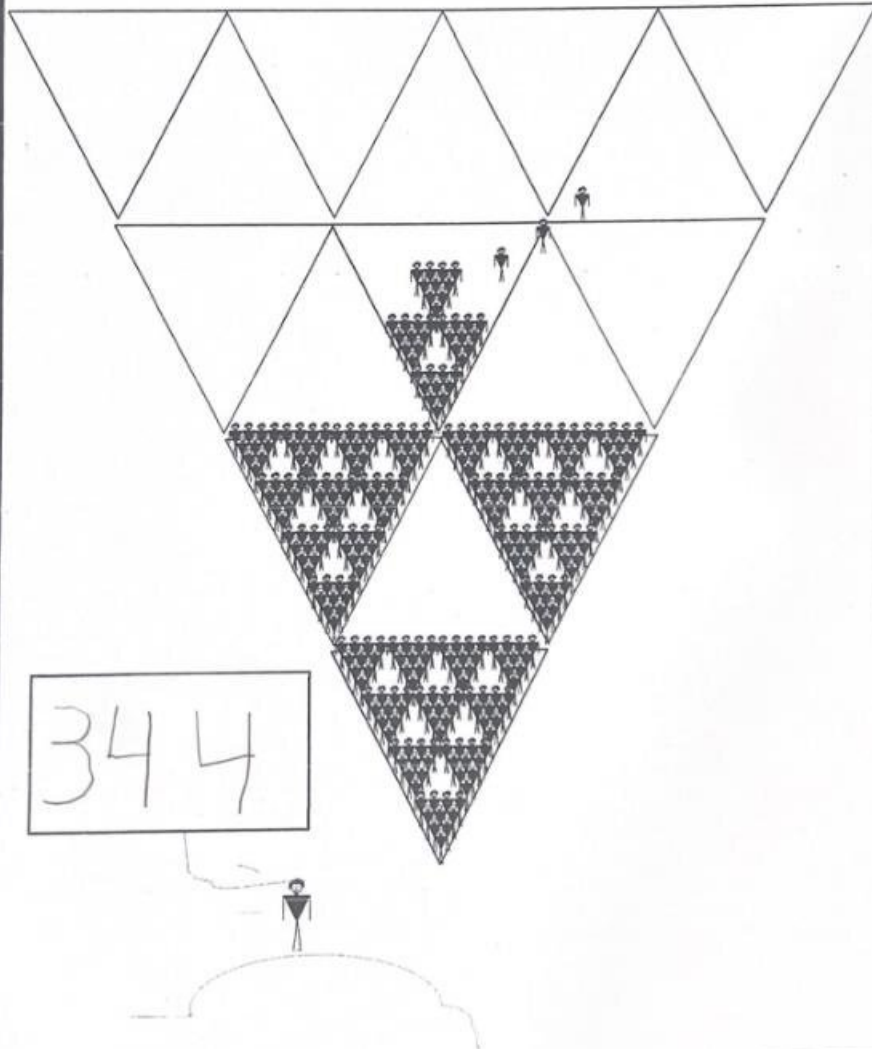
It was almost time to begin and Supra could not wait for the officials to begin their count. On the ground in front of the temple were drawn ten triangles. One huge triangle was made up of the ten smaller triangles. The top row was a single triangle, then below that were two more, then the next row was three triangles, and the bottom row was four triangles. ($1 + 2 + 3 + 4 = 10$)

The people were beginning to gather around the triangles for the count to start. The officials began to lead the people to stand on the smaller triangles. Supra stood on a hill top and watched the people fill in the triangles. The first person to enter a triangle stood at the top of the triangle. Then, the other people were guided to fill in the remaining 9 spaces. On each of the smaller triangles, first a person stood at the top row, then two in the second row, then three in the third row, and four in the fourth row.

