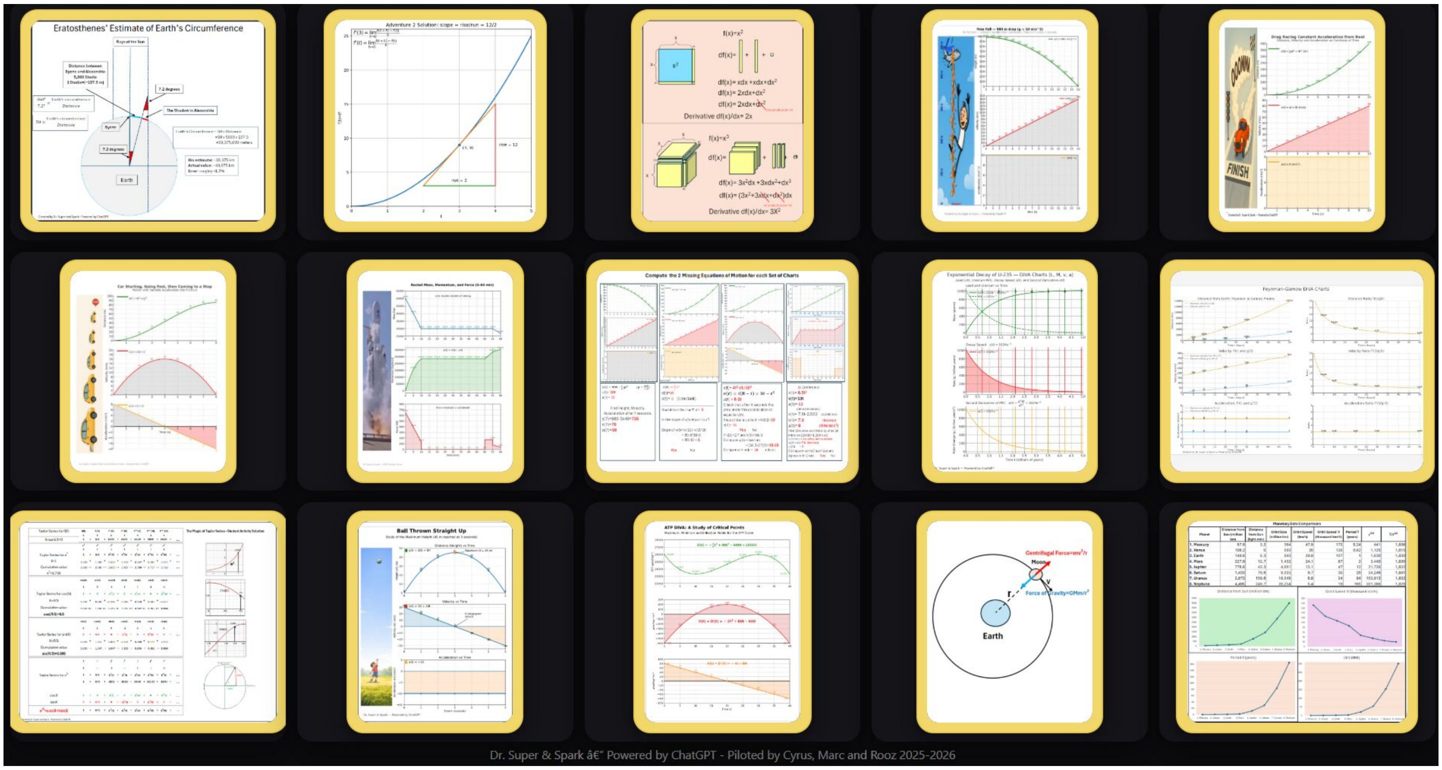
Math Circle Notes

(2025–2026 Calculus & Mechanics Series)

Adventures in Calculus & Mechanics for Young and Curious Minds



Dr. Super & Spark â€" Powered by ChatGPT - Piloted by Cyrus, Marc and Rooz 2025-2026

Original Math Circle Notes, Discussions, and Completed Worksheets

Behrouz B. Aghevli, PhD

(Dr. Super)

Companion Notes Volume for
Adventures in Calculus & Mechanics for Young and Curious Minds

These Math Circle Notes were created to be shared. Teachers, parents, homeschool educators, and Math Circle leaders are encouraged to print and distribute portions of these materials for non-commercial educational and home use.

This companion volume contains the original Math Circle Notes, discussions, and student work that contributed to the development of *Adventures in Calculus & Mechanics for Young and Curious Minds*.

First Edition

Tucson, Arizona

www.drsuper.com

MATH CIRCLE NOTES

(2025–2026 Calculus & Mechanics Series)

Companion Volume to

ADVENTURES IN CALCULUS & MECHANICS FOR YOUNG AND CURIOUS MINDS

A Story-Powered Introduction to Calculus Through Motion, Graphs, and Discovery

Behrouz B. Aghevli, PhD

(Dr. Super)

Based on the Wednesday Math Circles

Academic Year 2025–2026

Tucson, Arizona

with editorial assistance from Spark (ChatGPT)

drsuper.com

Introduction to the Math Circle Notes

Adventures in Calculus & Mechanics for Young and Curious Minds did not begin as a book. It began as a series of weekly Math Circle sessions conducted during the 2025–2026 academic year with my grandchildren, Cyrus and Rooz, and their friend Marc. The purpose of these sessions was not to teach a formal calculus course, but to explore the ideas of calculus and mechanics through stories, videos, experiments, discussion, and guided discovery.

The notes collected in this volume represent the working record of that process. They were never intended for publication. Most began as rough outlines, sketches of activities, references to books and videos, questions for discussion, and ideas intended to help students discover important concepts for themselves. Some sessions were carefully planned; others evolved in response to questions raised during earlier meetings.

The development of each Adventure typically followed several stages. First came the original idea, often inspired by a historical story, a mathematical puzzle, a scientific question, a student request, or a video such as those in the excellent *Essence of Calculus* series by 3Blue1Brown. Next came preliminary activities, which I would work through myself, looking for gaps, ambiguities, and opportunities for clearer visual explanations. The activities were then tested during the Math Circle sessions, where students frequently discovered unexpected approaches, asked challenging questions, or suggested better ways to think about a problem.

After each session, the notes, worksheets, and activities were revised. Explanations were clarified, diagrams improved, questions rewritten, and new activities added. In many cases, several rounds of revision took place before an activity reached its final form. The Teacher Guides and Adventures found in the main volume represent the culmination of this process of design, testing, observation, and refinement.

These notes therefore provide a view of the Adventures in their natural habitat. They include original session notes, completed worksheets by Cyrus, Rooz, and Marc, discussion points, alternative approaches, side explorations, and occasional dead ends. They reveal not only what was ultimately taught, but also how the ideas developed and evolved over time.

Readers should not expect these notes to have the polished structure of the Adventures themselves. They are working documents, preserved because they capture something valuable: the process of mathematical discovery. They show how questions lead to new questions, how misunderstandings often lead to deeper understanding, and how some of the best ideas emerge from unexpected conversations.

It is my hope that teachers, parents, Math Circle leaders, and curious students will find these notes useful not only as a companion to the Adventures, but also as encouragement to experiment, adapt, revise, and create activities of their own. Mathematics is rarely discovered in a straight line, and neither were these Adventures.

— Behrouz B. Aghevli, PhD (Dr. Super)

Table of Contents for Math Circle Notes

🧑‍🏫 Adventure 1 – Area Under a Line, Slope, & the Big Idea of Calculus

🧩 1. Overview of the Session

Date: 10/15/2025**Duration:** 1:45–2:40 pm (55 minutes)
Students present: Cyrus, Marc, Rooz
Main objective:
To introduce the students to the idea that calculus is fundamentally about *understanding change* — growth, motion, area, and slope — and to launch the long-term series built around 3Blue1Brown's *Essence of Calculus*.

This session served as an orientation to the visual style, intuition-first approach, and the flow of the 9+ sections to come.

Summary of student discoveries:

- They recognized that area can be built from many thin slices — an idea central to integrals.
- They saw that the derivative measures how fast something is changing "right now."
- They realized that calculus is not a set of formulas but a way to think visually and conceptually.
- They connected the video's circle-area idea to the distance–velocity–acceleration charts they've seen in earlier activities.

🚀 2. Warm-Up Conversation / Storytelling / Videos

We open the session with the remarkable story of Eratosthenes, who measured the entire circumference of the Earth using nothing more than:

- a stick,
- a shadow,
- a tiny angle,

After hearing the story, students estimate Earth's circumference themselves using the activity sheet titled: **Your Estimate of Earths Circumference,** and the solution sheet following it titled: **Eratosthenes' Estimate of Earth's Circumference**.

Then we introduced *Essence of Calculus* series (12 videos) and explained that this will be the conceptual backbone of our entire Calculus & Mechanics program.

We then watched Video 1 named: The Essence of Calculus, which introduces:

- area of a circle as area under a curve,
- how slicing leads to integrals,
- and how slopes/derivatives appear naturally from motion.

The kids immediately connected to the visuals:

- Cyrus leaned forward during the circle-unfolding demonstration, recognizing symmetry.
- Marc commented that the animation looked “like slicing a pizza thinner and thinner.”
- Rooz asked if the circle-area method also works for triangles — a great early insight.

We framed the series as a *story* they will follow — where chart reading, physical intuition, and calculus ideas all blend together.

📊 3. Activities Completed

Activity #1 — Worksheets on Area & Slope

I gave them two worksheets (front and back) inspired by the video.
They repeated the logic from the visual demonstration and applied it to **simple linear functions**.

- Cyrus and Marc followed the reasoning quickly.
- Rooz needed some encouragement but completed the exercises independently.
- Dr. Super participated and supervised the early discussion.

This activity reinforced the idea that integrals are “accumulated area” and derivatives are “instantaneous slope,” even though we didn’t use formal notation yet.

Activity #2 — Connecting Video Concepts to Real Motion

We briefly related the worksheet answers to the distance–velocity–acceleration charts that will come late and they were told that:

- distance curves have area meaning,
- velocity curves have slope meaning,
- and calculus explains why those relationships are linked.

This served as an important conceptual bridge to the later sections (Newton's Laws, rockets, drag racing, and exponentials).

🧮 4. Mini-Discussions & Side Topics

A short but meaningful discussion emerged around the question:
"Why does slicing something thinner make calculations more accurate?"

This opened the door to:

- the idea of limits (to be formalized in Section 8),
- infinitesimals,
- and why calculus feels magical when combined with visualization.

We kept the tone light, knowing this was their *first real taste* of calculus.

🧑‍🏫 5. What Worked Well

- Estimating the circumference of the Earth worked really well.
- The video's pacing was perfect — short, visual, and intuitive.
- The worksheets reinforced the concept without overwhelming them.
- The balance of story, visuals, charts, and hands-on work matched their attention spans.
- All students had at least one "aha" moment.

🧠 6. Adjustments for Next Time

For future introductory sessions:

- Provide a clearer bridge between the video and the worksheets.
- Add one or two guiding questions ("What do you notice about the slices?" "How does slope show up in real life?").
- Leave space for them to draw simple curves by hand.
- Consider a warm-up mini-chart next time to anchor their intuition.

🪄 7. Student Quotes (Optional)

- Marc: *"So area is like adding up lots of little sticks?"*
- Rooz (after finishing the problems): *"Okay… this is actually fun now."*
- *Rooz " it so cool when it fills the area under the line completely!"*

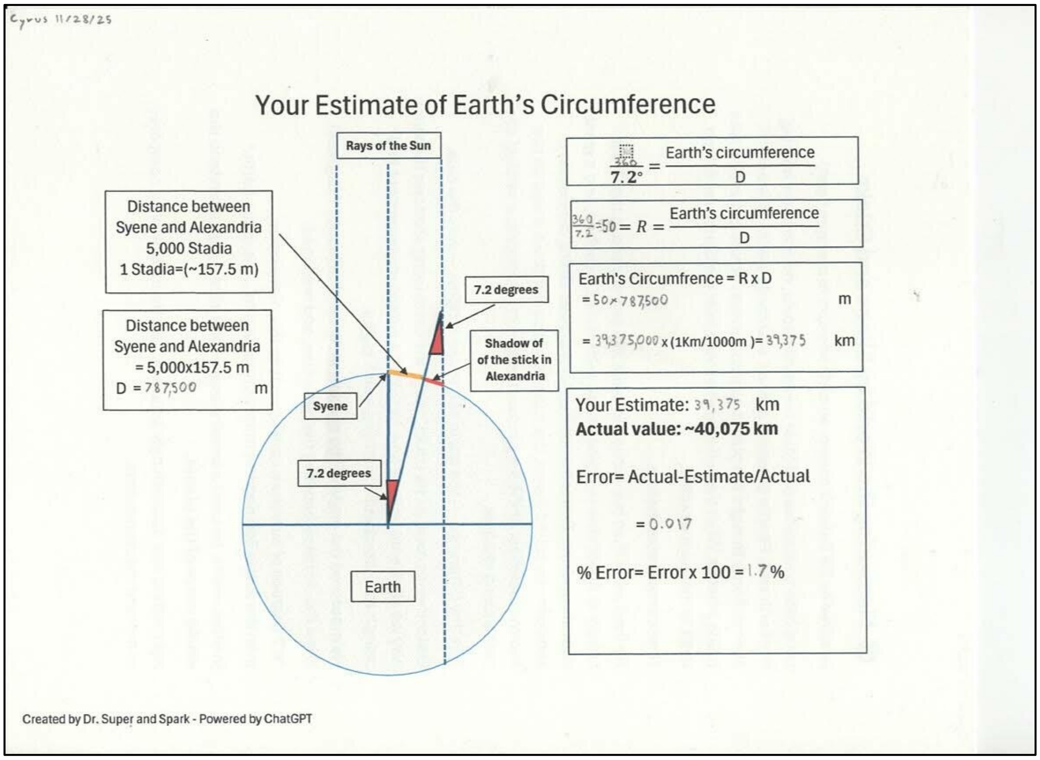
- Cyrus: *“I like when the circle turns into a line.”*

🧱 8. Closing Reflection by Dr. Super

This was a strong beginning to the Calculus & Mechanics journey. The kids engaged well with the story and the video, handled the worksheets confidently, and showed curiosity about how the ideas connect to motion. This introduction set the tone for the rest of the Calculus and Mechanics Series — building intuition first, formalism second, and always grounding abstract calculus ideas in physics and everyday motion.

