Math Circle 2024-2025

Rooz and Adam

Dr. Super Math Circles

Table of Contents

Math Circle 1 Polyominoes 1 8/13/2024 1:50-3:00

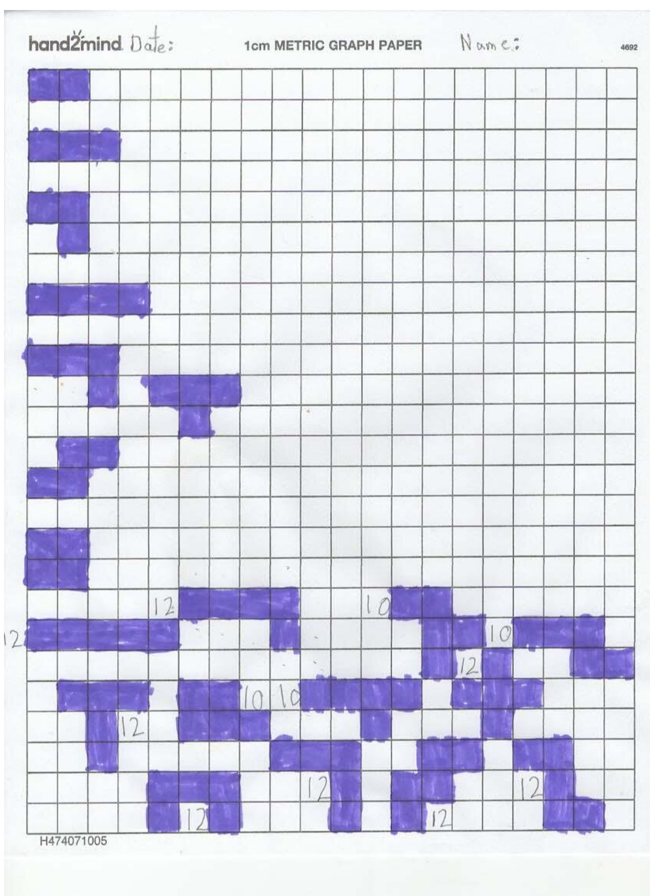
Rooz, Adam, and Dar

We started with an Activity called Polyominoes from the Book "" that you may wish to download. I am going to be using it for Cyrus and Marc and may do a few for Rooz and Adam also. I am attaching the relevant pages. There is a game called that is great for Adam and Benjamin.

First they made dominoes (1 configuration) then triaminoes (2) ,tetraminoes (5) and finally 12 pentaminoes. These are 2, 3, 4, or 5 squares or cubes that are attached to each other on one edge. They had found only 4 tetraminoes but Adam found the fifth one! Rooz made them all with cubes then colored them on the centimeter paper but Adam liked to work faster and just colored most of them. Next, we talked about the fact the Area for all the pentaninoes is 5 but the perimeter is different. They both found that the perimeter was either 10 or 12 and marked it on their paper.

Next, came the real work of extending this problem and finding the shortest and longest perimeter with Polynominoes with areas going to 25. Their first reaction was no way this is too hard. Soon they realized that the longest perimeter was twice the area plus 2 so they did all 25 quickly. The shortest was harder and the logic is not straight forward. I had them find it for 4, 9, 16 and 25 and they saw that this would be 8, 12, 16, and 20! Then I told them that for others the number will repeat. We will have to review the logic here. For example, for:21,222,23, and 24 the shortest perimeter will be 20 and for 17, 18, 19, 20 it ill be 18.

It was a great first session. They have both matured over the summer, nobody went on the table and no one stood on their head! I did have to get them to focus a few times but as soon as they started to fill in the table they both became very focused. We will finish this next time. They also wanted to do the Fraction, Percentage, Decimal an Ratio Flower game that we all also do next time.



Name:

Date:

	Perimeter	
Area	Shortest	Longes
1	4	4
2	6	6
3	8	8
4	8	10
5	10	12
6	10	14
7	12	16
8	12	18
9	12	20
10	14	22
11	16	24
12	16	26
13	18	28
14	16	30
15	16	32
16	16	34
17	18	36
18	18	38
19	20	40
20	20	42
21	20	44
22	20	46
23	20	48
24	20	50
25	20	52

1. What Patterns do you notice?

2. Predict the longest perimetes for polyominoe having the folling Areas:

Area	Longest Perimeter
36	
40	
100	
99	
101	
1,000	

3. Predict the shortest possible peremeters for Polyominoes have thes Are

Area	shortest perimeter
36	
100	
40	
99	
121	
10,000	

Math Circle 2 Polyominoes 2 Fraction Flower Game 8/21/2024 3:00-3:45

Rooz, Adam, and Dar

We continued with Polyominoes Activity from the Book "".

First we did two rounds of the Fraction, Percentage, Ratio, Decimal Flower game that they had requested. They really like this game, and they know all the flowers well now. They are happy to make the flowers, but they are not that competitive about it which make the game more interesting and about matching the 4 values rather than just making the most flowers.