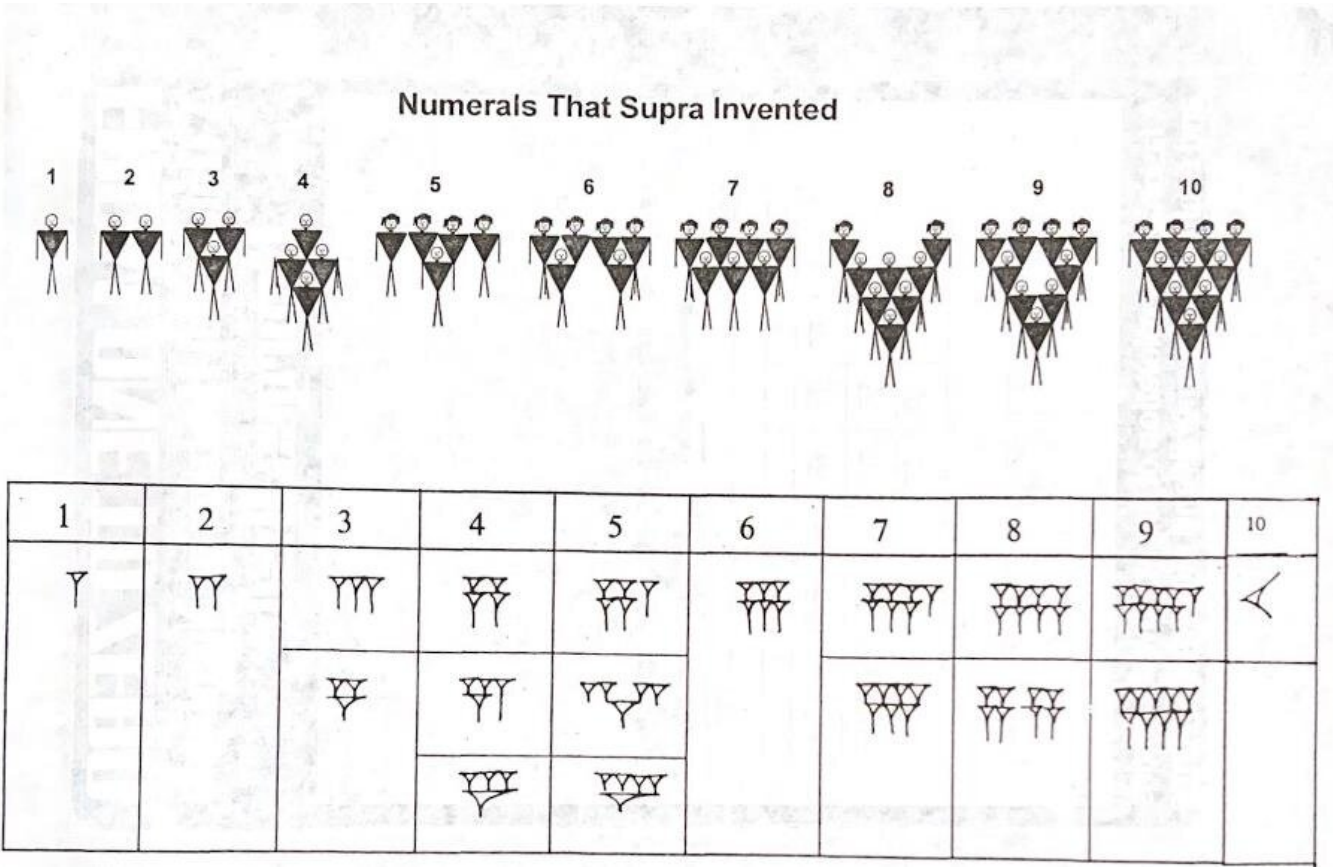
People; men, women, and children, continued to assemble on the smaller triangles filling each with ten people. This made it very easy for the officials to count the number of people, because as each triangle filled the officials knew there were ten people on the triangle. Thus, the officials were able to count all of the people by counting 10's. When the first triangle filled, they counted 10, when the second triangle filled they counted, 20, the third, 30, the fourth, 40, the fifth, 50, the sixth, 60, the seventh, 70, the eighth, 80, the ninth, 90. That meant there were 100 people in this larger triangle. Then, people started to fill up the second larger triangle. (Display the transparency for *Counting the Number of People in the Village* p. 47.)

Ask the students "How many people are in the village?" This was the question that Aryan's father asked. Aryan went to sleep wondering how many people were in the village and how Supra decided how many people were in the triangles without actually counting them one-by-one. She wondered how many more people would be living there in 5 or 10 years.

How many people were in the village?
Write the answer in the space provided.



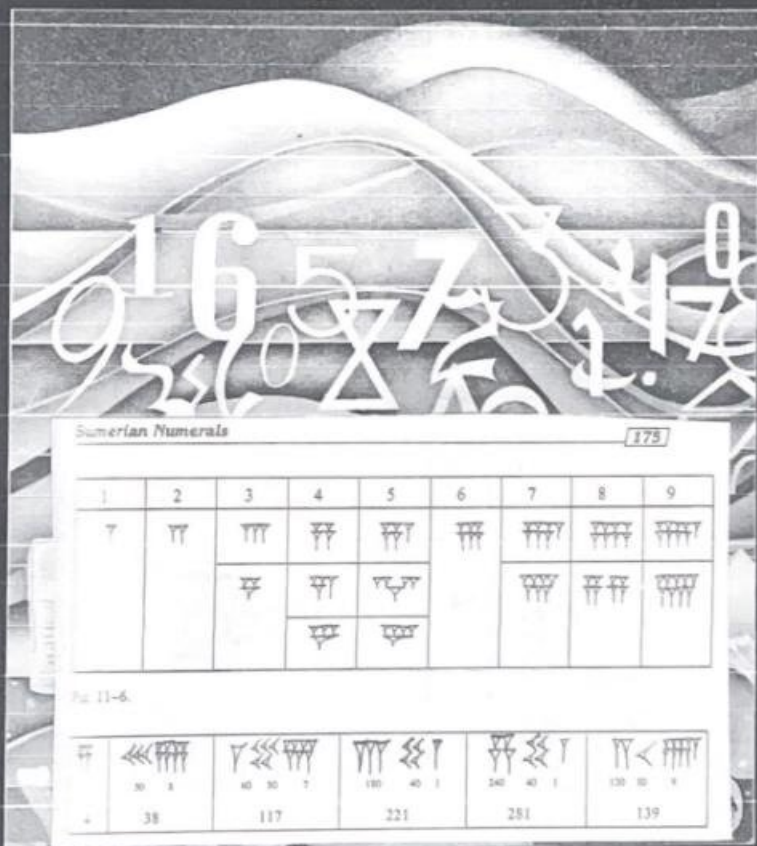
Next, I told them about Cuneiform Numeral and gave them the page that compares these with Super Numerals. I also told them and showed them the book from “One to Zero” that has many interesting and important stories and facts about numbers an counting and ends with 9 tables that show the progression of the Indo-Arabic Numeral (that should really be called Indo-Iranian-Arabic Numerals) as I believe that the Iranian scalers taught the Arabs these numerals that was then carried to Europe by them.