📘 Student Completed Worksheets

🌍 Measuring the World by Slices and Sticks – Student Worksheets



Cyrus 11/28/25

Your Estimate of Earth's Circumference

Rays of the Sun

Distance between Syene and Alexandria
5,000 Stadia
1 Stadia=(~157.5 m)

Distance between Syene and Alexandria
= 5,000x157.5 m
D = 787,500 m

7.2 degrees

Shadow of of the stick in Alexandria

Syene

7.2 degrees

Earth

$\frac{360}{7.2^\circ} = \frac{\text{Earth's circumference}}{D}$

$\frac{360}{7.2} = 50 = R = \frac{\text{Earth's circumference}}{D}$

Earth's Circumfrence = R x D
= 50×787,500 m
= 39,375,000 x (1Km/1000m)= 39,375 km

Your Estimate: 39,375 km
Actual value: ~40,075 km

Error= Actual-Estimate/Actual

= 0.017

% Error= Error x 100 = 1.7 %

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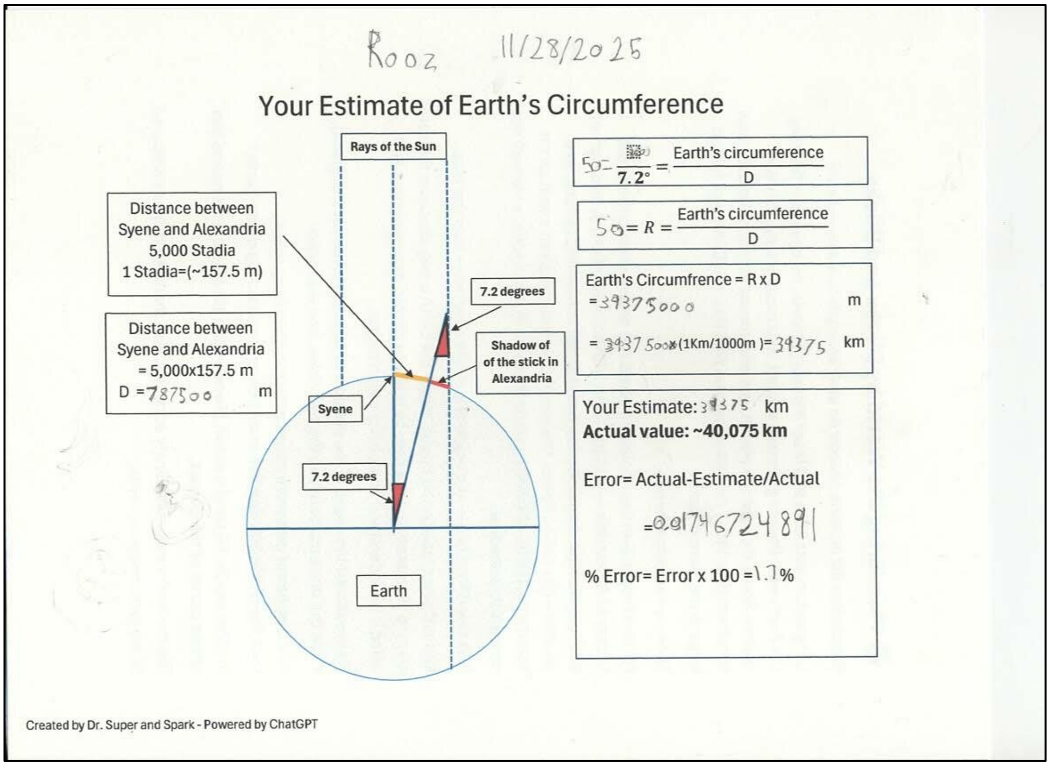


Rooz 11/28/2025

Your Estimate of Earth's Circumference

Rays of the Sun

Distance between Syene and Alexandria
5,000 Stadia
1 Stadia=(~157.5 m)

Distance between Syene and Alexandria
= 5,000x157.5 m
D = 787500 m

7.2 degrees

Shadow of of the stick in Alexandria

Syene

7.2 degrees

Earth

$50 = \frac{\ }{7.2^\circ} = \frac{\text{Earth's circumference}}{D}$

$50 = R = \frac{\text{Earth's circumference}}{D}$

Earth's Circumfrence = R x D
= 39375000 m
= 39375000x(1Km/1000m)= 39375 km

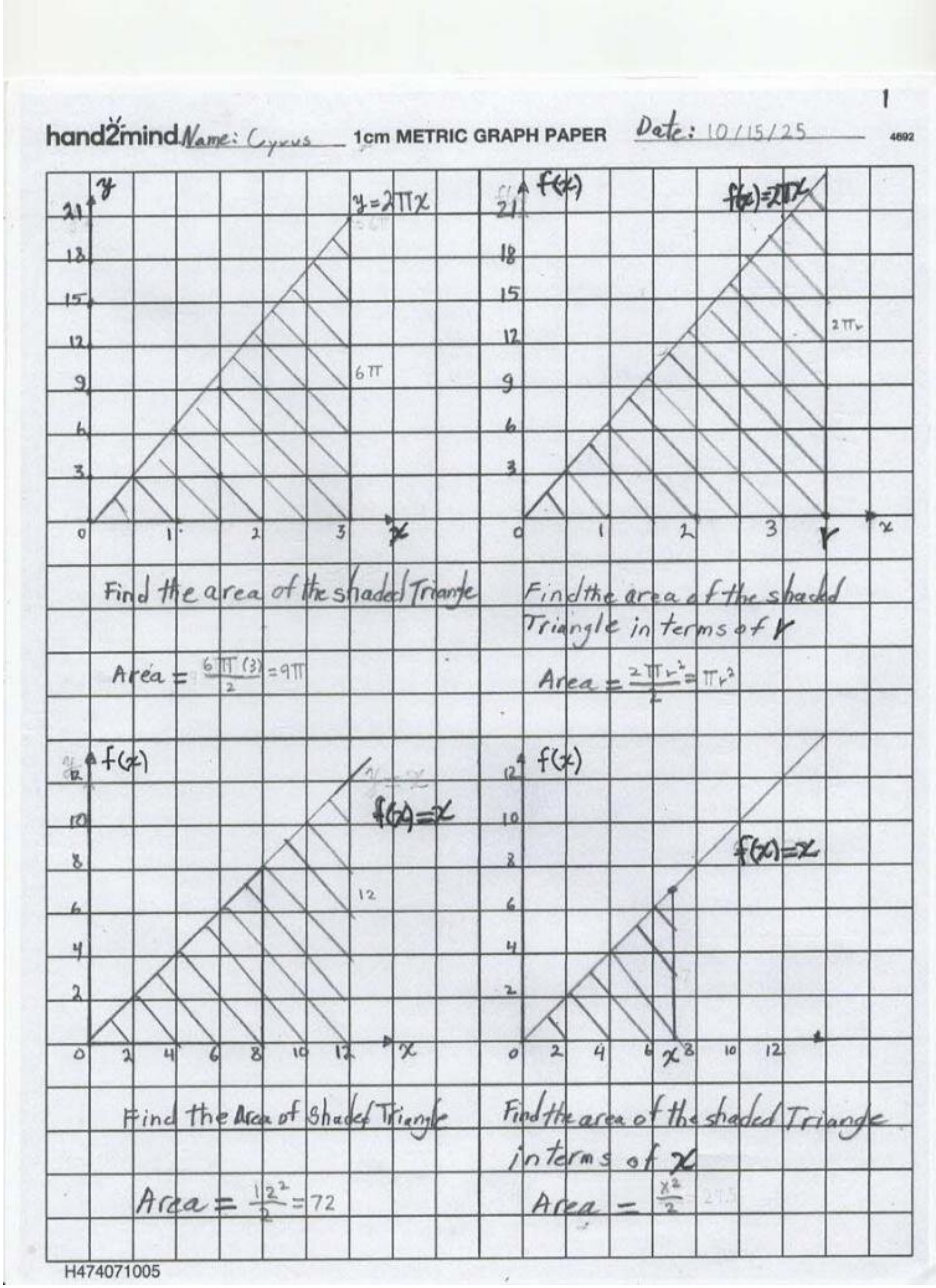
Your Estimate: 39375 km
Actual value: ~40,075 km

Error= Actual-Estimate/Actual

=0.01746724891

% Error= Error x 100 = 1.7%

Created by Dr. Super and Spark - Powered by ChatGPT



hand2mind Name: Cyrus 1cm METRIC GRAPH PAPER Date: 10/15/25 4692

Find the area of the shaded Triangle

Area $= \frac{6\pi(3)}{2} = 9\pi$

Find the area of the shaded Triangle in terms of r

Area $= \frac{2\pi r^2}{2} = \pi r^2$

Find the Area of Shaded Triangle

Area $= \frac{12^2}{2} = 72$

Find the area of the shaded Triangle in terms of x

Area $= \frac{x^2}{2}$ 24.5

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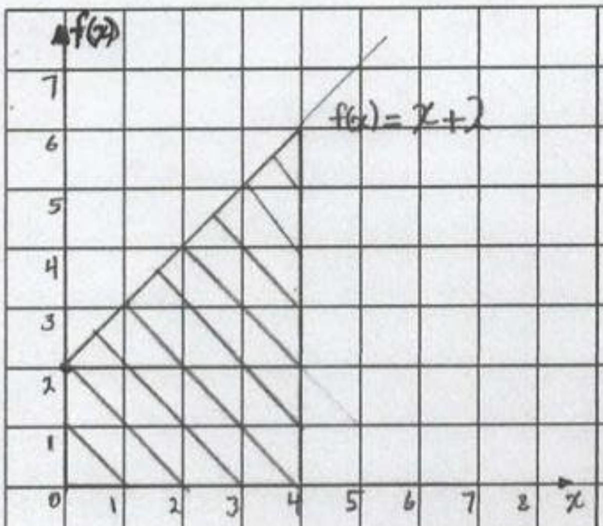
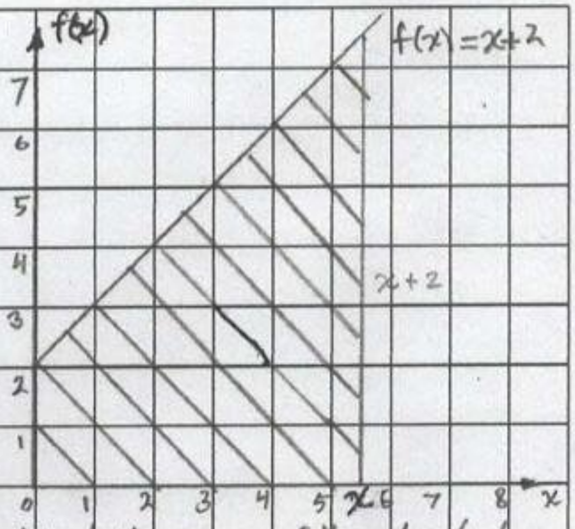
2

hand2mind Name: Cyrus 1cm METRIC GRAPH PAPER Date: 10/15/25 4692





Find the area of the shaded Trapezoid

Total Area = Area of △ + Area of ▭

= 8 + 8 = 16

Find the area of the shaded Trapezoid in terms of x

Total Area = Area of triangle + Area ▭

$= 2x + \frac{x^2}{2}$

Integrals

$\int 2\pi r\,dr = \pi r^2$

$\int x\,dx = \frac{x^2}{2}$

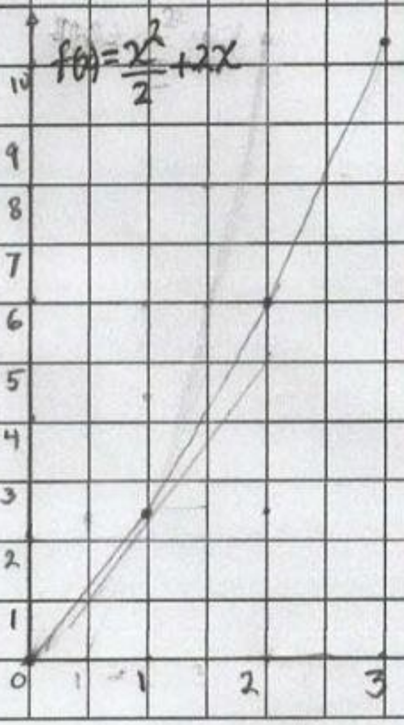
$\int (x+2)\,dx = \frac{x^2}{2} + 2x$

Derivates

$\frac{d(\pi r^2)}{dr} = 2\pi r$

$d\left(\frac{x^2}{2}\right) = x$

$d\left(\frac{x^2}{2} + 2x\right) = x+2$



x	$f(x)$
0	0
1	2.5
2	6
3	10.5
4	

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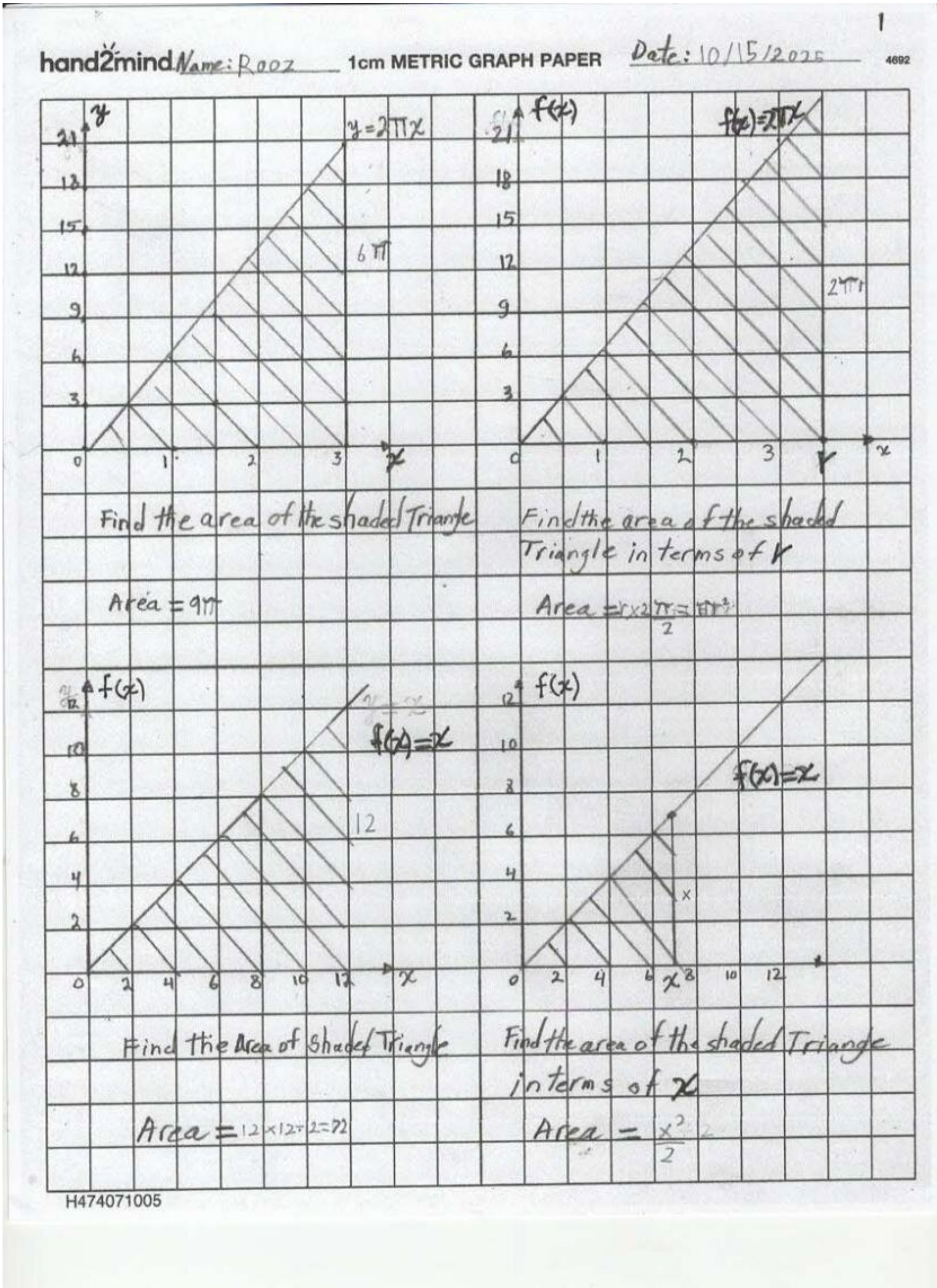


hand2mind Name: Rooz 1cm METRIC GRAPH PAPER Date: 10/15/2025 4692

Find the area of the shaded Triangle

Area = 9π

Find the area of the shaded Triangle in terms of r

Area = $\frac{r \times 2\pi r}{2} = \pi r^2$

Find the Area of Shaded Triangle

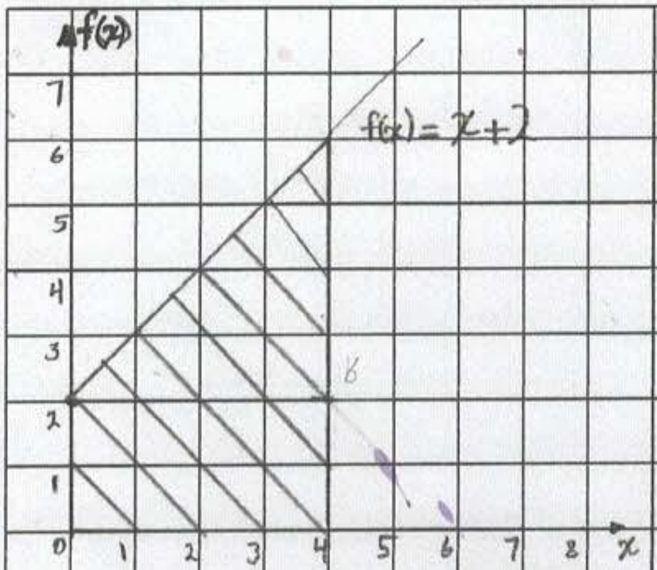
Area = 12×12÷2=72

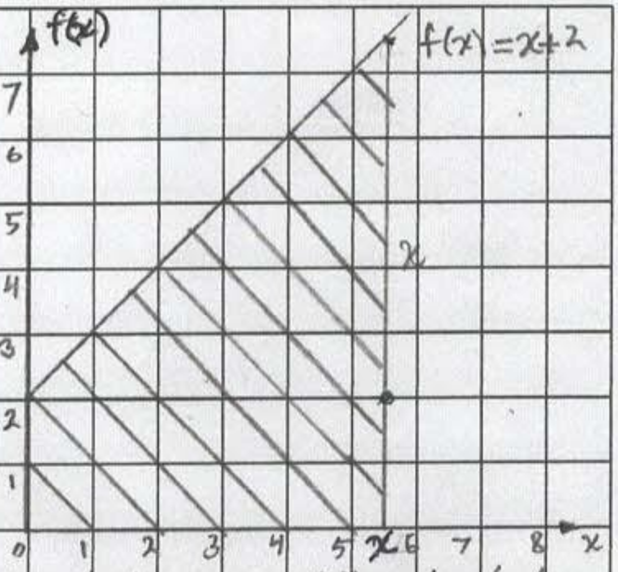
Find the area of the shaded Triangle in terms of x