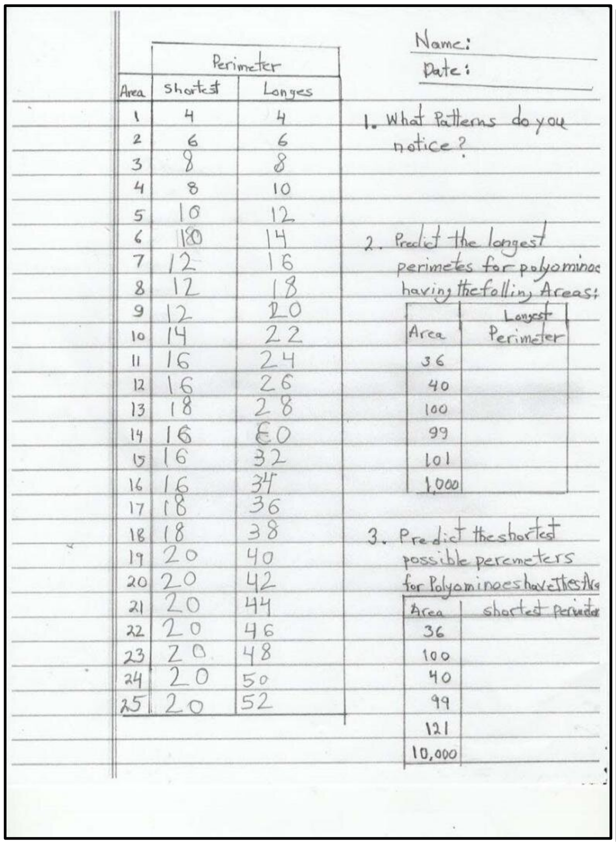
I had them complete the table that they had made for the smallest and longest perimeter for polyominoes with Areas from 1 to 25 that they had done last time. The worked with Rooz' table. First, I went over some of the errors they had in their table for the smallest Perimeter and a technique working back from the nearest square (or rectangle) with a given area. For example, 15, 14, and 13 will have the value 16 for their perimeter equal to the perimeter for 16 that is 4 by 4. The same way 11 and 10 will have the smallest perimeter 14 the same as for 12.

Then they completed together the tables that required them to use similar reasoning to what they had used in their table to predict the value of the longest and smallest perimeters for larger values of the Areas. They liked this part and Rooz wrote in the values. After they would agree on the right value.

Next, I told them that they were going to chart these values. They both objected that they did not want to chart! But we went ahead any ways for the Longest Perimeter and I had them come up with the formula Longest Perimeter = 2xArea +2 (LP=2A+2) we did discuss this some, but they were more comfortable finding the value than using the formula. They went down on the table and saw that this works for all the Areas for the longest perimeter.

Then they did chart. They both put the points with some help from me. Adam needed a bit more help but then they both saw that a line would go through all these points and drew it. I may have them complete the chart for the Shortest Perimeter also next time. (Please bring Adam's table so he can complete it also.) This is not as much fun as some for the other activities, but it is good for them. I will do something more interesting with them also next time.

This was a good session. Adam wanted recess but as it was a short session they got no recess.



Name:
Date:

Area	Perimeter Shortest	Perimeter Longes
1	4	4
2	6	6
3	8	8
4	8	10
5	10	12
6	10	14
7	12	16
8	12	18
9	12	20
10	14	22
11	16	24
12	16	26
13	18	28
14	16	30
15	16	32
16	16	34
17	18	36
18	18	38
19	20	40
20	20	42
21	20	44
22	20	46
23	20	48
24	20	50
25	20	52

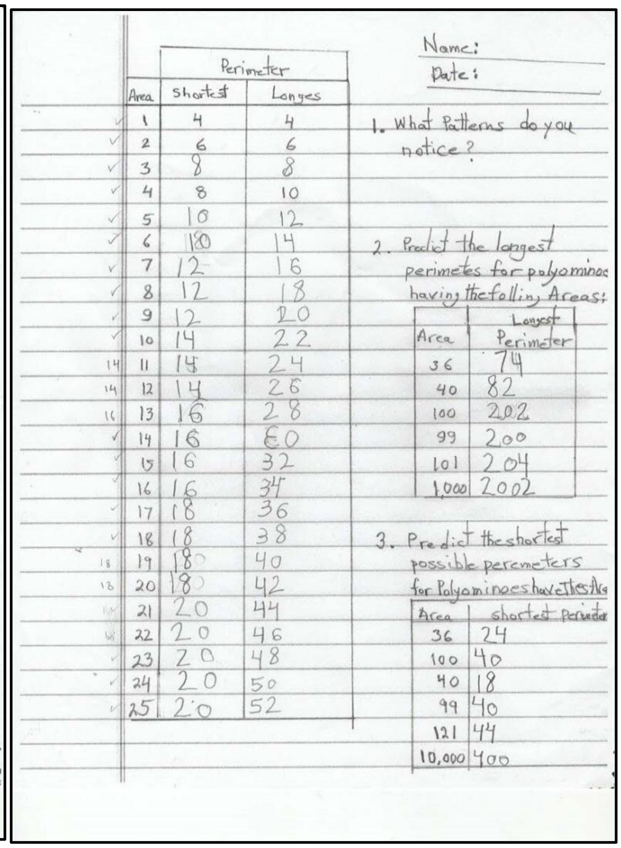
1. What Patterns do you notice?

2. Predict the longest perimetes for polyominoes having the folling Areas:

Area	Longest Perimeter
36	
40	
100	
99	
101	
1,000	

3. Predict the shortest possible peremeters for Polyominoes have these Area

Area	shortest perimeter
36	
100	
40	
99	
121	
10,000	



Name:
Date:

Area	Perimeter Shortest	Perimeter Longes
1	4	4
2	6	6
3	8	8
4	8	10
5	10	12
6	10	14
7	12	16
8	12	18
9	12	20
10	14	22
11	14	24
12	14	26
13	16	28
14	16	30
15	16	32
16	16	34
17	18	36
18	18	38
19	18	40
20	18	42
21	20	44
22	20	46
23	20	48
24	20	50
25	20	52