FROM ONE TO ZERO

A Universal History of Numbers

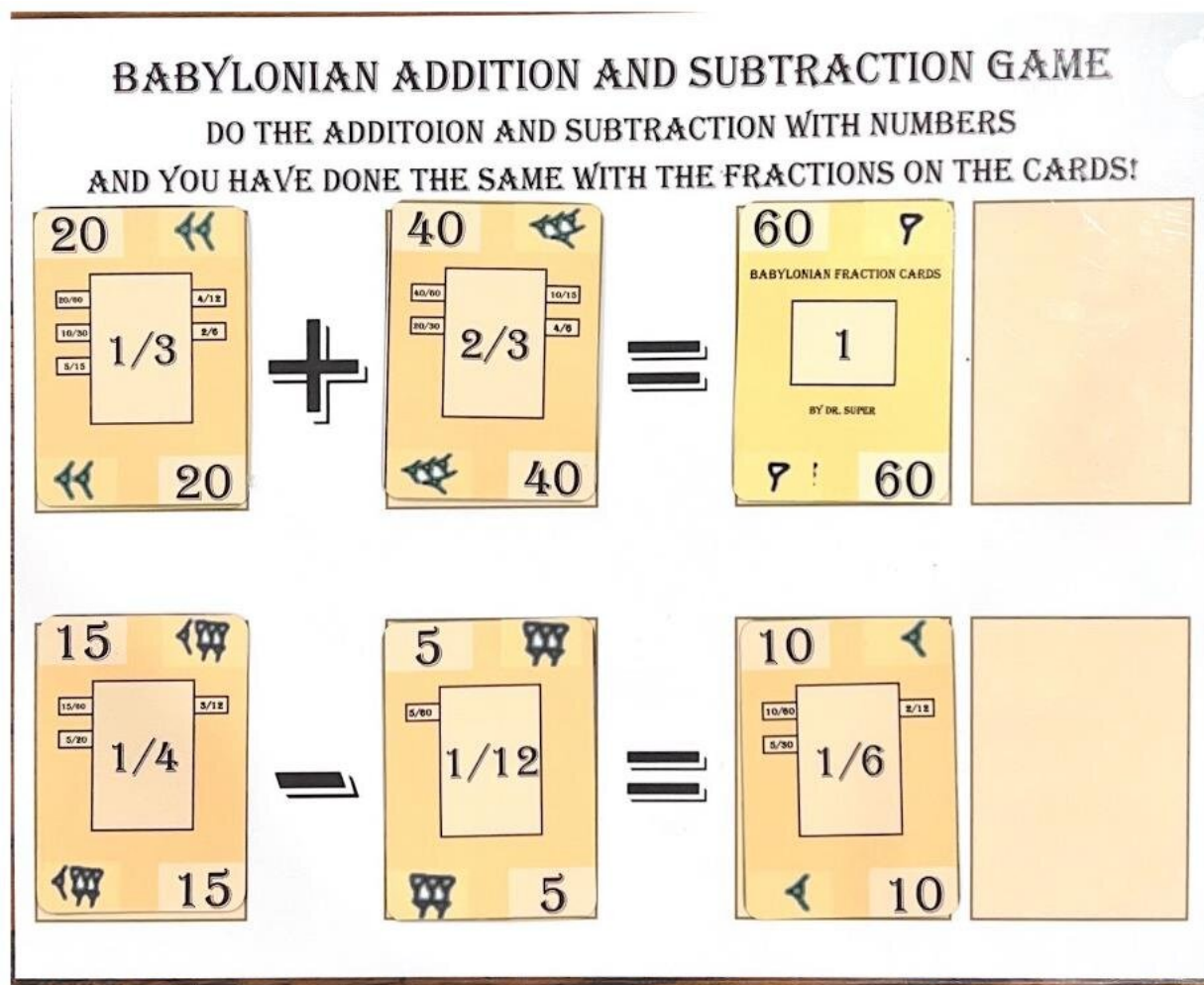
Georges Ifrah



A fascinating exploration of the worldwide origins and evolution of numerals and numerical systems

Next, I told them about the Babylonian Number system which is a base 60 number system (I asked them what the base was for our number system, and they were not sure, but they came up with 10 and we discussed it for a while and the Supers number system that used Super for 10 and had only ten number names that could name any number.

We played the game with Rooz and Adam against me and they won nearly all the tricks! To play you put down 6 cards on the board and if 3 of them can be rearranged as shown to make the right fraction equation after the addition or the subtraction the person wins that trick, and 3 new cards are placed. If no match can be found the team asks for a reshuffle and gets 3 or 6 new cards. They can take Tricks as long as they can make them but get only one reshuffle at each turn and then the other team gets the cards with an automatic reshuffle if they want it. When a trick is made which is easy to see with the numbers the team must explain how the addition or subtraction of the fractions gave the result. This was harder but it is the purpose of the game. I helped them a lot with this. For example for the subtraction problem below they need to reason that $\frac{1}{4}$ can be made to $\frac{3}{12}$ and $\frac{3}{12} - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$



We did finish one full deck and Dar got only 6 cards and they got the rest, so they were happy. This was a good session for me and for them. They got to play their games on the computer afterwards until Julia came. They were much more focused;=>

I have a version of this game on Scratch that would be great for them to play. [Babylonian Fraction Game + -](#)

Math Circle 21 Babylonian Fraction Game – Fraction, Percentage, Decimal, Ration Game 04-17-2024 3:30-4:30

Roosz, Adam, and Dar (Cyrus Joined in the end)

We played another version of the Babylonian Fraction Game. In this game that is similar to the [Babylonian Fraction Game + - in Scratch](#). Each player will roll 2 dice and makes a fraction less than or equal to 1 and finds the corresponding card from the available Fraction cards: $1/6$, $1/5$, $1/4$, $1/3$, $2/5$, $1/2$, $2/3$, $3/4$, $3/5$, $5/6$, 1 and places it on the board to find all the cards that will either add up to that card or when subtracted will give that cards value as a result. In the example on the next page the dice showed 1 and 4 and the fraction was $1/4$ which is equivalent to 15 ($15/60=1/4$). For fractions less than $1/2$ it is best to do the subtraction problem and for fractions larger than $1/2$ the addition. For $1/2$ you can go either way. For each pair of cards, they will get 2 to that power in monopoly money with a limit of 1024 ($2^{10}=1024$) in addition for explaining how the fractions added up and not just the numbers they get another \$50 for each pair.