Area = $\frac{x^2}{2}$

2

hand2mind Name: Rooz 1cm METRIC GRAPH PAPER Date: 4692





Find the area of the shaded Trapezoid

Total Area = Area of $\triangle$ + Area of $\square$

$\downarrow$ 8 $\quad$ $\downarrow$ 8

=

~~Find the area of the shaded~~ Trapezoid in terms of x

Total Area = Area of triangle + Area $\square$

$\downarrow$ $\frac{x^2}{2}$ $\quad$ $\downarrow$ $2x$

=

Integrals

$$\int 2\pi r\,dr = \pi r^2$$

$$\int x\,dx = \frac{x^2}{2}$$

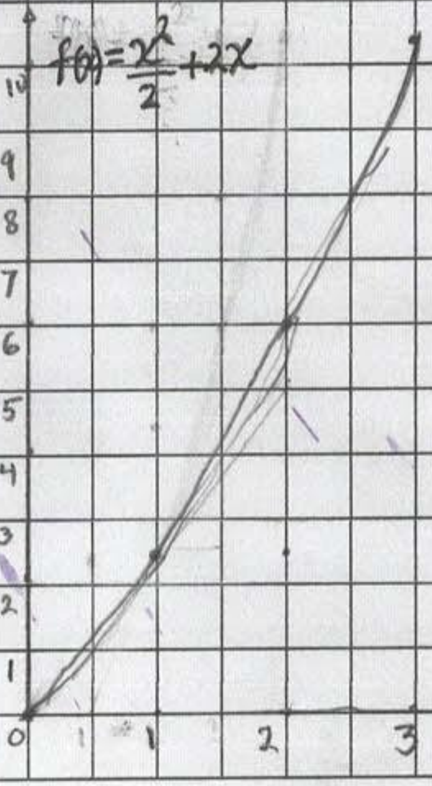
$$\int (x+2)\,dx = \frac{x^2}{2} + 2x$$

Derivates

$$\frac{d(\pi r^2)}{dr} = 2\pi r$$

$$d\left(\frac{x^2}{2}\right) = x$$

$$d\left(\frac{x^2}{2} + 2x\right) = x+2$$



x	$f(x)$
0	0
1	2.5
2	6
3	10.5

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Adventure 2 – Secret of Slope

1. Overview of the Session

Date: 10/21/2025

Duration: 11:40–1:00 (80 minutes)

Students present: Cyrus, Marc, Rooz

Main objective:
To introduce the core ideas behind derivatives and integrals by using hands-on worksheets, the 3Blue1Brown visuals, and a guided discovery of tangents and shaded areas. This session served as the kids' first structured encounter with the Power Rule and the idea that "slope tells a story."

Summary of student discoveries:

- They understood that a derivative tells how fast a function changes at a point.
- They successfully connected a tangent line's slope to movement along a curve.
- They rediscovered the Power Rule for $h(t) = t^2$ in a hands-on way.
- They began to see how shaded area connects to the idea of an "integral."

Math Circle CMR 10 Calculus 2 1...

2. Warm-Up Conversation / Story & Video Integration

We continued with the **3Blue1Brown Calculus series**, watching the second video.

The kids remained focused through the explanation of slopes, tangents, and curved motion.

Before handing out worksheets, Dr. Super paused to highlight key moments from the video:

- How Grant draws the secant lines getting closer
- How the slope "settles" into a single value
- Why the tangent at a point captures instantaneous behavior

This setup made the worksheet feel like a continuation of the video instead of a separate activity.

Rooz needed more redirection, but the 3B1B visuals helped him re-engage each time.

3. Activities Completed

Deriving the Derivative of $h(t) = t^2$

The first worksheet guided them to:

- Start from the difference quotient
- Reduce it step by step
- Arrive at the derivative formula

Then they had to **draw** the graph of $h(t) = t^2$, mark the point $(3, 9)$, and analyze the tangent.

They discovered:

- The slope at $t = 3$ is exactly **3**
- Therefore $d(x^2)/dx = 2x$

- The video had used t^3, so this felt familiar but empowering

A pre-drawn tangent line helped them avoid getting bogged down by drawing details.

Marc and Cyrus worked confidently;
Rooz required steady prompting but finished successfully.

4. Mini-Discussions & Side Topics

During and after the worksheets, Dr. Super introduced:

- Why tangent slope represents instantaneous speed
- How the Power Rule will generalize
- How the formulas the video shows connect to what they had just drawn

These side conversations planted seeds for the connection between calculus and mechanics that develops fully in later sections.

5. What Worked Well

- The pairing of **video → worksheet → discussion** created a smooth learning arc.
- The tangent line being pre-drawn made it accessible without removing the challenge.
- The kids handled the area problem well, even though it was their first real "integral."
- The discovery that $F(x)$was the area function produced genuine excitement.

Marc and Cyrus stayed focused; Rooz needed gentle pressure but stayed motivated and refused to give up.

6. Adjustments for Next Time

- Add one guiding question to help them identify the area-function pattern sooner.
- Include a check-in prompt for Rooz before each step to reduce wander.
- Introduce a small verbal cue about "area makes curves bend upward" to help connect intuition to graphs.

7. Student Quotes (Optional but Fun)

Rooz (after finishing the slope step): **"I did it — I actually did it!"**

Marc (looking at the tangent line): **"So this skinny line tells everything?"**

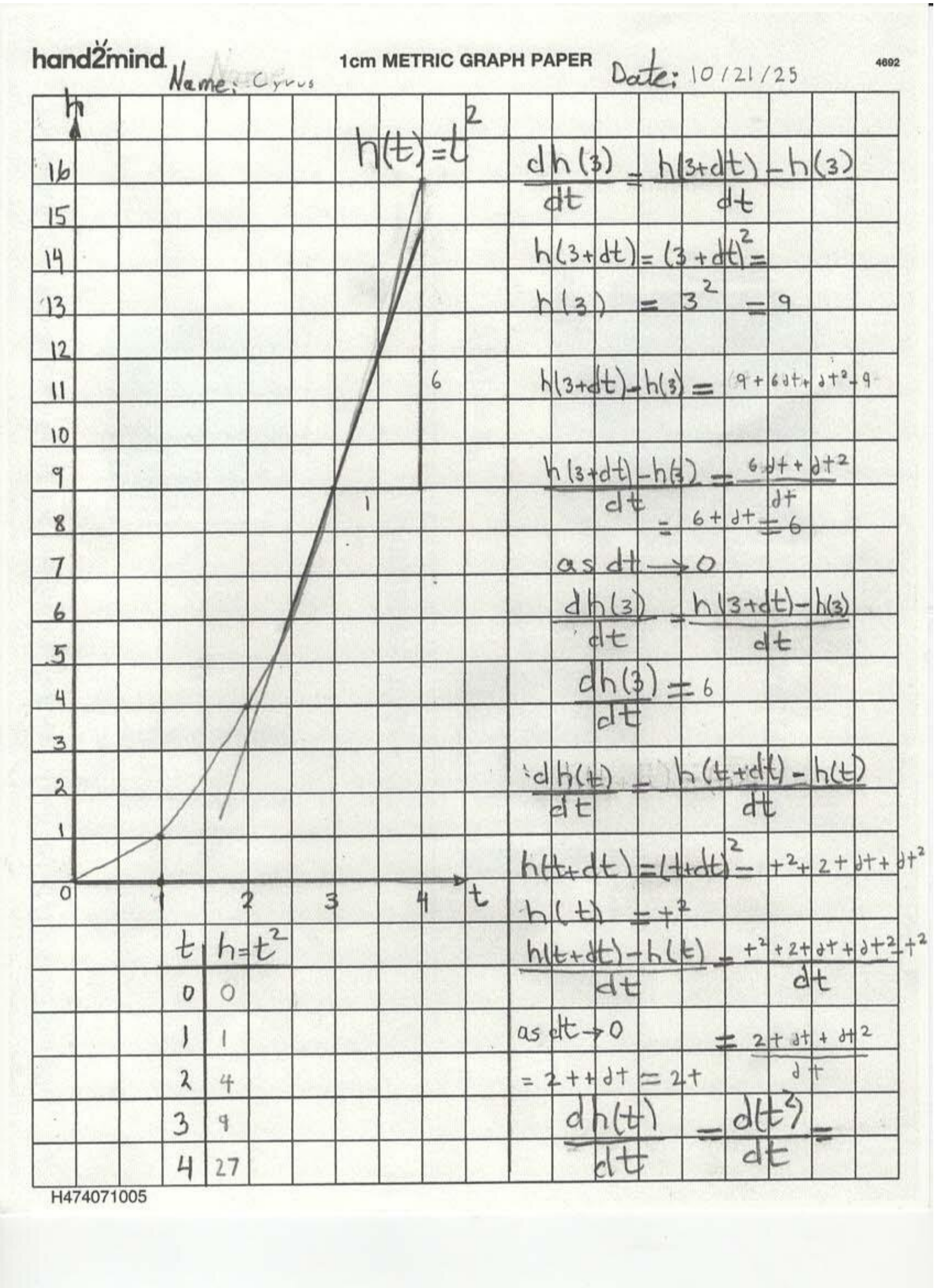
Cyrus (seeing the area function): **"It curves because the area keeps adding up!"**

8. Closing Reflection by Dr. Super

This early-calculus session was a strong foundation. The kids began to internalize the main story of calculus: slopes measure change. Given their age, this was challenging material — but they rose to it. The most encouraging sign was that even when difficulty increased, **they pushed through and stayed with the work**.

This session set the stage for the deeper connections to come in free fall, drag racing, and rocket launch activities.

9. Student's Worksheets



hand2mind 1cm METRIC GRAPH PAPER 4692

Name: Cyrus Date: 10/21/25

$h(t) = t^2$

t	$h = t^2$
0	0
1	1
2	4
3	9
4	27

$$\frac{dh(3)}{dt} = \frac{h(3+dt) - h(3)}{dt}$$

$$h(3+dt) = (3+dt)^2 =$$

$$h(3) = 3^2 = 9$$

$$h(3+dt) - h(3) = 9 + 6dt + dt^2 - 9$$

$$\frac{h(3+dt) - h(3)}{dt} = \frac{6dt + dt^2}{dt}$$

$$= 6 + dt = 6$$

as $dt \to 0$

$$\frac{dh(3)}{dt} = \frac{h(3+dt) - h(3)}{dt}$$

$$\frac{dh(3)}{dt} = 6$$

$$\frac{dh(t)}{dt} = \frac{h(t+dt) - h(t)}{dt}$$

$$h(t+dt) = (t+dt)^2 = t^2 + 2t\,dt + dt^2$$

$$h(t) = t^2$$

$$\frac{h(t+dt) - h(t)}{dt} = \frac{t^2 + 2t\,dt + dt^2 - t^2}{dt}$$

as $dt \to 0$

$$= \frac{2t\,dt + dt^2}{dt}$$

$$= 2t + dt = 2t$$

$$\frac{dh(t)}{dt} = \frac{d(t^2)}{dt} =$$

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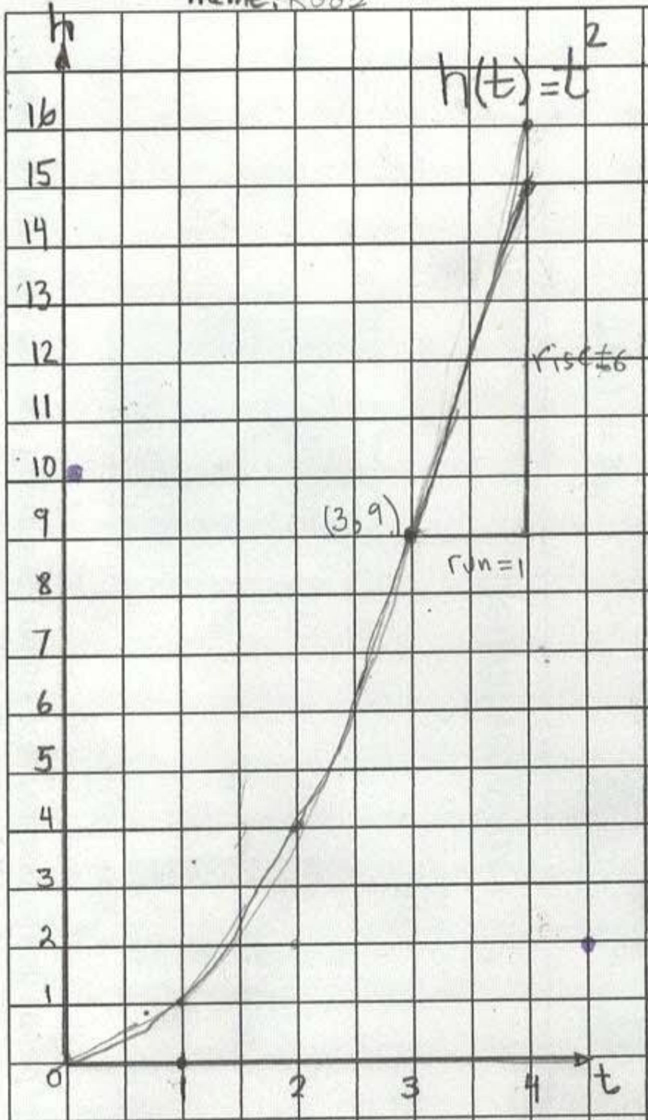
hand2mind 1cm METRIC GRAPH PAPER 4692

Name: Rooz

Date: 10/21/2025



$$\frac{dh(3)}{dt} = \frac{h(3+dt)-h(3)}{dt}$$

$$h(3+dt) = (3+dt)^2 = 9+6dt+dt^2$$

$$h(3) = 3^2 = 9$$

$$h(3+dt)-h(3) = 6dt+dt^2$$

$$\frac{h(3+dt)-h(3)}{dt} = \frac{6\cancel{dt}+dt^{\cancel{2}}}{\cancel{dt}} = 6+dt$$

as $dt \rightarrow 0$

$$\frac{dh(3)}{dt} = \frac{h(3+dt)-h(3)}{dt} = 6$$

$$\frac{dh(3)}{dt} = 6 = 2\times 3$$

$$\frac{dh(t)}{dt} = \frac{h(t+dt)-h(t)}{dt}$$

$$h(t+dt) = (t+dt)^2 = t^2+dt^2+2tdt$$

$$h(t) = t^2$$

$$\frac{h(t+dt)-h(t)}{dt} = \frac{2t\cancel{dt}+dt^{\cancel{2}}}{dt}$$

as $dt \rightarrow 0$ $\quad = 2t+dt$

$$\frac{dh(t)}{dt} = \frac{d(t^2)}{dt} = 2t$$

t	$h=t^2$
0	0
1	1
2	4
3	9
4	16

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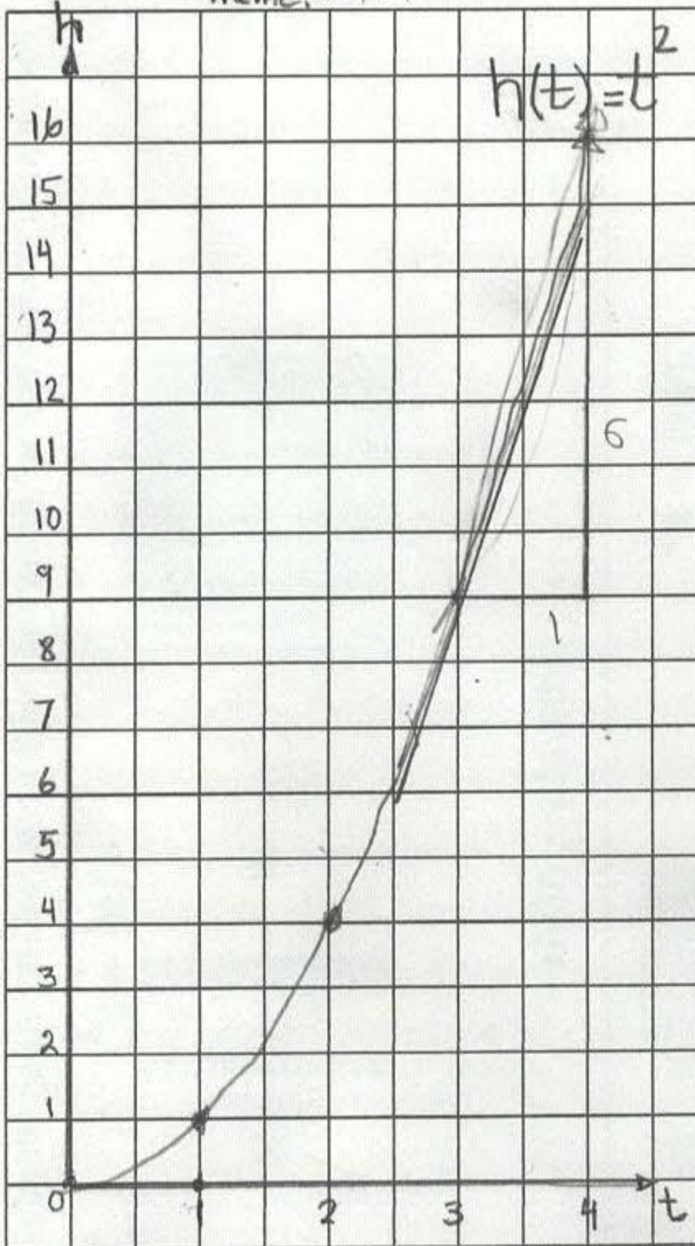
Name: Marc