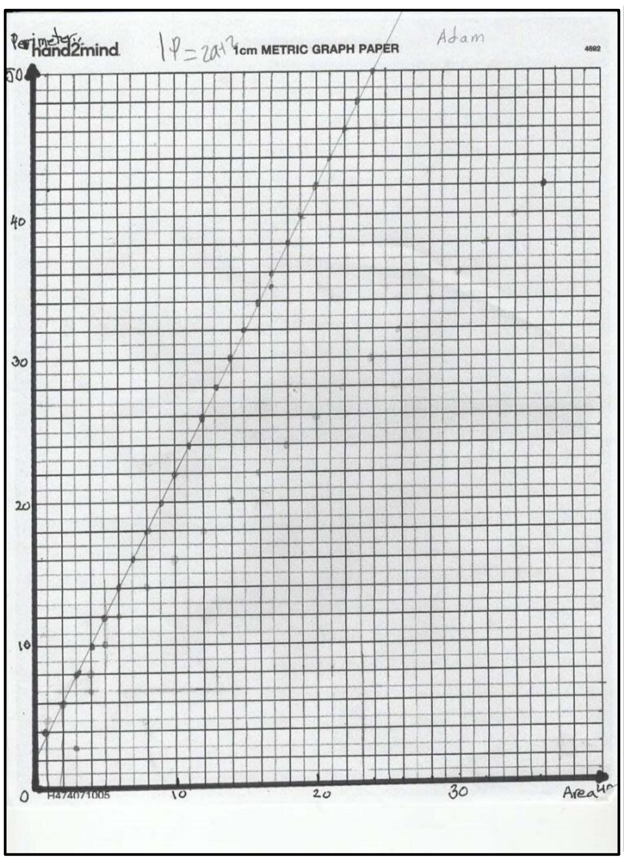
1. What Patterns do you notice?

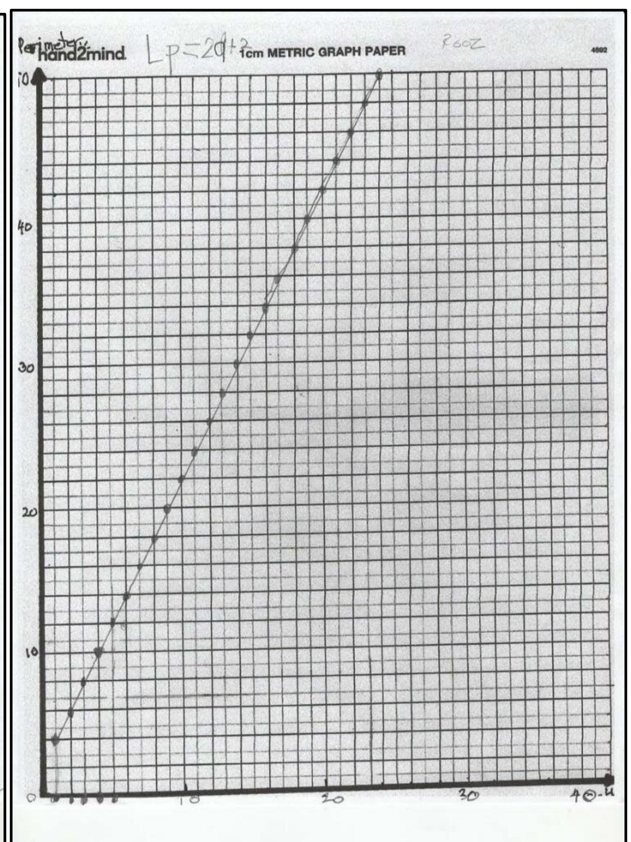
2. Predict the longest perimetes for polyominoes having the folling Areas:

Area	Longest Perimeter
36	74
40	82
100	202
99	200
101	204
1,000	2002

3. Predict the shortest possible peremeters for Polyominoes have these Area

Area	shortest perimeter
36	24
100	40
40	18
99	40
121	44
10,000	400





Math Circle 3 Polyominoes 3 Fraction Flower Game 8/28/2024 3:00-3:40

Rooz, Adam, and Dar

We did one more activity with Polyominoes Activity from the Book “” now making Hexominoes.

They absolutely wanted to play the Fraction, Percentage, Decimal and Ration game. So, we played three rounds of this game this time. As usual they enjoyed the game and by the end, they knew all the flowers and they were not very competitive about it which make the game more interesting and about matching the 4 values rather than winning more flowers. I think they are both getting to understand fractions better and how they relate to percentages, ratios or decimals.