Adam went first and had $2/3$ (40) and found all the corresponding cards in the addition part. He got 10 pairs but had a little trouble with making the common denominators (that is not trivial). I helped them with these, and he got his \$50 for each one. I think this will be a good way for them to make all the equivalent fractions that have a common denominator. In Scratch all you do is click on the cards and they will change their denominators and once all 3 denominators match the cards go to another pile, and you will get points. Next Roosz went and got 3, 3, equivalent to 1 (60) that is much easier to find the cards as all the pairs will have the matching denominators. He also stopped at 10 pairs. I think they will both benefit from this game for doing their fraction arithmetic.

Next, I showed a trick for comparing two fractions. I think they have both seen this but is good for them to review it. I gave them $7/11$ and $8/13$ that are not easy to compare. They guessed first but the trick is to cross multiply and see which cross product is bigger.

$$7 \times 13 = 91 \text{ but } 11 \times 8 = 88 \text{ so } 7/11 > 8/13.$$



BABYLONIAN ADDITION AND SUBTRACTION GAME
DO THE ADDITION AND SUBTRACTION WITH NUMBERS
AND YOU HAVE DONE THE SAME WITH THE FRACTIONS ON THE CARDS

$\square + \square = \square \square$

$\begin{matrix} 30 \\ 1/2 \\ 30 \end{matrix} - \begin{matrix} 15 \\ 1/4 \\ 15 \end{matrix} = \begin{matrix} 15 \\ 1/4 \\ 15 \end{matrix} \square$

Next, we played another game that I have come up with which helps them match equivalent Fractions, Percentages, Decimals, and Ratios to make a flower. Each player gets 4 cards initially and puts one of them down in turn we keep dealing and putting down cards until a flower is made with all 4 equivalent values then he will receive \$100 for each flower made. The game went well, and we played 2 complete rounds. Rooz was a bit more concerned with who got more money in the Babylonian Fraction Game and this game but fortunately Adam is more mature about the money, so he does not get into the argument of who is winning even though he enjoys receiving the money when he made the flowers.



Cyrus came at 4:15 and as the door was left open he came in and wanted to play a round with us. It is nice to see that they are all behaving better together and even though Rooz and Adam tease each other a little but everyone is focused on the games that we play. (Adam was on his head and Rooz on the table only a couple of times). Cyrus and Adam were nice together and I am happy about that. I think all the talking on both sides has worked well. They got their Pomegranate Rice after they finished the last round.

This was a good session and they all enjoyed it.



Math Circle 22 Area, Perimeter, Fraction patterns with Pattern Blocks 04-24-2024 3:15-4:15

Roosz, Adam, and Dar

I always wanted to come up with an interesting activity with Pattern Blocks. I really enjoyed coming up with this activity that covers Area and Perimeter Patterns as well as Fractions while they are engaged in creating nice symmetric patterns. I really enjoyed doing the activity myself and Roosz and Adam also enjoyed the activity very much to my delight.

First, they started with filling the hexagonal patterns. They rolled dice and Adam won and choose to do the hexagons with sides 2 and 5 and Roosz did 3 and 4. I had figured out some combinations of the shapes that would fill each shape, but they decided on their own and in particular Adam did a beautiful design of his own for the large hexagon. As they were completing each figure they filled in the Area and the Perimeter for these shapes on the work sheet and we discussed it. They quickly figured out that the area was the square of the sides, but they had a bit more difficulty with the perimeters (and seeing that it was 6 times the number of sides), but they got it eventually and started filling it in on their work sheets. Next, they had to find the area and the perimeter for Hexagons with sides 10, 100, 10,000 and n . This presented more of a problem but with some help from Dare y the found them. Writing the area as n^2 (n to power 2) and perimeter as $6n$ was done with my help and they were a bit shaky on it, but we talked about it several times, so it became somewhat clearer for both.



Next, they did the same activity for triangles, rhombi, and trapezoids. Rooz did triangles and rhombi and Adam did the trapezoids (that was more challenging) after some argument for who got to do which shape but they agreed that if Rooz did triangles and rhombi then Adam could do the trapezoids. First, for area they figured out the fractions that each shape represented in comparison with the hexagon that was one unit area then they started to fill in their work sheets. I encouraged them not to simplify the fractions for area and see the patterns that were repeating. After some discussion they both did well and got it. Rooz could not get the 100,000,000 area for the shapes as he does not write that small yet, so I wrote it for him. They did write the formula for all the areas and perimeters, and I think by the end they had a pretty good idea what these formulas represented.

This was an excellent session. Adam was very well behaved, and I stopped Rooz from teasing him a few times when he tried. They were really engaged, and Adam did not go on his head even once! I think if I can make the activities really engaging then their focus will increase also.

Julia, dan and Jerry,

I have included a blank work sheet and I encourage you to have Adam fill it in again from memory and discuss it with him. If he has [Pattern Blocks](#) he can make the shapes also if not get them as they will be good for Benjamin also.