Date: 10/21/25



t	$h=t^2$
0	0
1	1
2	4
3	9
4	16

$$\frac{dh(3)}{dt} = \frac{h(3+dt)-h(3)}{dt}$$

$$h(3+dt) = (3+dt)^2 = 3^2+6dt+dt^2$$

$$h(3) = 3^2 = 9$$

$$h(3+dt)-h(3) = 9+6dt+dt^2-9$$

$$\frac{h(3+dt)-h(3)}{dt} = \frac{6\cancel{dt}+dt^2}{\cancel{dt}} = 6+dt$$

as $dt \to 0$ = 6

$$\frac{dh(3)}{dt} = \frac{h(3+dt)-h(3)}{dt}$$

$$\frac{dh(3)}{dt} = 6$$

$$\frac{dh(t)}{dt} = \frac{h(t+dt)-h(t)}{dt}$$

$$h(t+dt) = (t+dt)^2 = t^2+2tdt+dt^2$$

$$h(t) = t^2$$

$$\frac{h(t+dt)-h(t)}{dt} = \frac{\cancel{t^2}+2t\cancel{dt}+dt^2-\cancel{t^2}}{\cancel{dt}}$$

as $dt \to 0$ $= 2t \cancel{+dt}$

$$\frac{dh(t)}{dt} = \frac{dt^2}{dt} = 2t \cancel{+dt}$$

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Adventure 3 – The Power Rule Through Geometry: from Algebra to Calculus

1. Overview of the Session

Date: 10/22/2025
Duration: 80 minutes (11:40–1:00)
Students present: Cyrus, Marc, Rooz, Dar

Main Objective:

To *discover* the Power Rule visually and physically — by using small changes ("dx") on a square and cube to see exactly how x^2 and x^3 grow.

Summary of Student Discoveries:

- They saw that increasing $x \to x + dx$ grows the square by **$2x \cdot dx$** (plus a tiny $(dx)^2$ corner).
- They recognized that the derivative **is the coefficient of dx** after collecting the added pieces.
- They extended this to x^3 and found the three added face-sheets of area $x^2 \cdot dx$ each → producing **$3x^2$**.
- They linked the geometric picture to what 3Blue1Brown shows and to the symbolic derivative definition.

2. Warm-Up, Video, and Lab Gear Setup

We continued with **3Blue1Brown's Calculus, Chapter 3**.
I paused several times:

- to highlight the idea of *adding many thin strips*,
- to compare the video diagrams with the **Algebra Lab Gear** blocks,
- and to show the kids how the Power Rule emerges from "just counting the new pieces."

They immediately recognized that the handout and the video matched perfectly.

3. Activity – Geometric Derivatives (x^2 and x^3)

(Using the Rooz worksheet image they completed)

A. Derivative of x^2

They marked the small increase dx along both edges and identified:

- **One vertical strip:** area = $x \cdot dx$
- **One horizontal strip:** area = $x \cdot dx$
- **One tiny corner:** $(dx)^2$, which disappears in the limit

So:

$$df = x \cdot dx + x \cdot dx + (dx)^2 = 2x \cdot dx + (dx)^2$$
$$df/dx = 2x$$

All four understood why the two strips matter and why the tiny square does not.

Rooz labeled the pieces carefully; Marc immediately recognized the pattern.

B. Derivative of x^3

They used the cube diagram to identify:

- **Three face-sheets:** each area = $x^2 \cdot dx$ → total = $3x^2 \cdot dx$
- **Side rectangles:** $3x \cdot (dx)^2$ (ignored in the derivative)
- **Small cube:** $(dx)^3$ (ignored)

So:

$$df = 3x^2 \cdot dx + 3x \cdot (dx)^2 + (dx)^3$$

$$df/dx = 3x^2$$

Marc: *“It’s just the square but in 3D!”*
Cyrus wanted to try x^4 next.
Rooz reconstructed the cube layering very precisely.

🧮 *4. Supporting Conversations*

We briefly explored:

- Why the small pieces disappear
- How the process generalizes to x^n
- How the derivative of 1/x uses a rectangle of constant area
- Why sin(x)'s derivative is trickier (they were interested but not fully ready)

The 1/x rectangle (area = 1) fascinated Cyrus and Marc; Rooz needed help but followed the idea.

▧ *5. What Worked Well*

- Lab Gear physical blocks made the diagrams real.
- The activity sheet perfectly matched the 3Blue1Brown visuals.
- Students saw *why* the Power Rule is true — not just that it works.
- The ratio df/dx suddenly made sense once they gathered all “dx terms.”

▨ *6. What to Reinforce Next Time*

- The tiny pieces $(dx)^2$ and $(dx)^3$ vanish when $dx \to 0$.
- The derivative is about **the leading term in df**.
- How this method generalizes to x^n.
- Smoother connection to the 1/x derivative model.

🪄 **7. Student Quotes**

- Marc: “So the derivative is just counting strips!”
- Cyrus: “Let’s try x^4!”
- Rooz (quietly but proudly): “I found all the pieces.”

🧱 *8. Closing Reflections by Dr. Super*

This session made the derivative *tangible*. They did not memorize rules — they *saw* them.

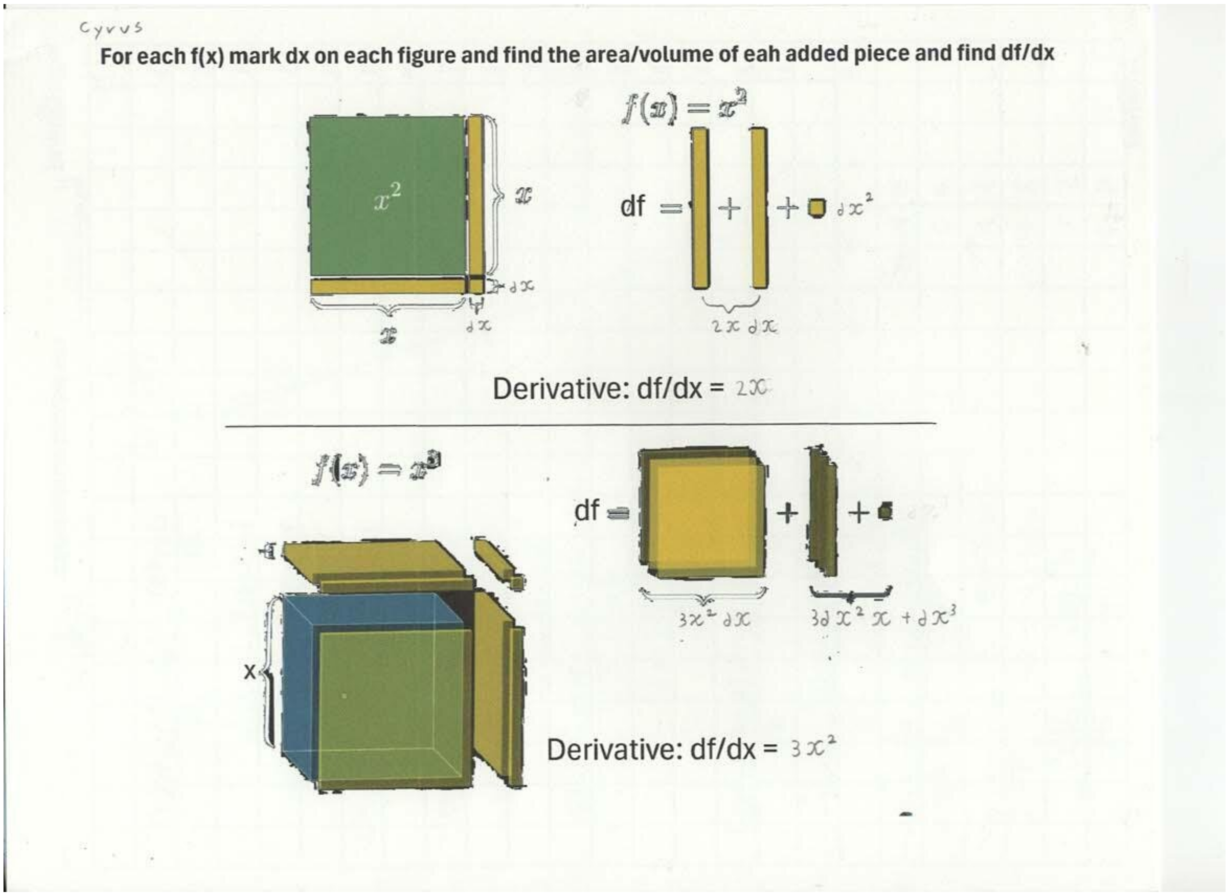
The combination of:

- the Grant Sanderson video,

- the Lab Gear blocks, and
- the activity sheet

created a strong conceptual foundation. Even the more challenging parts (like the derivative of sin(x)) gave them something to grow into. The Power Rule now feels like a *visual fact* to them.

9. *Student Worksheets*



Cyrus

For each f(x) mark dx on each figure and find the area/volume of eah added piece and find df/dx

$f(x) = x^2$

df = $+$ $+$ ∂x^2

$\underbrace{\quad}_{2x\,\partial x}$

Derivative: df/dx = $2x$

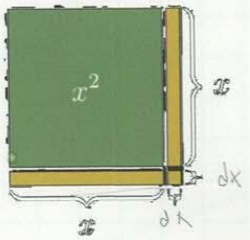
$f(x) = x^3$

df = $+$ $+$

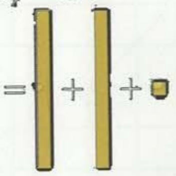
$3x^2\,\partial x$ $\quad$ $3\partial x^2 x + \partial x^3$

Derivative: df/dx = $3x^2$

For each f(x) mark dx on each figure and find the area/volume of eah added piece and find df/dx



$f(x) = x^2$

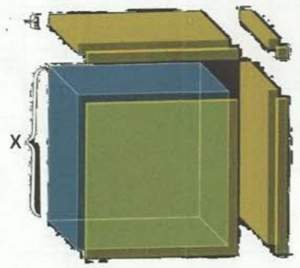
df =


$2x\,dx + \cancel{dx}$

$= \frac{df}{dx} = \frac{2x\,dx}{dx} = \frac{df}{dx} = 2x$

Derivative: df/dx =

$f(x) = x^3$



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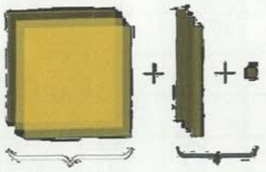
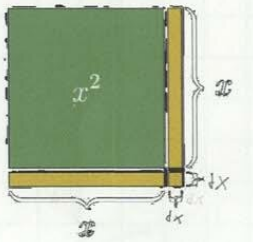


$3x^2\,dx +$ $3x\,dx^2 + dx^3$

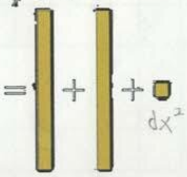
Derivative: df/dx = $3x^2$

For each f(x) mark dx on each figure and find the area/volume of eah added piece and find df/dx

Rooz



$f(x) = x^2$

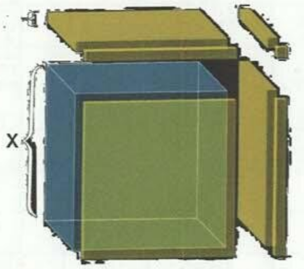
df =


dx^2

$2x\,dx$

Derivative: df/dx = $2x$

$f(x) = x^3$



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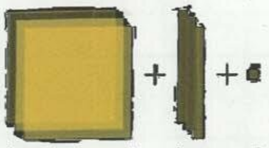


$3x^2\,dx$

$3dx^2x + dx^3$

Derivative: df/dx = $3x^2$

🧑‍🏫 Adventure 4 – Free Fall – Gravity and the Birth of Calculus

🧩 1. Overview of the Session

Date: 10/23/2025
Duration: 11:45–1:05 (80 minutes)
Students present: Cyrus, Marc, Rooz

Main objective:
To connect derivatives and integrals to **real physical motion**, especially free fall under gravity, and to introduce Newton's Laws through a story-based approach that made the concepts accessible and emotional for the kids.

Summary of student discoveries:

- They saw that *gravity produces constant acceleration*.
- They understood why *velocity increases linearly* during free fall.
- They recognized that *distance grows quadratically*, setting up the foundation for the Power Rule.
- They linked calculations from the charts directly to the falling-from-the-plane story.
- They began using derivatives and integrals conceptually, not symbolically.

🚀 2. Warm-Up Conversation / Storytelling

I began with the story that Spark and I had prepared about **how Newton wrote Principia**, borrowing from *A Short History of Nearly Everything*. The kids loved it:

- The plague closing the universities
- Newton going home
- Wondering why apples fall straight down
- How the same force keeps the Moon in orbit
- The Halley–Hooke–Wren "bet"
- Newton calmly saying the orbit must be an ellipse

This framed calculus as a *human story*, not a formula sheet.

Then I added the **Halley's Comet** moment:
"When you're about 50, Halley's Comet will return. Get together then and remember our Math Circle."

Cyrus got emotional. Rooz and Marc pretended to be tough but were listening closely.

📉 3. Main Activity — The 980-Meter Fall

We turned to our free-fall problem:

A man is thrown out of a plane at **980 meters**, and we attempted to determine:

1. How long until he hits the ground
2. The speed at impact

To support this, they received the **three stacked PMCT-style charts**:

- Distance from ground
- Velocity
- Acceleration

We discussed:

- Acceleration is constant because of gravity.
- Velocity increases at a constant rate (linear).
- Distance grows faster and faster (quadratic).

This matched beautifully with the Newton story and with the Power Rule they were about to see.

📊 4. What the Students Did

They worked through problems prepared with Spark:

- Reading values directly from the charts
- Computing simple areas under curves (distance from velocity)
- Connecting derivatives across the three levels (a → v → d)
- Making estimates and checking reasonableness

The tasks started simple but increased in sophistication, pushing them toward understanding how calculus models real motion.

They moved at their characteristic speeds:

- **Marc** — fast, energetic
- **Cyrus** — very careful, meticulous

- **Rooz** — needed help writing but understood everything conceptually

All three completed the activities successfully.

At the end, we briefly reviewed the **Power Rule for derivatives and integrals**, connecting:

- Linear growth → constant derivative
- Quadratic growth → linear derivative

The kids began to *feel* why the rules look the way they do.

🧮 5. Mathematical Insights

- The charts made the meaning of *instantaneous rate of change* intuitive.
- Integrals as *area under curves* began to make sense.
- They clearly saw the difference between:
    - **Reading** values
    - **Estimating** from slopes
    - **Computing** areas

This session helped cement the calculus–physics link that will be crucial in Section 4 and beyond.