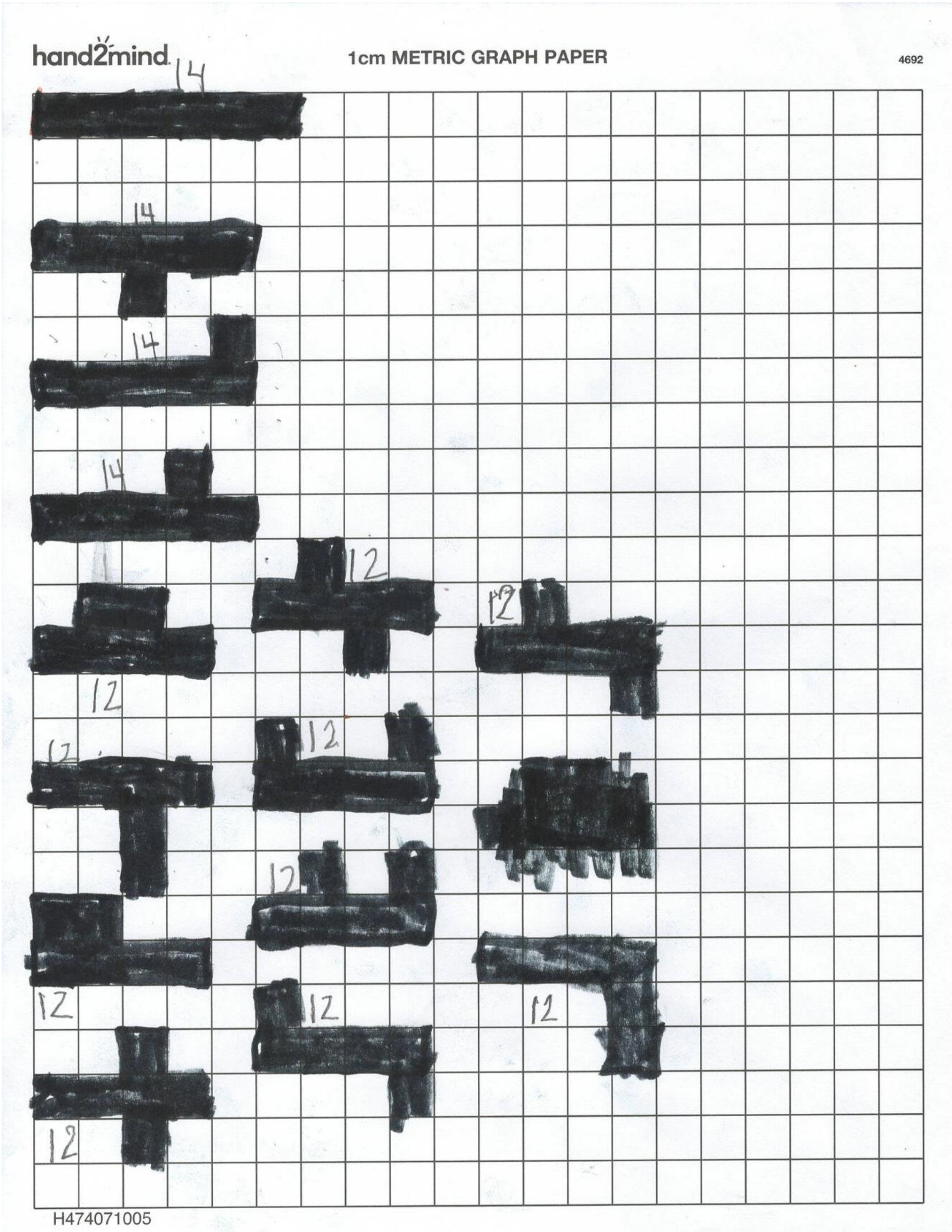
I wanted them to complete the chart from the previous time, but they were a bit agitated, so I gave them a break and challenged them to find all the Hexominoes and their Perimeters instead.

Adam was to find all the Hexominoes that did not have more than 3 in a row horizontally vertically and Rooz was to find all the rest that had to have 6 or 5 or 4 in a row. We had to discuss exactly what the assignments meant.

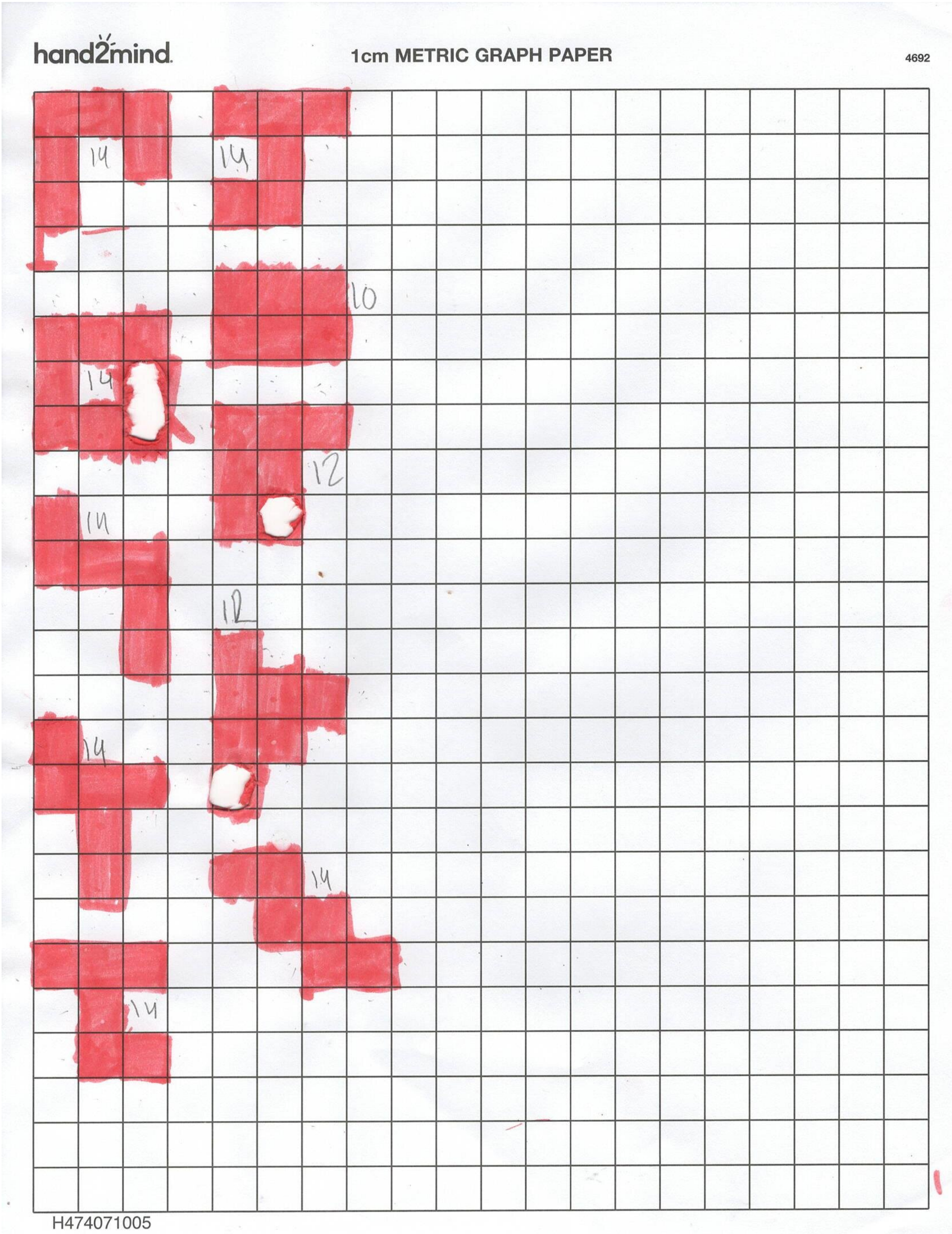
After the discussion they got started quickly. Adam did some with 7 rather than six squares (he usually rushes to get the work done and I try to slow him down a bit and to think more) but then he promptly made holes in the extra places! Rooz goes about the problems a bit more methodically and slower, but they talk to each other and help each other and that is good for both. Then they found all the perimeters and the longest Perimeter that was 14 (From Rooz) and shortest perimeter that was 10 (from Adam). And we discussed again why that is and that they had seen this in the previous week when they were making their table for the longest and shortest perimeters for all Polyominoes with Area from 1 to 25.