Date _____

Name _____

								
Side Length	Area	Perimeter	Area	Perimeter	Area	Perimeter	Area	Perimeter
1	1	6	$\frac{1}{2}$	5	$\frac{1}{3}$	4	$\frac{1}{6}$	3
2	4	12	$\frac{4}{2}$	10	$\frac{4}{3}$	8	$\frac{4}{6}$	6
3	9	18	$\frac{9}{2}$	15	$\frac{9}{3}$	12	$\frac{9}{6}$	9
4	16	24	$\frac{16}{2}$	20	$\frac{16}{3}$	16	$\frac{16}{6}$	12
5	25	30	$\frac{25}{2}$	25	$\frac{25}{3}$	20	$\frac{25}{6}$	15
10	100	60	$\frac{100}{2}$	50	$\frac{100}{3}$	40	$\frac{100}{6}$	30
100	10,000	6,000	$\frac{10,000}{2}$	5,000	$\frac{10,000}{3}$	4,000	$\frac{10,000}{6}$	3,000
10,000	100,000,000	600,000	$\frac{100,000,000}{2}$	50,000,000	$\frac{100,000,000}{3}$	40,000,000	$\frac{100,000,000}{6}$	30,000,000
n	n^2	$6n$	$\frac{n^2}{2}$	$5n$	$\frac{n^2}{3}$	$4n$	$\frac{n^2}{6}$	$3n$

								
Side Length	Area	Perimeter	Area	Perimeter	Area	Perimeter	Area	Perimeter
1								
2								
3								
4								
5								
10								
100								
10,000								
n								

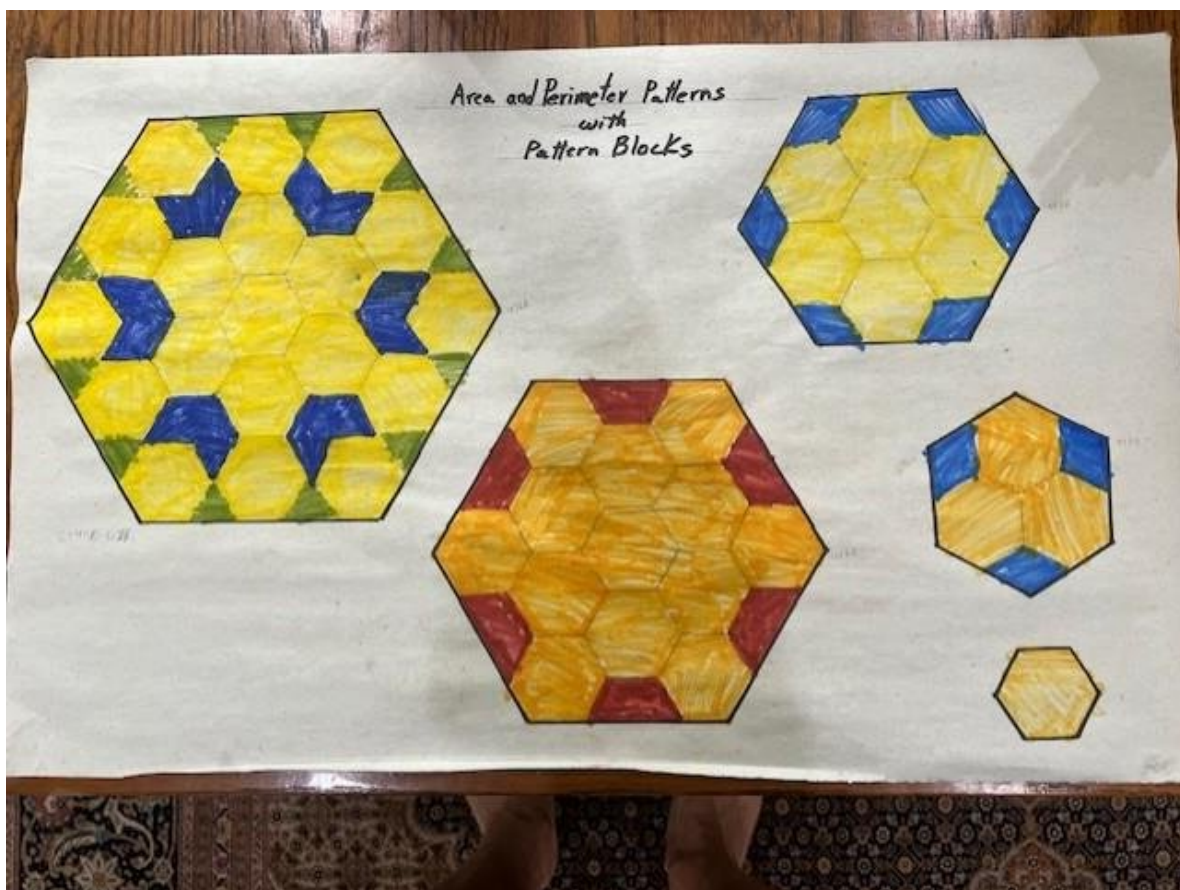
Math Circle 23 Coloring Designs Pattern Blocks – Flower Fraction game 05-01-2024 3:30-4:15