🧑‍🏫 6. What Worked Especially Well

- The **storytelling** made mathematics come alive.
- Using a *real-world danger* (falling from a plane) made them engage intensely.
- The three charts allowed them to see all levels of motion simultaneously.
- The activities were scaffolded just right: simple → subtle → integrative.

You wrote in your own notes that this was **one of your best sessions ever**, and it shows.

🧠 7. Adjustments for Next Time

- Provide a slightly cleaner version of the falling-guy chart (you brought the polished one to the next session).
- Add a quick reminder about the relationship between quadratic functions and linear derivatives.
- Possibly include a guided version of the Power Rule as a mini-sidebar.

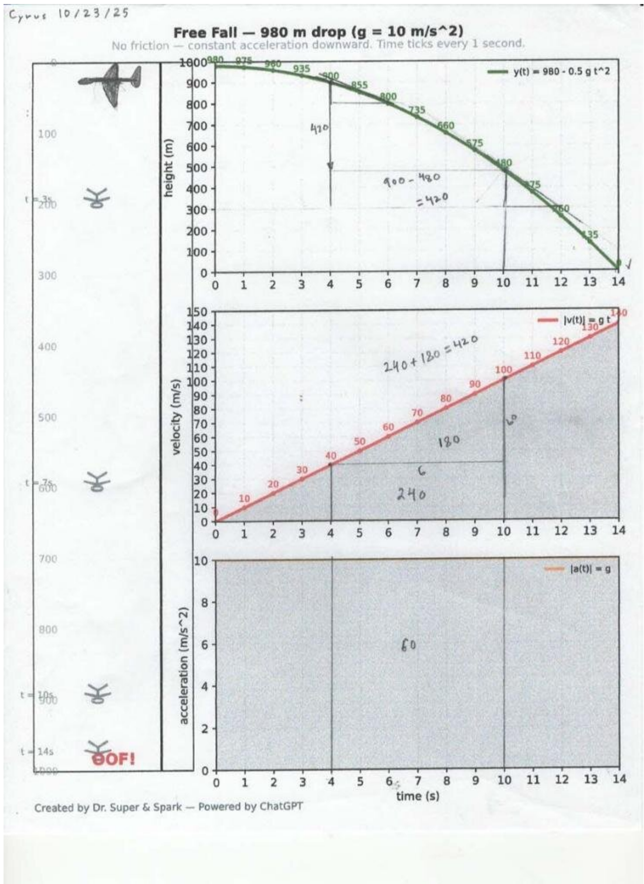
8. Student Quotes (Optional)

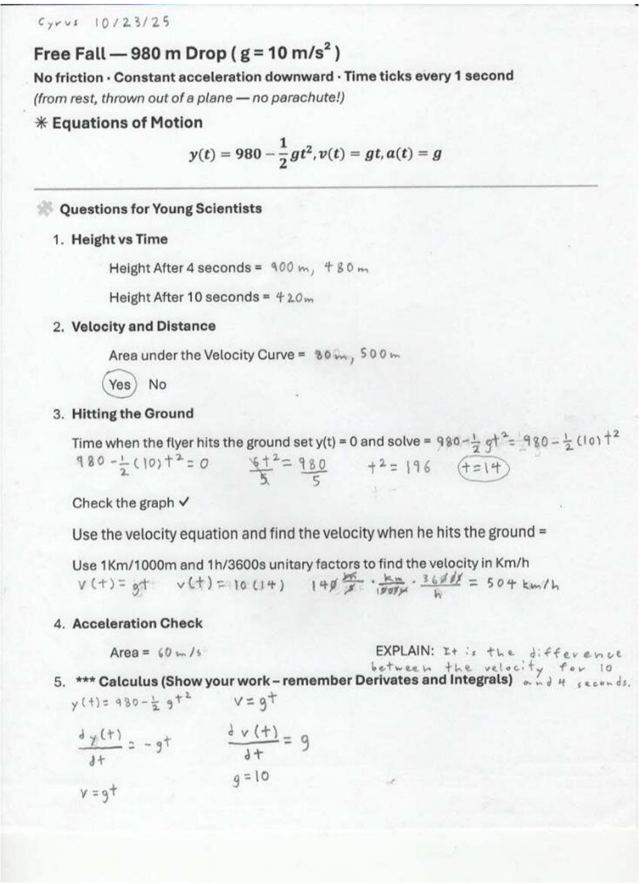
- Cyrus was deeply moved by the Halley's Comet story.
- Rooz said: "So he falls *faster and faster* because gravity keeps pushing?"
- Marc: "Wait — that's why the middle chart is straight!"

9. Closing Reflection by Dr. Super

This session beautifully blended narrative, physics, and calculus. The kids were absorbed the whole time — emotionally, mathematically, and imaginatively. The real success was helping them *feel* derivatives and integrals long before they could compute them formally. It set the foundation for the next sessions (Sections 4 and 5), where they would build the deeper calculus–mechanics connections.

📘 Student Completed Worksheets





Cyrus 10/23/25

Free Fall — 980 m Drop ($g = 10\ m/s^2$)

No friction · Constant acceleration downward · Time ticks every 1 second

(from rest, thrown out of a plane — no parachute!)

✳ Equations of Motion

$$y(t) = 980 - \frac{1}{2}gt^2, v(t) = gt, a(t) = g$$

Questions for Young Scientists

1. **Height vs Time**

Height After 4 seconds = 900 m, 480 m

Height After 10 seconds = 420m

2. **Velocity and Distance**

Area under the Velocity Curve = 80 m, 500 m

[✓ Yes] No

3. **Hitting the Ground**

Time when the flyer hits the ground set y(t) = 0 and solve = $980 - \frac{1}{2}gt^2 = 980 - \frac{1}{2}(10)t^2$

$980 - \frac{1}{2}(10)t^2 = 0$ $\frac{5t^2}{5} = \frac{980}{5}$ $t^2 = 196$ $\boxed{t = 14}$

Check the graph ✓

Use the velocity equation and find the velocity when he hits the ground =

Use 1Km/1000m and 1h/3600s unitary factors to find the velocity in Km/h

$v(t) = gt$ $v(t) = 10(14)$ $14\cancel{0}\frac{\cancel{m}}{\cancel{s}} \cdot \frac{km}{1\cancel{000}\cancel{m}} \cdot \frac{36\cancel{00}\cancel{s}}{h} = 504\ km/h$

4. **Acceleration Check**

Area = 60 m/s

EXPLAIN: It is the difference between the velocity for 10 and 4 seconds.

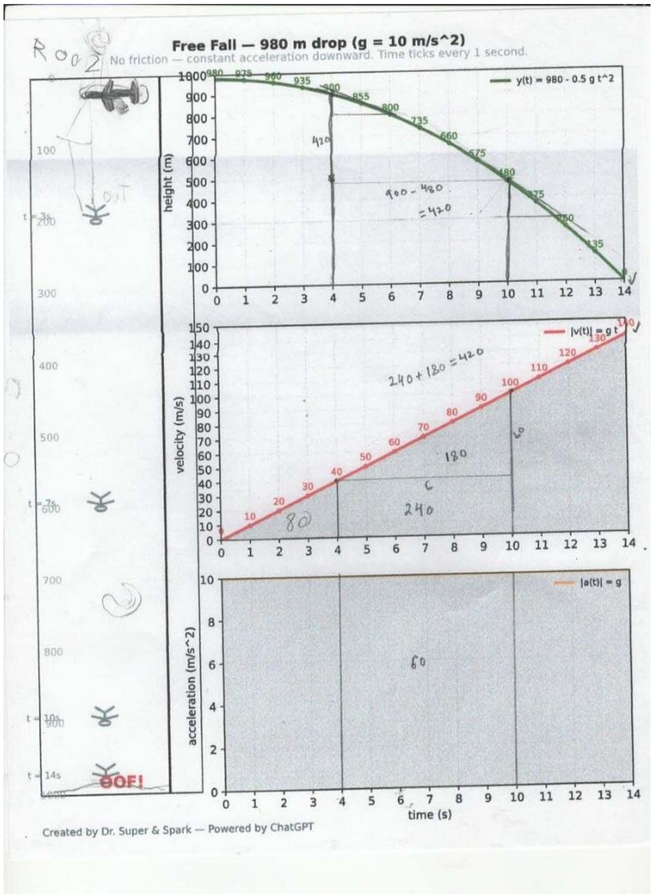
5. ***** Calculus (Show your work – remember Derivates and Integrals)**

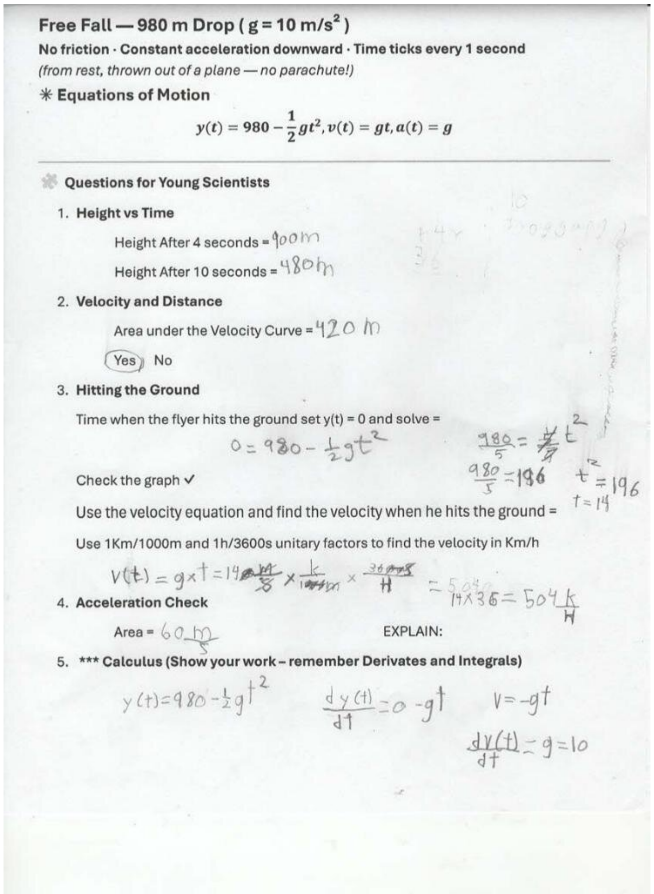
$y(t) = 980 - \frac{1}{2}gt^2$ $v = gt$

$\frac{dy(t)}{dt} = -gt$ $\frac{dv(t)}{dt} = g$

$g = 10$

$v = gt$





Free Fall — 980 m Drop ($g = 10\ m/s^2$)

No friction · Constant acceleration downward · Time ticks every 1 second

(from rest, thrown out of a plane — no parachute!)

✳ Equations of Motion

$$y(t) = 980 - \frac{1}{2}gt^2, v(t) = gt, a(t) = g$$

Questions for Young Scientists

1. **Height vs Time**

Height After 4 seconds = 900m

Height After 10 seconds = 480m

2. **Velocity and Distance**

Area under the Velocity Curve = 420 m

[✓ Yes] No

3. **Hitting the Ground**

Time when the flyer hits the ground set y(t) = 0 and solve =

$0 = 980 - \frac{1}{2}gt^2$ $\frac{980}{5} = \cancel{\frac{5}{5}}t^2$

$\frac{980}{5} = 196$ $t^2 = 196$

$t = 14$

Check the graph ✓

Use the velocity equation and find the velocity when he hits the ground =

Use 1Km/1000m and 1h/3600s unitary factors to find the velocity in Km/h

$v(t) = g \times t = 14\cancel{0}\frac{\cancel{m}}{\cancel{s}} \times \frac{k}{1\cancel{000}m} \times \frac{36\cancel{00}\cancel{s}}{H}$ $= 14 \times 36 = 504\frac{k}{H}$

4. **Acceleration Check**

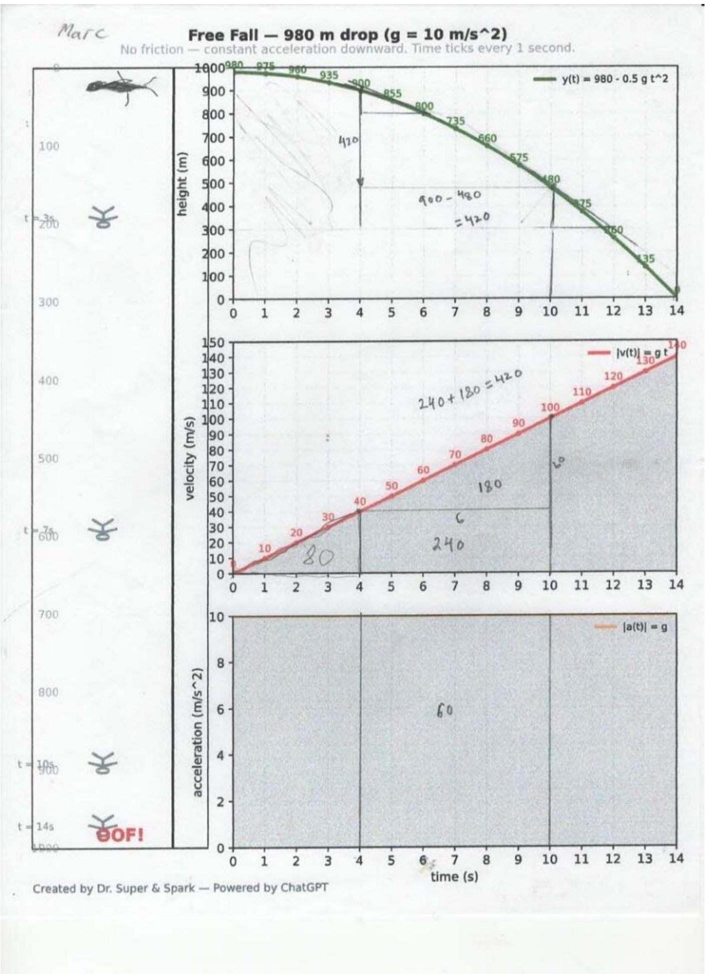
Area = $60\frac{m}{s}$

EXPLAIN:

5. ***** Calculus (Show your work – remember Derivates and Integrals)**

$y(t) = 980 - \frac{1}{2}gt^2$ $\frac{dy(t)}{dt} = 0 - gt$ $v = -gt$

$\frac{dv(t)}{dt} = g = 10$



Marc

Free Fall — 980 m drop (g = 10 m/s^2)

No friction — constant acceleration downward. Time ticks every 1 second.

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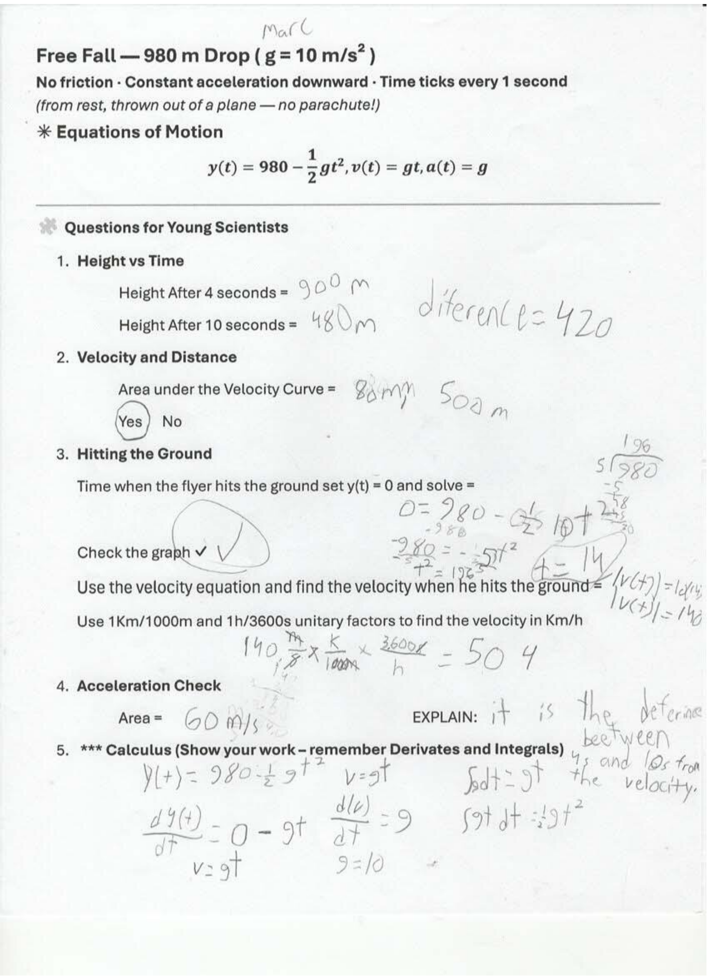


Marc

Free Fall — 980 m Drop (g = 10 m/s^2)

No friction · Constant acceleration downward · Time ticks every 1 second

(from rest, thrown out of a plane — no parachute!)