This was a good session but a little shorter than our usual 45 minutes. They were both a bit agitated but behaved. No standing on the head and no sitting on the table and they were listening to Dar better.

Rooz


hand2mind
1cm METRIC GRAPH PAPER
4692
14
14
14
14
12
12
12
12
12
12
12
12
12
H474071005

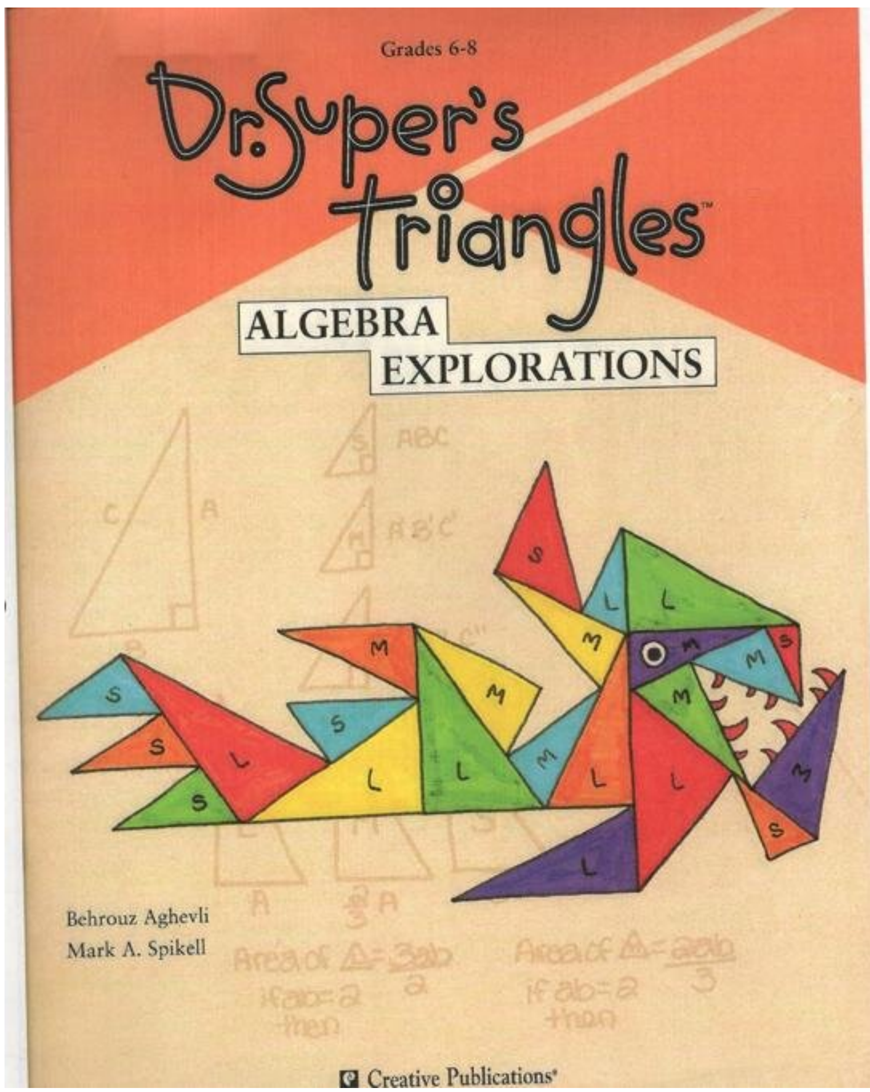
Adam


hand2mind
1cm METRIC GRAPH PAPER
4692
14
14
10
14
12
14
12
14
14
14
H474071005

Math Circle RA 4 Fitz Da Pieces Geometry and Algebra 9-11-20243:00-3:50

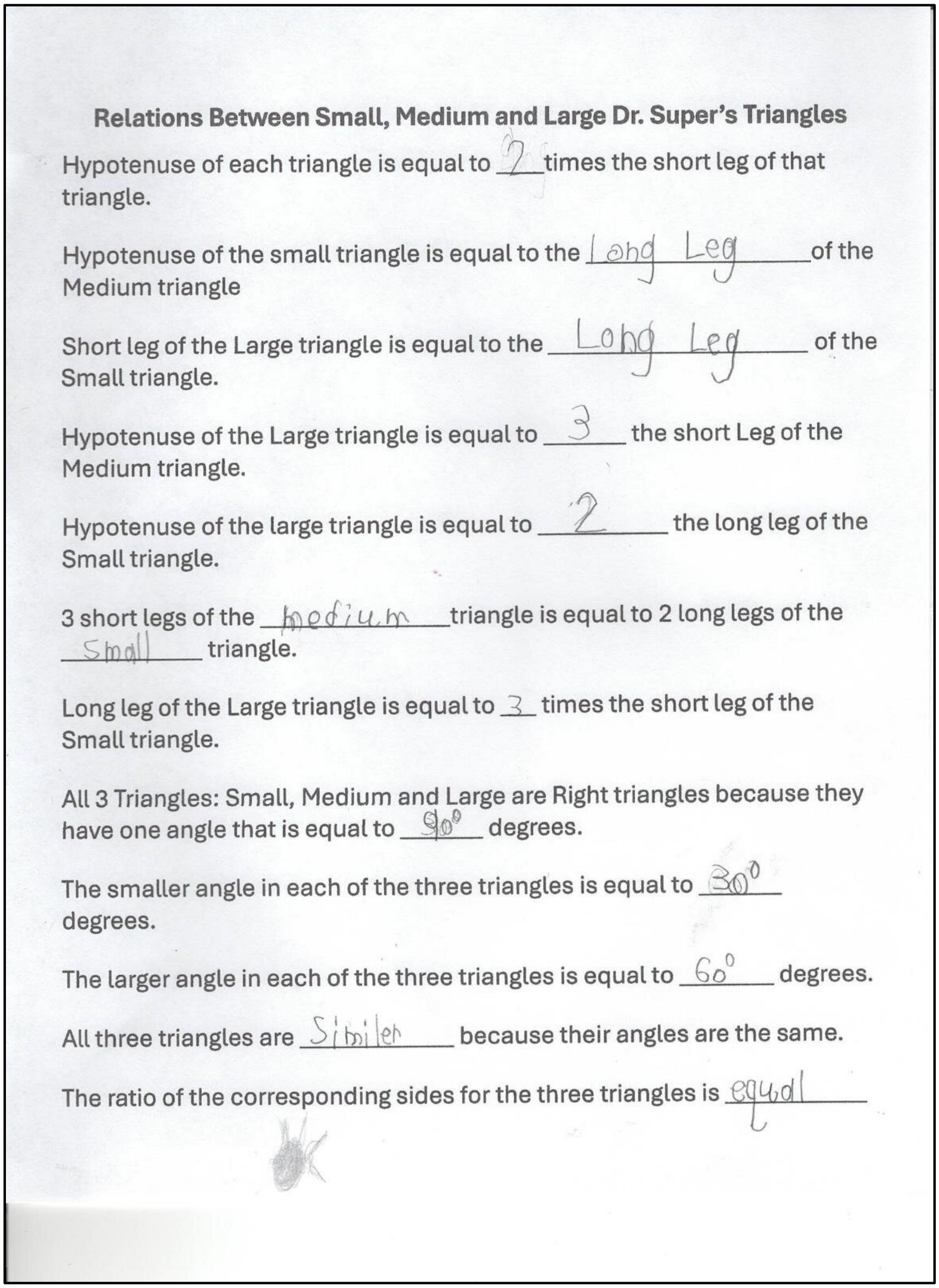
Rooz, Adam, and Dar

In this session they completed a series of small puzzles from which they were to find the relations between the different size triangles from Dr. Super's Triangles. This activity I had adapted from Dr. Super's Triangle's: Algebra Explorations, Generally for 6th grade students, but The adapted activity that I did with Cyrus and Marc last year is a good one for Rooz and Adam at this point. It combines geometric and a little algebraic notation with a lot of hands on and visual reasoning. Last year they did a simpler activity with Dr. Super's Triangles called Horatio FD Rectangle that they liked.



Initially I left the room and gave them a few minutes to make the puzzles and think about the relationships between the sides of the triangles. I had given half of the pieces to each one of them, so they had to work together. When I came back, I helped them complete all the puzzles and discussed with them some of the relations that they could see. Initially They were not sure about the relationships, and I pointed out to them first the names of the sides Hypotenuse (which Adam called the Hippopotamus immediately and it was funny but got a bit too much after many repetitions), the long leg and the short leg.

Next, I gave the sheet where they needed to plug in the right names or numbers that describe these relations. Adam, as usual, started answering quickly and sometimes without too much thinking. Eventually they both completed and the relations. Adam was good with the angles at the end of the sheet. Each time I would have them remake and find the puzzle that would help them answer the question.



Relations Between Small, Medium and Large Dr. Super's Triangles

Hypotenuse of each triangle is equal to 2 times the short leg of that triangle.

Hypotenuse of the small triangle is equal to the Long Leg of the Medium triangle

Short leg of the Large triangle is equal to the Long Leg of the Small triangle.

Hypotenuse of the Large triangle is equal to 3 the short Leg of the Medium triangle.

Hypotenuse of the large triangle is equal to 2 the long leg of the Small triangle.

3 short legs of the medium triangle is equal to 2 long legs of the Small triangle.

Long leg of the Large triangle is equal to 3 times the short leg of the Small triangle.

All 3 Triangles: Small, Medium and Large are Right triangles because they have one angle that is equal to 90° degrees.