Roosz, Adam, and Dar

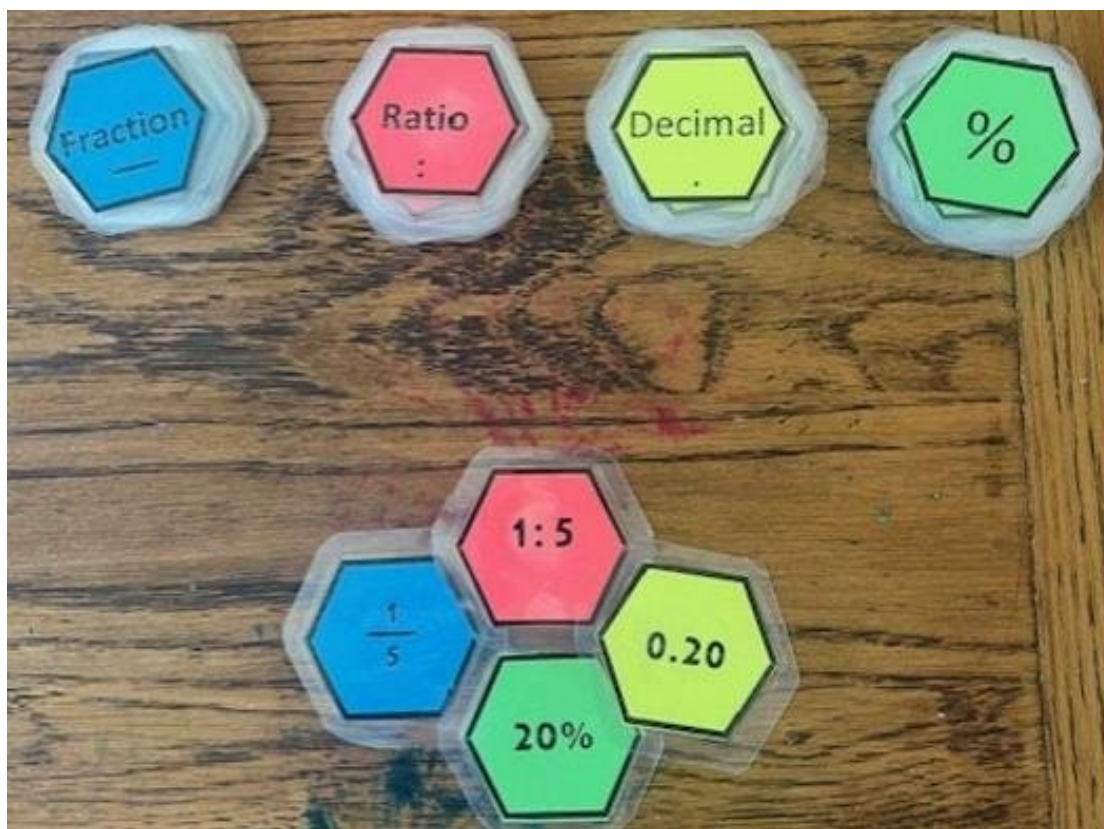
I had them color the designs that they had made last time. This was more of a fun activity, but it was in some ways reminding them of the Areas and Perimeters that they had found last time. They both enjoyed it. Adam started fast but I slowed him down a bit and made sure that he colored each piece in a way that the designs came out. They took their time and put in nearly $\frac{1}{2}$ hour doing it.

Next, we had another round of the Fraction Flower game that relates Fractions, Percentages, Ratios and Decimals. They really like this game. We also started another round when Cyrus got back but by then Rice with Pomegranate arrived and we stopped.

This was a much lighter session, but I was hoping that the painting would remind them of all the hard work they did with Areas and Perimeters. I should have made the connection a lot clearer but they were enjoying their drawings so much that I held back. They were both so calm doing the coloring and Adam helped Roosz to complete his coloring. While they were coloring Ava also came in and wanted to “Do Math” so I made her a Super Triangle and she did a good Job coloring part of it and chatted with the boys some. She was very impressed by their work and so was I.







Date _____ Name _____

								
Side Length	Area	Perimeter	Area	Perimeter	Area	Perimeter	Area	Perimeter
1	1	6	$\frac{1}{2}$	5	$\frac{1}{3}$	4	$\frac{1}{6}$	3
2	4	12	$\frac{4}{2}$	10	$\frac{4}{3}$	8	$\frac{4}{6}$	6
3	9	18	$\frac{9}{2}$	15	$\frac{9}{3}$	12	$\frac{9}{6}$	9
4	16	24	$\frac{16}{2}$	20	$\frac{16}{3}$	16	$\frac{16}{6}$	12
5	25	30	$\frac{25}{2}$	25	$\frac{25}{3}$	20	$\frac{25}{6}$	15
10	100	60	$\frac{100}{2}$	50	$\frac{100}{3}$	40	$\frac{100}{6}$	30
100	10,000	6,000	$\frac{10,000}{2}$	5,000	$\frac{10,000}{3}$	4,000	$\frac{10,000}{6}$	3,000
10,000	100,000,000	60,000,000	$\frac{10,000,000}{2}$	50,000,000	$\frac{10,000,000}{3}$	40,000,000	$\frac{10,000,000}{6}$	30,000,000
n	n^2	6n	$\frac{n^2}{2}$	5n	$\frac{n^2}{3}$	4n	$\frac{n^2}{6}$	3n



Math Circle 24 Preparing for the Class Demo 05-08-2024 3:30-4:15

Rooz, Adam, and Dar

In this session we discussed what we would do for the class on Monday the 13 when I come. I also asked them to explain the Super numbers to their friends in class before and after the session using the big board that I left there last Friday. Also, I asked them to play the Triango! Game and the Super Match as they will leave the material there until the end of the year.