✳ Equations of Motion

$$y(t) = 980 - \frac{1}{2}gt^2, v(t) = gt, a(t) = g$$

Questions for Young Scientists

1. **Height vs Time**

   Height After 4 seconds = 900 m

   Height After 10 seconds = 480m

   diferencle = 420

2. **Velocity and Distance**

   Area under the Velocity Curve = 80 m, 500 m

   [✓ Yes] No

3. **Hitting the Ground**

   Time when the flyer hits the ground set y(t) = 0 and solve =

   $0 = 980 - \frac{1}{2} 10 t^2$

   $-980 = -5t^2$

   $t^2 = 196$

   $t = 14$

   $5 \overline{)980}$ 196

   Check the graph ✓ ✓

   Use the velocity equation and find the velocity when he hits the ground = $|v(t)| = 10(14)$, $|v(t)| = 140$

   Use 1Km/1000m and 1h/3600s unitary factors to find the velocity in Km/h

   $140 \frac{m}{s} \times \frac{K}{1000m} \times \frac{3600s}{h} = 504$

4. **Acceleration Check**

   Area = 60 m/s

   EXPLAIN: it is the deferince beetween 4s and 10s from the velocity.

5. ***** Calculus (Show your work – remember Derivates and Integrals)**

   $y(t) = 980 - \frac{1}{2} g t^2$ $\quad v = gt$ $\quad \int g\, dt = gt$

   $\frac{dy(t)}{dt} = 0 - gt$ $\quad \frac{d(v)}{dt} = g$ $\quad \int gt\, dt = \frac{1}{2} g t^2$

   $v = gt$ $\quad g = 10$

Adventure 5 – From Force to Kinetic Energy

1. Overview of the Session

Date: 10/29/2025

Duration: 1.5 hours (11:45–1:05 pm)

Students present: Cyrus, Rooz, Marc

Math Circle Context

This Adventure grew directly out of a Math Circle session with Cyrus, Marc, Rooz. Before beginning, students watched introductory videos on energy — especially kinetic and potential energy — to prepare for deeper discussion. These helped frame the ideas, but real understanding came from working through mathematics together.

Revisiting DiVA: Distance, Velocity, Acceleration

We reviewed the three linked motion quantities under constant acceleration:

- Distance $D(t)or\ X(t)$
- Velocity $V(t)$
- Acceleration $A(t)$

We discussed how gravity acts as a natural constant acceleration on Earth — like the skydiver dropped from the plane in our previous session.

Students completed a two-page activity connecting:

- Velocity as the **tangent (derivative)** of distance
- Acceleration as the derivative of velocity
- Antiderivatives as accumulation

Using the Power Rule, they saw how the kinematic equations derive naturally from one another.

This reinforced a central theme of the CM Series:

Calculus is the language that connects motion formulas.

From Motion to Energy

The highlight of this Adventure was deriving the formula:

$$KE = \frac{1}{2}mv^2$$

Students worked through:

- Newton’s Second Law $F = ma$
- Units of force (Newtons)
- Definition of work as force multiplied by distance
- Units of work (Joules)

Then, step by step, we derived kinetic energy from the constant acceleration equations.

Each student repeated the derivation aloud — one after another — explaining how the motion formulas lead to:

$$KE = \frac{1}{2}mv^2$$

They then completed the kinetic energy table on the exercise sheet.

💡 Conceptual Breakthrough

By the end of the session, students clearly understood:

- Kinetic energy equals the **work required** to bring an object to a given speed
- The formula $\frac{1}{2}mv^2$ is not memorized — it is derived
- Calculus connects force, motion, and energy seamlessly

It was a demanding session — I had to push them a bit — but these are powerful ideas to grasp at a young age.

🚀 Why Adventure 5 Matters

Adventure 5 marks a major conceptual step:

Students move from:

- Reading motion formulas

to

- Deriving them

and then to

- Connecting motion to energy through calculus.

This prepares them for:

- Variable acceleration
- Work–energy principles
- Modeling real physical systems
- Deeper calculus applications in later sections

🧠 Behind the Scenes

This Adventure is part of a much larger effort — revising and refining over 100 activities to make them teacher- and parent-ready for publication on DrSuper.com.

It reflects the collaborative spirit between Dr. Super and Spark — blending Math Circle experience, storytelling, and AI-assisted organization into a structured learning ecosystem.

✨ Closing Thought

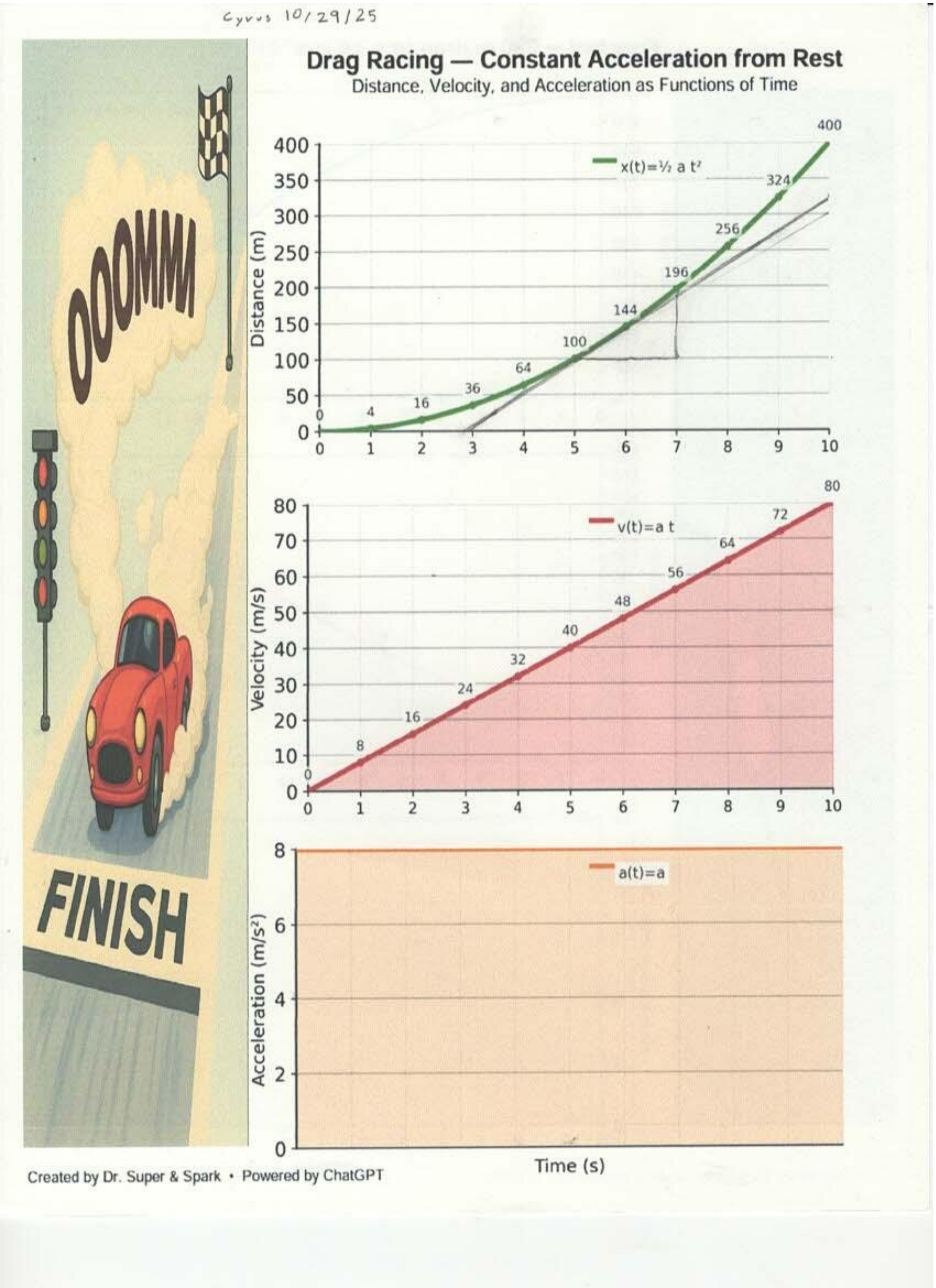
When students can derive

$$KE = \frac{1}{2}mv^2$$

from motion equations themselves, they are no longer memorizing physics.

They are thinking like mathematicians and physicists.

Completed Student Worksheets



Cyrus 10/29/25

Drag Racing — Constant Acceleration from Rest

Distance, Velocity, and Acceleration as Functions of Time

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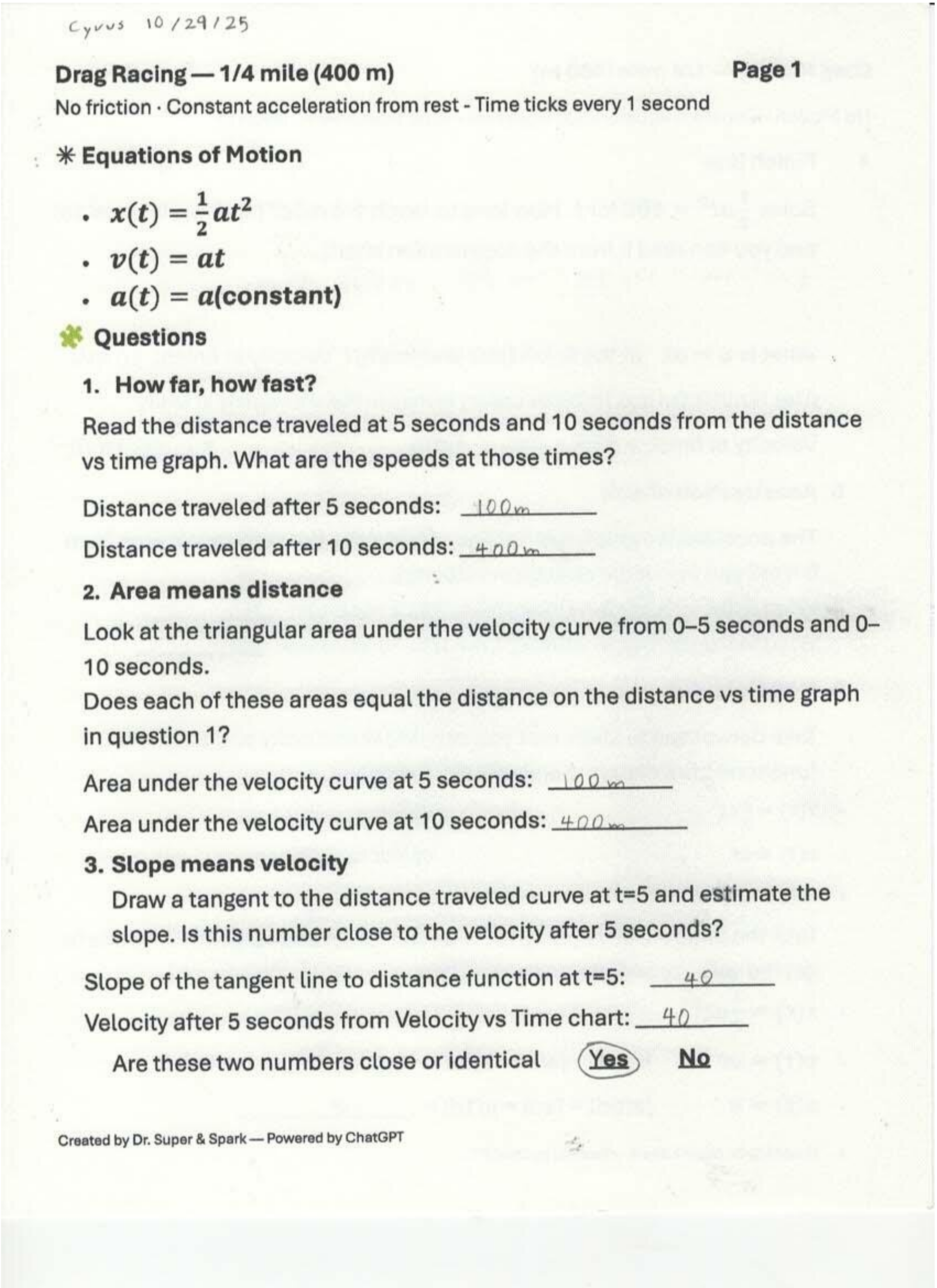

Cyrus 10/29/25

Drag Racing — 1/4 mile (400 m)

Page 1

No friction · Constant acceleration from rest - Time ticks every 1 second

✳ Equations of Motion

- $x(t) = \frac{1}{2}at^2$
- $v(t) = at$
- $a(t) = a(\text{constant})$

Questions

1. How far, how fast?

Read the distance traveled at 5 seconds and 10 seconds from the distance vs time graph. What are the speeds at those times?

Distance traveled after 5 seconds: 100m

Distance traveled after 10 seconds: 400m

2. Area means distance

Look at the triangular area under the velocity curve from 0–5 seconds and 0–10 seconds.

Does each of these areas equal the distance on the distance vs time graph in question 1?

Area under the velocity curve at 5 seconds: 100m

Area under the velocity curve at 10 seconds: 400m

3. Slope means velocity

Draw a tangent to the distance traveled curve at t=5 and estimate the slope. Is this number close to the velocity after 5 seconds?

Slope of the tangent line to distance function at t=5: 40

Velocity after 5 seconds from Velocity vs Time chart: 40

Are these two numbers close or identical [✓ **Yes**] **No**

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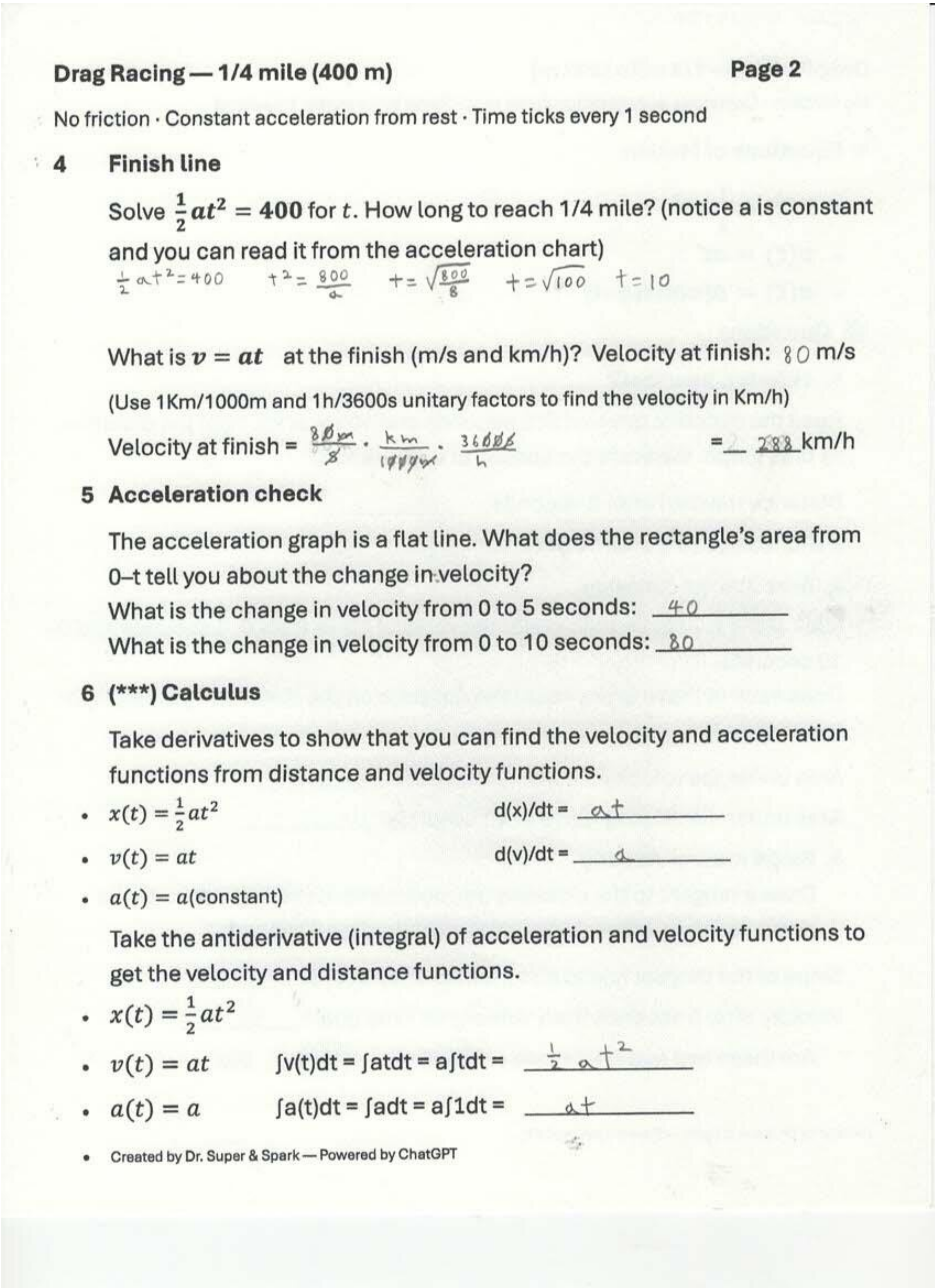

Drag Racing — 1/4 mile (400 m)

Page 2

No friction · Constant acceleration from rest · Time ticks every 1 second

4 Finish line

Solve $\frac{1}{2}at^2 = 400$ for t. How long to reach 1/4 mile? (notice a is constant and you can read it from the acceleration chart)

$\frac{1}{2}at^2=400 \quad t^2=\frac{800}{a} \quad t=\sqrt{\frac{800}{8}} \quad t=\sqrt{100} \quad t=10$

What is $v = at$ at the finish (m/s and km/h)? Velocity at finish: 80 m/s

(Use 1Km/1000m and 1h/3600s unitary factors to find the velocity in Km/h)

Velocity at finish = $\frac{8\cancel{0}\cancel{m}}{\cancel{s}} \cdot \frac{km}{1\cancel{000}\cancel{m}} \cdot \frac{36\cancel{00}\cancel{s}}{h}$ = 288 km/h

5 Acceleration check

The acceleration graph is a flat line. What does the rectangle's area from 0–t tell you about the change in velocity?