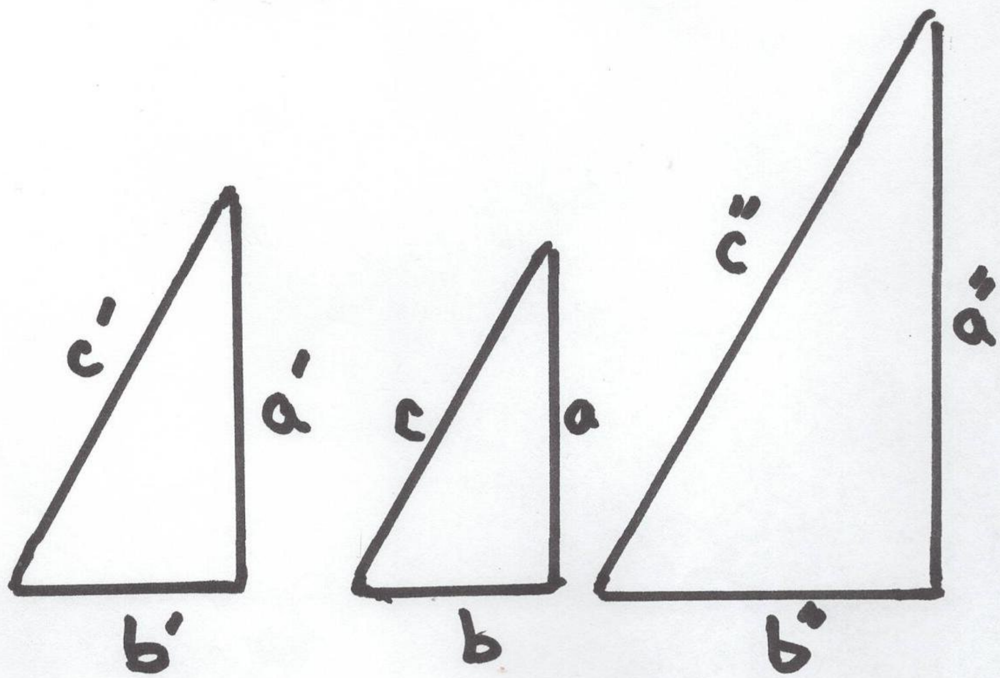
The smaller angle in each of the three triangles is equal to 30° degrees.

The larger angle in each of the three triangles is equal to 60° degrees.

All three triangles are Similer because their angles are the same.

The ratio of the corresponding sides for the three triangles is equal

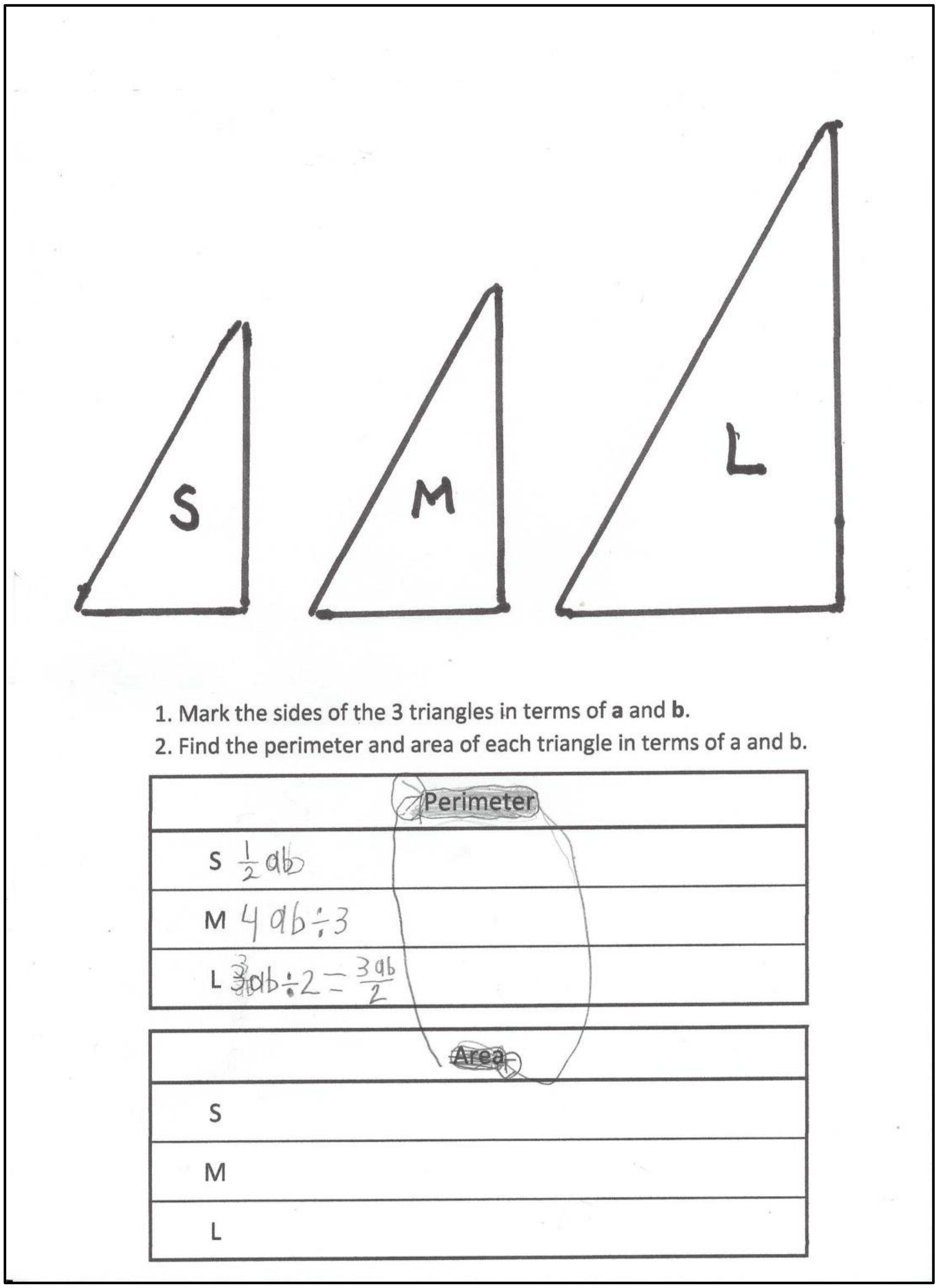
Next they found the sides of the three triangles: Small, Medium and Large in terms of a (the long leg of the small triangle) and b(the short leg of the small triangle).I had them do the easier ones first. The sides of the Small and Large Triangles that are easier to find. Finding the short leg and Hypotenuse for the Medium was more challenging as doing fractions with symbols is a difficult task and writing 2a/3 or (2/3)a is not a very simple concept at this point for them.