First, we discussed the Magical Mind Reader: Rooz and Adam would take turn and leave the room and we would come up with a number and when they came back, we went over the cards and they guessed the number. In class all 3 of us will go out and only one of us will guess in turn. Some of the additions become harder if there are 4 or 5 of them so I told them they could subtract the numbers that are not included from 63. They liked that say for 62 only the 1 card is not included so they subtract it. In any case I will be there to help. I also told them that they should make a show of it by looking at everybody and pretending that they were reading their minds. This would also give them more time to add the numbers. I think they will enjoy it in class.

Next, we practiced making numbers with powers of 2. And I told them that we would start really easy say with 1, 2, and 4 cards and have the first graders only make a number like 6 or 3. Then we will make it harder step by step and probably not go over 6 cards like the Magical Mind Reader Trick. We did several numbers, and they are both very good at this game even with 6 cards 1, 2, 4, 8, 16 and 32.

Next, I told them that I will tell the story of Super Lady. In class I may have them come up and play with some of the boards and pieces as I am telling the story. I will also try to teach the class counting by Supers in Super language and end up the class say Super, Super, Super, ... for Infinity;=>

Next, they will help make the big triangles with rope with me as I read the story and usher the kids for their first census. After the story we will distribute the handout that has the story and the picture of Supra counting the villagers. We will go around and help the students, in particular the first graders if they have a problem with that. Next, I will talk about how the Super numerals resemble the Sumerian Numerals that were supposedly invented by Supra.

Next, we practiced the Triango! Game and I told them that they would each have 2 tables. On each table they will play 2 games as the kids were going to partner up. I will also play 2 tables. Last year some of the kids started playing against each other and that is OK too. I may do 4 tables and only leave 2 tables to them if I see that it is getting too much for them. I played several games against them and even with the cheat sheet they do not know all the good moves yet but that is OK as some of the kids will win.

Finally, we played a complete round of Super Match with the Super Cards, and I said that if there was time the students would play the Super Match and Rooz and Adam may play on 2 tables and I will supervise the rest. We shall see how it will go.

Next, Cyrus arrived and also the Pomegranate Rice (maybe we should call it Pomrice) and they enjoyed it. It was a good last working session they were very well behaved.

The First Census-A Sumerian Folk Legend

Aryan had just finished brushing her teeth and had climbed into bed when she heard her father walking down the hallway. Even though Aryan was now in second grade, she still loved having her daddy tell her bedtime stories that she had listened to since she could remember. "Daddy, Daddy tell me the story about our relative, the king. That's one of my favorites." So her father sat down next to her and began telling the story about many, many, many, many, many hundreds, even thousands of years ago, when her great, great, ..., great grandfather, the king, Supra, ruled a far away land called Sumer.

King Supra let out a sigh as he stood on the steps in front of the great temple of Sumer. He looked out at all the people of the town scurrying about to prepare for the spring celebration. This was a very special time of year because it's when everyone would be counted in the census to find out how much the tribe had grown since the spring census the year before. After the official count took place, the people would celebrate by eating, playing games, and dancing.

It was almost time to begin and Supra could not wait for the officials to begin their count. On the ground in front of the temple were drawn ten triangles. One huge triangle was made up of the ten smaller triangles. The top row was a single triangle, then below that were two more, then the next row was three triangles, and the bottom row was four triangles. ($1 + 2 + 3 + 4 = 10$)

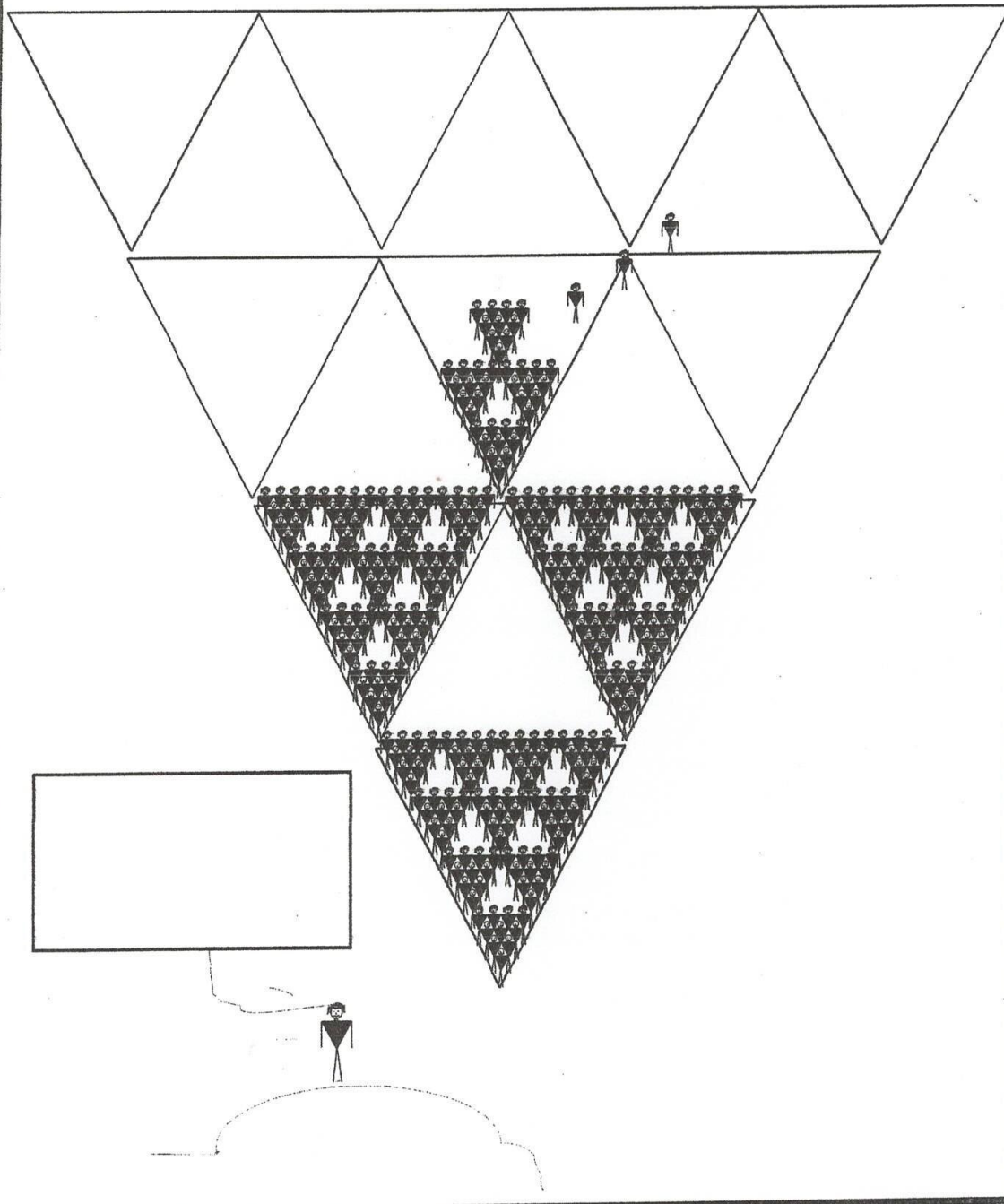
The people were beginning to gather around the triangles for the count to start. The officials began to lead the people to stand on the smaller triangles. Supra stood on a hill top and watched the people fill in the triangles. The first person to enter a triangle stood at the top of the triangle. Then, the other people were guided to fill in the remaining 9 spaces. On each of the smaller triangles, first a person stood at the top row, then two in the second row, then three in the third row, and four in the fourth row.