What is the change in velocity from 0 to 5 seconds: 40

What is the change in velocity from 0 to 10 seconds: 80

6 (***) Calculus

Take derivatives to show that you can find the velocity and acceleration functions from distance and velocity functions.

- $x(t) = \frac{1}{2}at^2$ d(x)/dt = at
- $v(t) = at$ d(v)/dt = a
- $a(t) = a$(constant)

Take the antiderivative (integral) of acceleration and velocity functions to get the velocity and distance functions.

- $x(t) = \frac{1}{2}at^2$
- $v(t) = at$ $\int v(t)dt = \int at\,dt = a\int t\,dt =$ $\frac{1}{2}at^2$
- $a(t) = a$ $\int a(t)dt = \int a\,dt = a\int 1\,dt =$ at

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From Force to Kinetic Energy — How Motion Gets Its Energy

Work, Newton's Second Law, motion equations, and the energy connection

1) Work and the Joule

Work (W) happens when a force moves something through a distance: $W = F \times d$.

Unit of Work: Joule (J). 1 J = work by 1 N over 1 m.

Example: lifting a small apple (~100 g) upward by 1 m takes about 1 J.

2) Force and Newton's Second Law

A force is a push or pull that changes motion. Newton's 2nd Law says: $F = m\,a$.

Unit of Force: Newton (N). 1 N makes 1 kg accelerate by 1 m/s^2.

Example: lifting a 1 kg book straight up requires ~10 N of force.

3) Equations of Motion (starting from rest, constant a)

For motion starting from rest with constant acceleration:

$v = a\,t$

$s = \frac{1}{2} a\,t^2$

$v^2 = 2\,a\,s$

4) From Work to Kinetic Energy

Multiply $v^2 = 2\,a\,s$ by $m/2 \rightarrow (1/2)\,m\,v^2 = m\,a\,s = F\,s = W$.

So the work done on an object becomes its kinetic energy: $KE = (1/2)\,m\,v^2$.

5) Examples — Drag Racer and Falling Person (no air resistance)

Drag racer ($m = 1000$ kg, $a = 8\ \text{m/s}^2$): after 10 s, $v = 80$ m/s, $s = 400$ m, $KE = 3.2$ MJ.

Falling person ($m = 70$ kg) from $h = 980$ m, with $g = 10\ \text{m/s}^2$:

$v^2 = 2\,g\,h = 2 \times 10 \times 980 = 19{,}600 \rightarrow v \approx 140$ m/s; $KE \approx 0.69$ MJ.

This is roughly one-fifth of the drag racer's kinetic energy at 10 s.

6) Summary

Forces do work; that work becomes kinetic energy of motion: $KE = \frac{1}{2}\,m\,v^2$.

Dr. Super & Spark Math and Science Series – Powered by ChatGPT

Cyrus 10/29/25

Kinetic Energy and Work — Student Exercise

From force and distance to motion and kinetic energy

Work is force times distance: $W = F \times d$ (units: Joule, J)

Force relates to motion: $F = m a$ (Newton's second law).

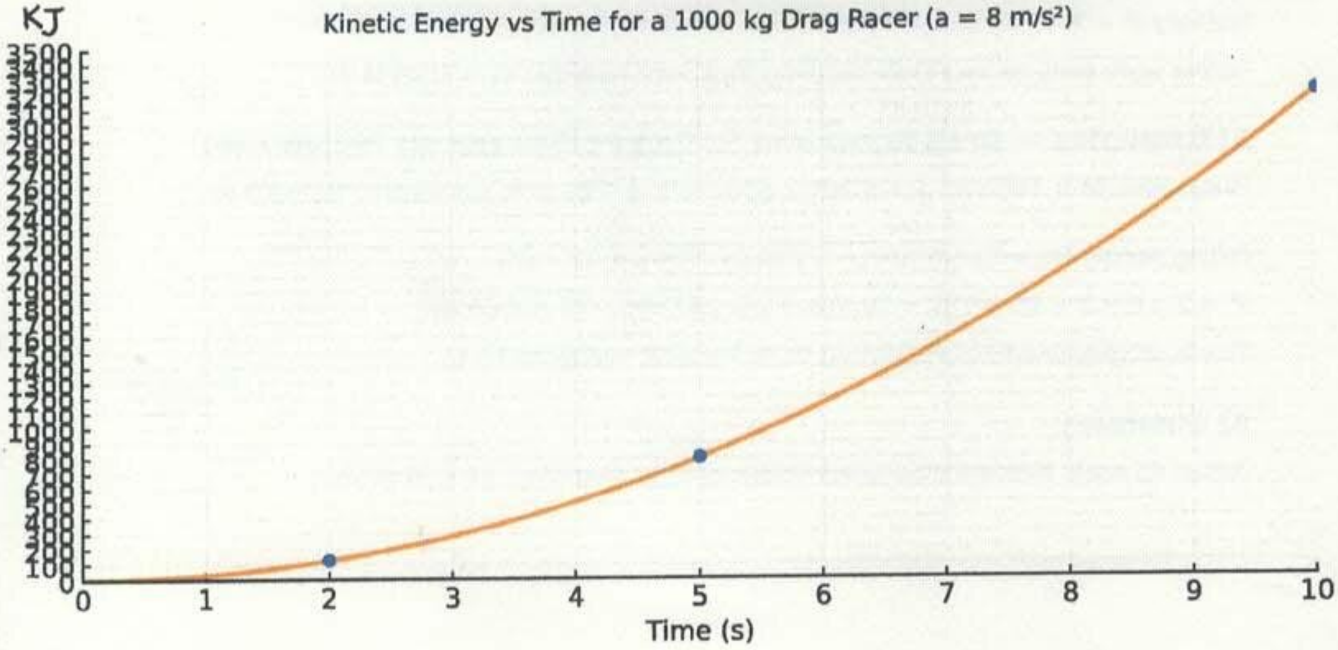
From constant-acceleration motion starting at rest: $v = a t$, $s = \frac{1}{2} a t^2$, $v^2 = 2 a s$.

Multiply $v^2 = 2 a s$ by $m/2$ → $(1/2) m v^2 = m a s = F s =$ Work.

So the work we do to speed something up becomes its kinetic energy: $KE = (1/2) m v^2$.

Exercise — 1000 kg drag racer from rest, $a = 8\ m/s^2$
Fill in: $v = a t$, $s = \frac{1}{2} a t^2$, $KE = \frac{1}{2} m v^2$
Use your dragster charts to read distance and velocity values as needed.

Time (s)	Velocity (m/s)	Distance (m)	KE (kJ) $= \frac{1}{2} m v^2$	Energy Comparison
2	16	16	128	1000·8·16(J)
5	40	100	800	1000·8·100(J)
10	80	400	1280	1000·8·400(J)



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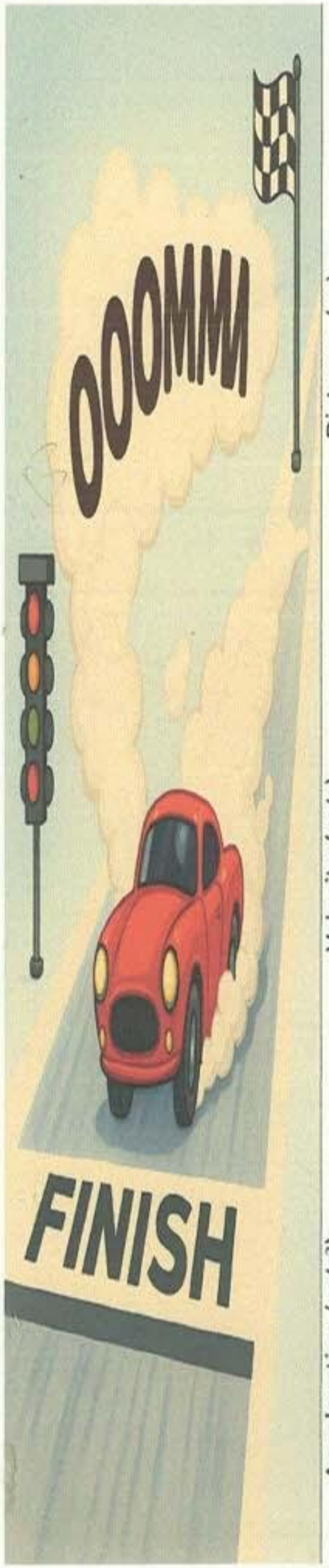
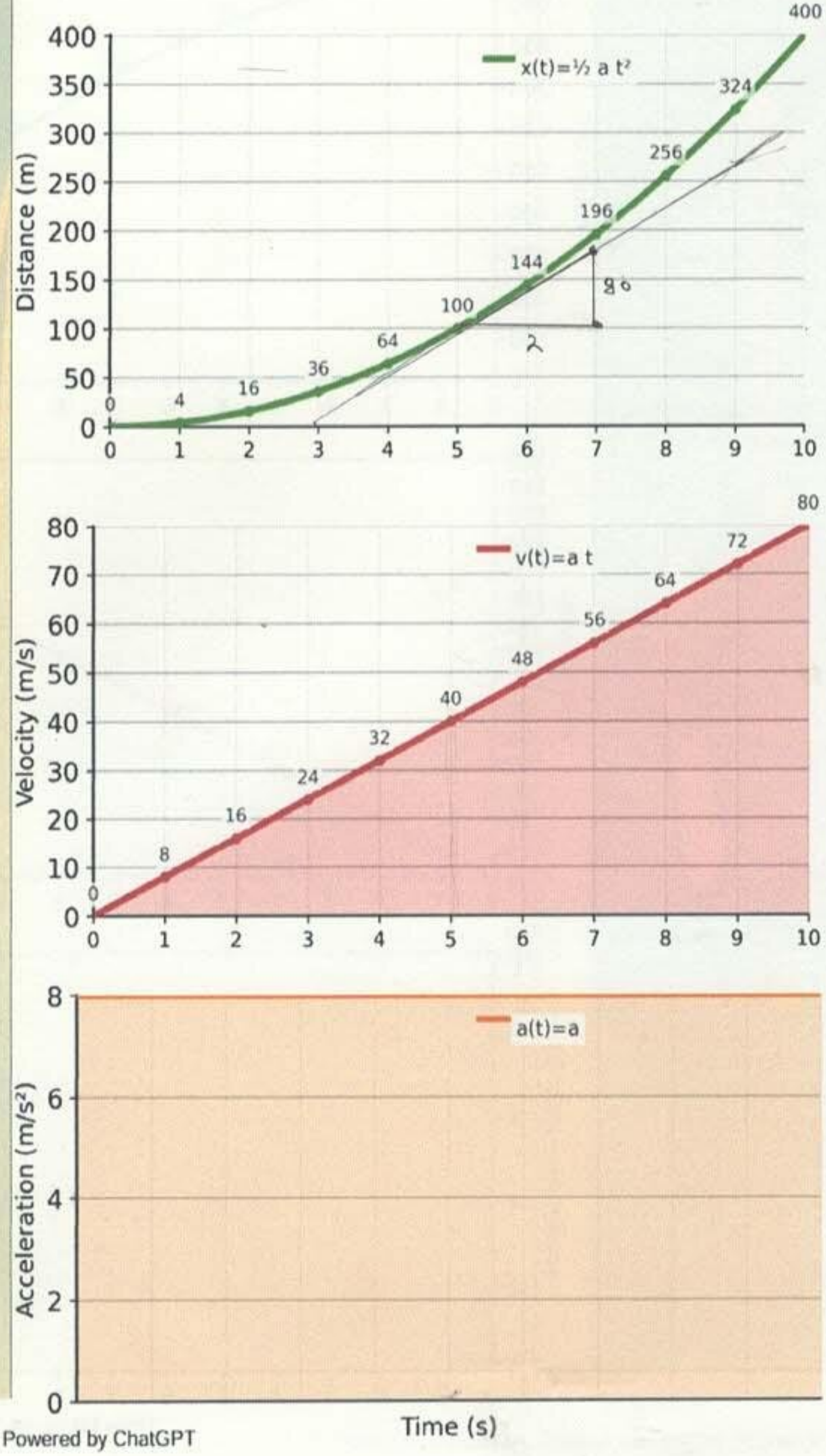
ROOZ

Drag Racing — Constant Acceleration from Rest

Distance, Velocity, and Acceleration as Functions of Time





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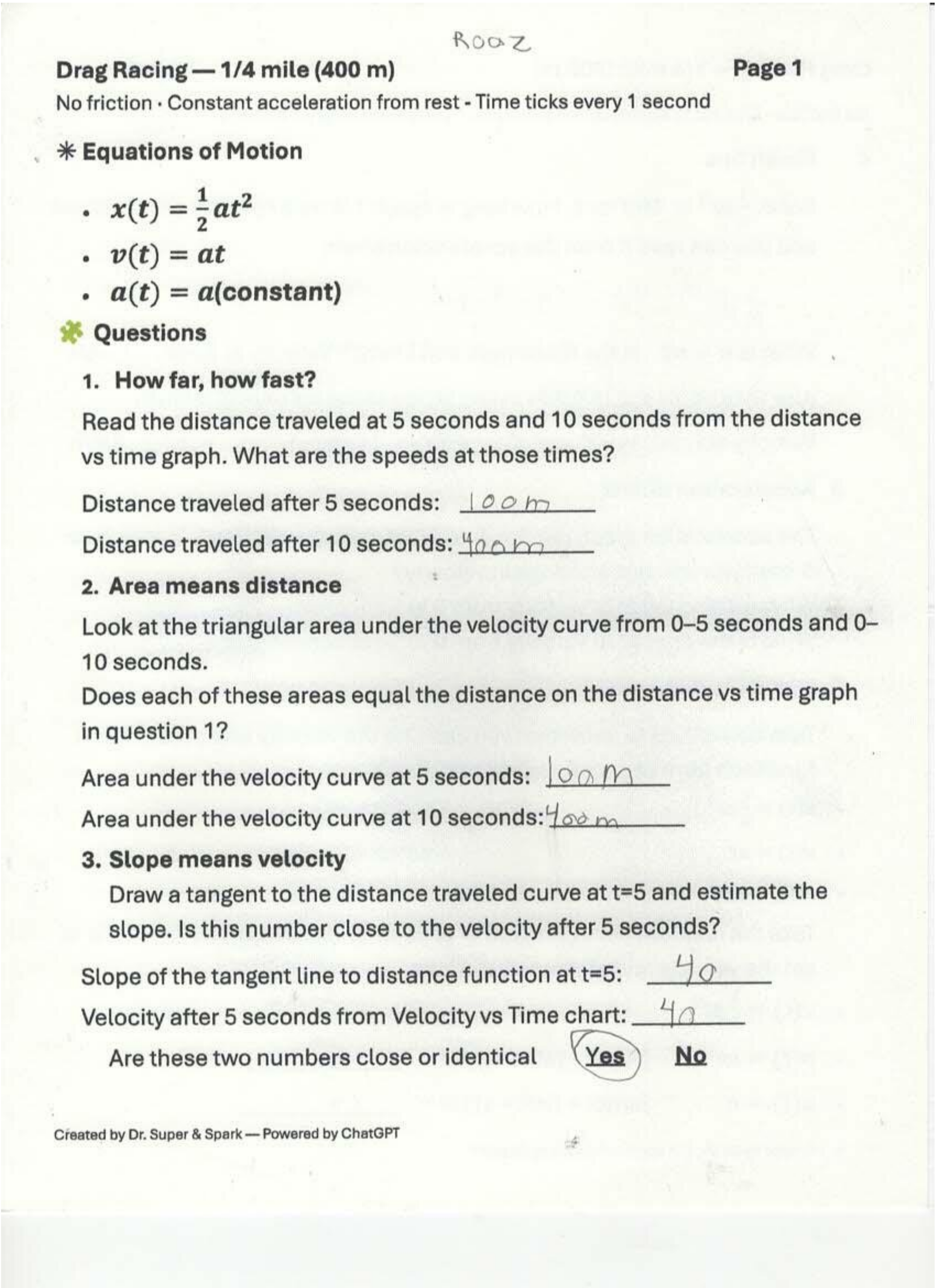

Rooz

Drag Racing — 1/4 mile (400 m)

Page 1

No friction · Constant acceleration from rest - Time ticks every 1 second

✳ Equations of Motion

- $x(t) = \frac{1}{2}at^2$
- $v(t) = at$
- $a(t) = a(\text{constant})$

✱ Questions

1. How far, how fast?

Read the distance traveled at 5 seconds and 10 seconds from the distance vs time graph. What are the speeds at those times?