Find all the sides of the 3 triangles in terms of **a** and **b:**

	Long Leg	Short Leg	Hypotenuse
S	**a**	**b**	c= $2b$
M	a'= $2b$	3b'= $2a$ b'= $2a \div 3 = \frac{2a}{3}$	c'= $\frac{4a}{3}$
L	a''= $3b$	b''= a	c''= $2a$

Next, they tried to find the Area and Perimeter of these 3 triangles. For the small and Large triangles, it was easier for them to see that they could make a rectangle and divide the area by 2. This was a harder task for the medium triangle that we will review next time. Perimeters will also involve adding fractions with symbols and will be more challenging.


S M L

1. Mark the sides of the 3 triangles in terms of **a** and **b**.
2. Find the perimeter and area of each triangle in terms of a and b.

	Perimeter
S $\frac{1}{2}ab$	
M $4ab \div 3$	
L $3ab \div 2 = \frac{3ab}{2}$	

	Area
S	
M	
L	

We will review these activities next time and they will start to make their Fitz Da Pieces Monster and color it and eventually find it's area and perimeter.

Math Circle RA 5-6 Math League Continued 9-18 & 25-2024 3:05-3:50

Rooz, Adam , and Dar

I continued with some of the basic numbers sense tricks that they can use for the their Math League qualification and also it is good for them to have in their Mental Math knowledge. I also gave them some quick quizes to practice doing the problems fast but accuratly.

I gave them first the cards that I had made based on the draft that I shared with them for finding remainders. I have found that if the cards are nicely made and laminated they pay more attention to them! We discussed the rules in particular for 7 and 11 and also for 8 that I also had a card made for them for divisibility by 8. Then I gave them the quiz that they did in 5 minutes. They finished before time. They got most of the answers but they both had problems with remainders for 7 and 11 that are the hardest.

Divisibility Check and Finding the Remainder
For 2) If the Number is Odd the remainder is 1. R=1
For 3) Subtract 1 or 2 to make the sum of the digits divisible by 3. For example: For 377 subtract 2 to get 375. R=2
For 4) Subtract 1, 2, or 3 to make the last 2 digits divisible by 4. (odd+2,6 - even+0,4,8) For example: For 51,939 subtract 3 to from 39 to get 36. R=3
For 5) Subtract 1,2,3, or 4 to make the last digit 0 or 5. e.g. for 69,427 subtract 2 to get 69,425. R=2
For 6) Subtract 1 to 5 to make the result even and divisible by 3. e.g. For 5263 subtract 1 to get 5262 that is even with sum of digits=15, R=1
For 7) Subtract 1 to 6 to make the result divisible by 7 (a-2b divisible by 7) e.g. for 14626 ignore 14 then for 625 subtract 2 to get 623 since 62-6=56=7x8 R=2
For 8) Subtract 1 to 7 to make the last 3 digits of the result divisible by 8 (e xx div by 8, oxx div by 4 not 8). e.g. for 35,671 subtract 7 from 671 to get 664 that is divisible by 8. R=7
For 9) Subtract 1 or 8 to make the sum of the digits divisible by 9. For example: For 36,782 subtract 8 to get 36,774 (sum of digits=27). R=8
For 11) Subtract 1 to 10 to make the alternating sum divisible by 11. e.g. for 35,237 3-5+2-3+7=4 subtract 4 to get 35,233 3-5+2-3+3=0. R=4
For 12) Subtract 1 to 11 so the result is divisible by 3 and 4. e.g.for 3615 subtract 3 to get 3612 sum of digits 12 and 12 is divisible by 4. R=3

Divisibility Rule for 8: Look at last 3 Digits For 200,400,800 last 2 digits should be green. For 100,300,500,700,900 last 2 digits should be red.				
00		04		08
	12		16	
20		24		28
	32		36	
40		44		48
	52		56	
60		64		68
	72		76	
80		84		88
	92		96	

Next, I gave them the cards for the 20x20 Multiplication table that had the squares of the 2-digit numbers on the back and went over the fact that they needed to know these results pretty much like the 10x10 multiplication table. We went over the multiplication of one-digit numbers by numbers from 11to 20 and the squares of numbers from 11 to 19. I also went over the multiplication method for numbers between 11-19 that will help them find these values quickly. I went over the rules and we each asked a few of them. Rooz is familiar with this method, but Adam must practice it a bit more to completely master it.

Name: Rooz

the remainder of 362 ÷ 3 is 2

the remainder of 4,562 ÷ 5 is 2

The remainder of 36,323 ÷ 4 is 3

the remainder of 52,799 ÷ 10 is 9

The remainder of 391,631 ÷ 2 is 1

the remainder of 65,323 ÷ 6 is 1

the remainder of 655 ÷ 7 is 6 4

the remainder of 999,817 ÷ 9 is 6

the remainder of 71,410 ÷ 8 is 2

the remainder of 657,651 ÷ 12 is 3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

1s Digit / 10s Value	0	1	2	3	4	5	5	7	8	9
0	0	1	4	9	16	25	36	49	64	81
10	100	121	144	169	196	225	256	289	324	361
20	400	441	484	529	576	625	676	729	784	841
30	900	961	1024	1089	1156	1225	1296	1369	1444	1521
40	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
50	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
60	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
70	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
80	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
90	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801

I also wanted to show them how using the algebraic identity:

(a+b)(a-b)=a2-b2

(a+b)(a-b)+b2=a2

They can find the squares of two-digit numbers mentally much easier. I will do this in the next session maybe without the mention of the algebra behind it. For example to find 582you can think of it as (58-2)(58+2)+4=56x50+4=2,804 The trick is to make one of the numbers a multiple of 10 so they can do the multiplication easier in their head and add the square of the number they had to add to find the multiple of 10. This was a good session, and they were both pretty focused on the work. These mental tricks as well as some others that I will teach them should help them in the Math League if they get selected.