People; men, women, and children, continued to assemble on the smaller triangles filling each with ten people. This made it very easy for the officials to count the number of people, because as each triangle filled the officials knew there were ten people on the triangle. Thus, the officials were able to count all of the people by counting 10's. When the first triangle filled, they counted 10, when the second triangle filled they counted, 20, the third, 30, the fourth, 40, the fifth, 50, the sixth, 60, the seventh, 70, the eighth, 80, the ninth, 90. That meant there were 100 people in this larger triangle. Then, people started to fill up the second larger triangle. (Display the transparency for *Counting the Number of People in the Village* p. 47.)

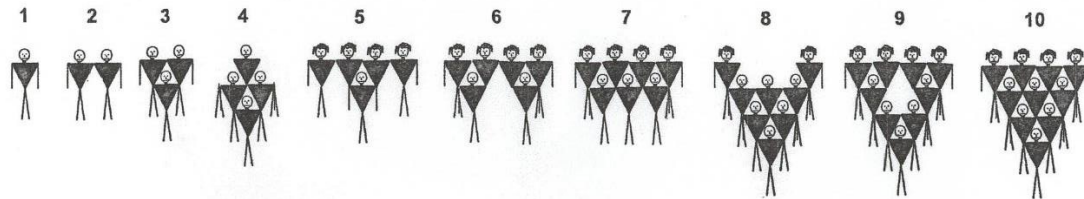
Ask the students "How many people are in the village?" This was the question that Aryan's father asked. Aryan went to sleep wondering how many people were in the village and how Supra decided how many people were in the triangles without actually counting them one-by-one. She wondered how many more people would be living there in 5 or 10 years.

Distribute a copy of *Counting the Number of People in the Village* (p. 47) to each student. Have the students guess what the number is by looking at the full triangles that contain 10 or 100 people. Encourage them to count by 100 and by 10. Discuss how the population might have grown in 5 or 10 years. Have students suggest various amounts.

How many people were in the village?
Write the answer in the space provided.



Numerals That Supra Invented



1	2	3	4	5	6	7	8	9	10
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			𐎶𐎶𐎶	𐎶𐎶𐎶𐎶𐎶					

The Sumerian Cuneiform Numeral 2700 B.C.

Math Circle 23 CM 25 RA End of Year Celebration 5-15-2024 1:50-2:45

Cyrus, Marc, Rooz, Adam, Hedi, Julia, Aryan, Laura, and Dar

This was our final celebration for the end of Math Circles for Cyrus and Marc and Rooz and Adam. I will Airdrop all the pictures and a couple of videos that were taken.

We watched a couple of Fibonacci and Golden Ratio YouTube videos before Adam got here at 2:15. We only saw a few minutes of the second one (Julia and Laura you may want to watch these with Adam and Marc they are excellent and fun)

[The magic of Fibonacci numbers | Arthur Benjamin | TED](#)

[The Golden Ratio: Is It Myth or Math?](#)

Dar gave a little speech and thanked Maman Gol, on behalf of all present, for all the great food she has given the kids to get them ready for the serious but fun math that they do in the Math Circle every week: Shakes, Vegetables, Fruits and most importantly Pomegranate Rice (Pomrice). In addition, he mentioned that the students have been mostly well behaved except for standing on their heads or fidgeting constantly sometimes. No names were mentioned, but a lot of giggles were heard from the guilty parties.

Then Dar awarded the trophies and the certificates, and we took pictures. Mamman Gol gave the kids Pomegranate Rice, and we had the Chocolate Mousse Cake and celebrated.

It was a great year for our Math Circles and all the students did such an incredible job. I am so proud of all of them, and you should be also be very proud of them.