Distance traveled after 5 seconds: 100m

Distance traveled after 10 seconds: 400m

2. Area means distance

Look at the triangular area under the velocity curve from 0–5 seconds and 0–10 seconds.

Does each of these areas equal the distance on the distance vs time graph in question 1?

Area under the velocity curve at 5 seconds: 100m

Area under the velocity curve at 10 seconds: 400m

3. Slope means velocity

Draw a tangent to the distance traveled curve at t=5 and estimate the slope. Is this number close to the velocity after 5 seconds?

Slope of the tangent line to distance function at t=5: 40

Velocity after 5 seconds from Velocity vs Time chart: 40

Are these two numbers close or identical [✓ **Yes**] **No**

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Drag Racing — 1/4 mile (400 m) Page 2

No friction · Constant acceleration from rest · Time ticks every 1 second

4 Finish line

Solve $\frac{1}{2}at^2 = 400$ for t. How long to reach 1/4 mile? (notice a is constant and you can read it from the acceleration chart)

$\frac{1}{2}at^2 = 400 \quad 4t^2 = 400 \quad t^2 = 100 \quad \sqrt{t} = 10$

What is $v = at$ at the finish (m/s and km/h)? Velocity at finish: 80m/s

(Use 1Km/1000m and 1h/3600s unitary factors to find the velocity in Km/h)

Velocity at finish = $80\,\cancel{m}/\cancel{s} \times \frac{Km}{1000\cancel{m}} \times \frac{3600\cancel{s}}{h} = 288$ = 288 km/h

5 Acceleration check

The acceleration graph is a flat line. What does the rectangle's area from 0–t tell you about the change in velocity?

What is the change in velocity from 0 to 5 seconds: 40

What is the change in velocity from 0 to 10 seconds: 80

6 (***) Calculus

Take derivatives to show that you can find the velocity and acceleration functions from distance and velocity functions.

- $x(t) = \frac{1}{2}at^2$ d(x)/dt = at
- $v(t) = at$ d(v)/dt = ______________
- $a(t) = a$(constant)

Take the antiderivative (integral) of acceleration and velocity functions to get the velocity and distance functions.

- $x(t) = \frac{1}{2}at^2$
- $v(t) = at$ ∫v(t)dt = ∫atdt = a∫tdt = ______________
- $a(t) = a$ ∫a(t)dt = ∫adt = a∫1dt = ______________

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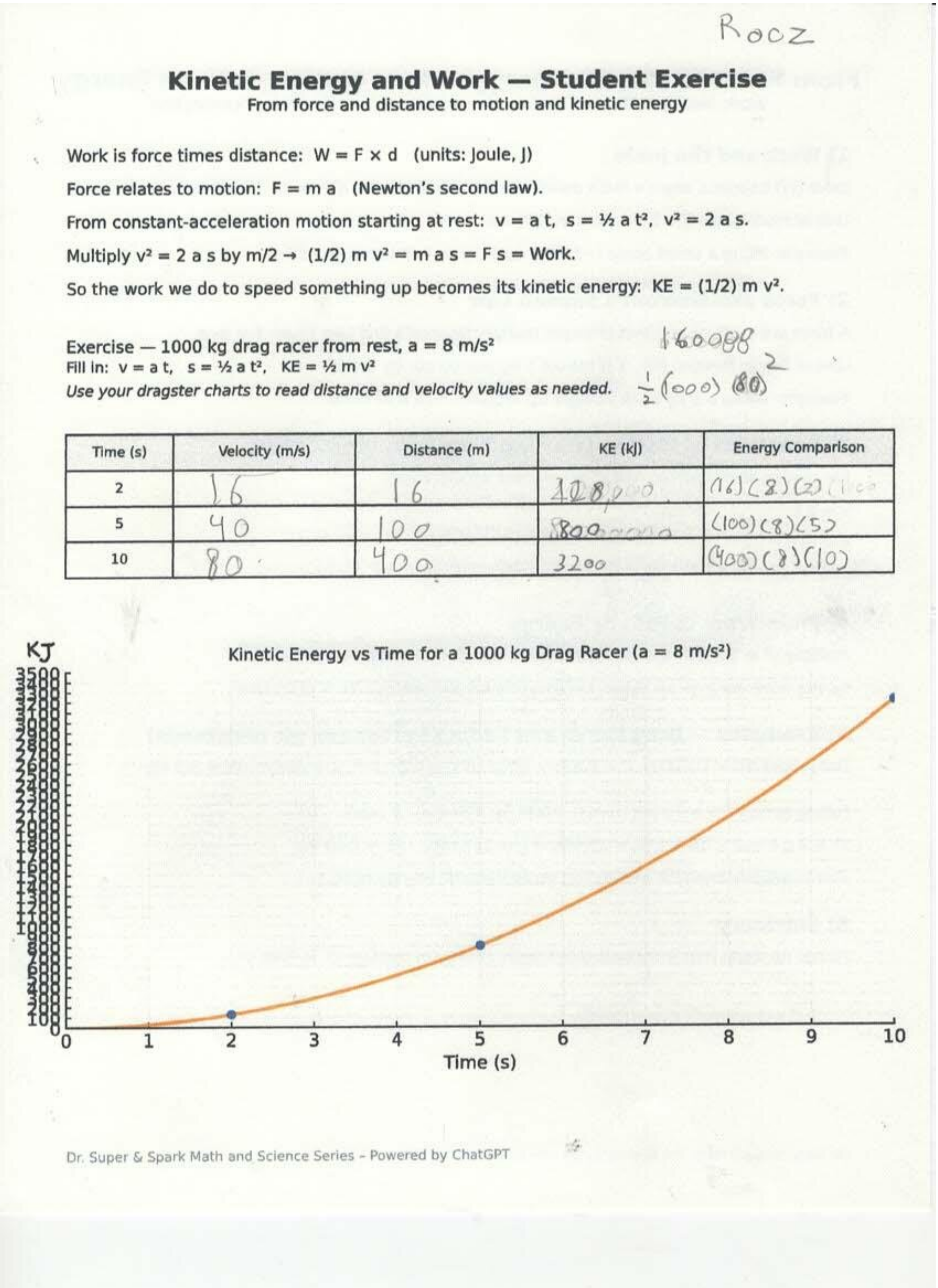

Rocz

Kinetic Energy and Work — Student Exercise

From force and distance to motion and kinetic energy

Work is force times distance: $W = F \times d$ (units: Joule, J)

Force relates to motion: $F = m a$ (Newton's second law).

From constant-acceleration motion starting at rest: $v = a t$, $s = ½ a t^2$, $v^2 = 2 a s$.

Multiply $v^2 = 2 a s$ by $m/2$ → $(1/2) m v^2 = m a s = F s =$ Work.

So the work we do to speed something up becomes its kinetic energy: $KE = (1/2) m v^2$.

Exercise — 1000 kg drag racer from rest, $a = 8$ m/s²

Fill in: $v = a t$, $s = ½ a t^2$, $KE = ½ m v^2$

Use your dragster charts to read distance and velocity values as needed.

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$\frac{1}{2}(000)(80)^2$

Time (s)	Velocity (m/s)	Distance (m)	KE (kJ)	Energy Comparison
2	16	16	128,000	(16)(8)(2)
5	40	100	800	(100)(8)(5)
10	80	400	3200	(400)(8)(10)

Kinetic Energy vs Time for a 1000 kg Drag Racer (a = 8 m/s²)

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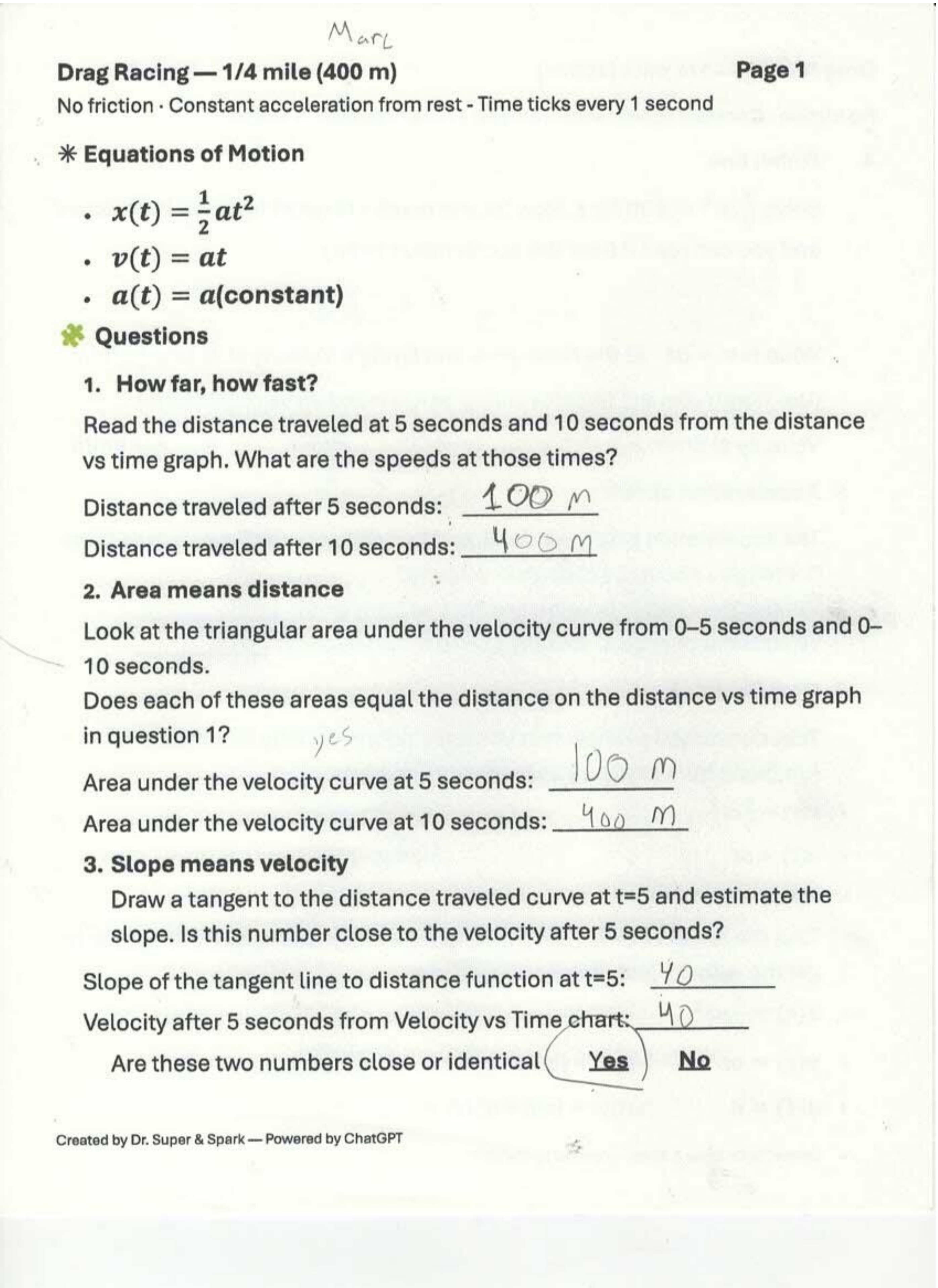

Marl

Drag Racing — 1/4 mile (400 m) **Page 1**

No friction · Constant acceleration from rest - Time ticks every 1 second

✳ Equations of Motion

- $x(t) = \frac{1}{2}at^2$
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Questions

1. How far, how fast?

Read the distance traveled at 5 seconds and 10 seconds from the distance vs time graph. What are the speeds at those times?

Distance traveled after 5 seconds: 100 m

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2. Area means distance

Look at the triangular area under the velocity curve from 0–5 seconds and 0–10 seconds.

Does each of these areas equal the distance on the distance vs time graph in question 1?

yes

Area under the velocity curve at 5 seconds: 100 m

Area under the velocity curve at 10 seconds: 400 m

3. Slope means velocity

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Slope of the tangent line to distance function at t=5: 40

Velocity after 5 seconds from Velocity vs Time chart: 40

Are these two numbers close or identical [✓ **Yes**] **No**

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Drag Racing — 1/4 mile (400 m)

No friction · Constant acceleration from rest · Time ticks every 1 second

4 Finish line

Solve $\frac{1}{2}at^2 = 400$ for t. How long to reach 1/4 mile? (notice a is constant and you can read it from the acceleration chart)

$2 \cdot \frac{1}{2}at^2 = 400 \cdot 2$

$at^2 = 800$ $\quad t^2 = \frac{800}{a}$ $\quad t = \sqrt{100} = t = 10$

What is $v = at$ at the finish (m/s and km/h)? Velocity at finish: 80 m/s

(Use 1Km/1000m and 1h/3600s unitary factors to find the velocity in Km/h)

Velocity at finish = $\frac{8\cancel{0}}{1} \times \frac{\cancel{m}}{\cancel{s}} \times \frac{km}{1\cancel{000}} \times \frac{36\cancel{00}}{hr}$ = 288 km/h

5 Acceleration check

The acceleration graph is a flat line. What does the rectangle's area from 0–t tell you about the change in velocity?

What is the change in velocity from 0 to 5 seconds: 40

What is the change in velocity from 0 to 10 seconds: 80

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- $v(t) = at$ d(v)/dt = a
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- $x(t) = \frac{1}{2}at^2$
- $v(t) = at$ $\int v(t)dt = \int at\,dt = a\int t\,dt =$ $\frac{1}{2}at^2$
- $a(t) = a$ $\int a(t)dt = \int a\,dt = a\int 1\,dt =$ at

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Marc

Kinetic Energy and Work — Student Exercise

From force and distance to motion and kinetic energy

Work is force times distance: $W = F \times d$ (units: Joule, J)

Force relates to motion: $F = m a$ (Newton's second law).

From constant-acceleration motion starting at rest: $v = a t$, $s = \frac{1}{2} a t^2$, $v^2 = 2 a s$.

Multiply $v^2 = 2 a s$ by $m/2 \rightarrow (1/2) m v^2 = m a s = F s =$ Work.

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Exercise — 1000 kg drag racer from rest, $a = 8$ m/s²
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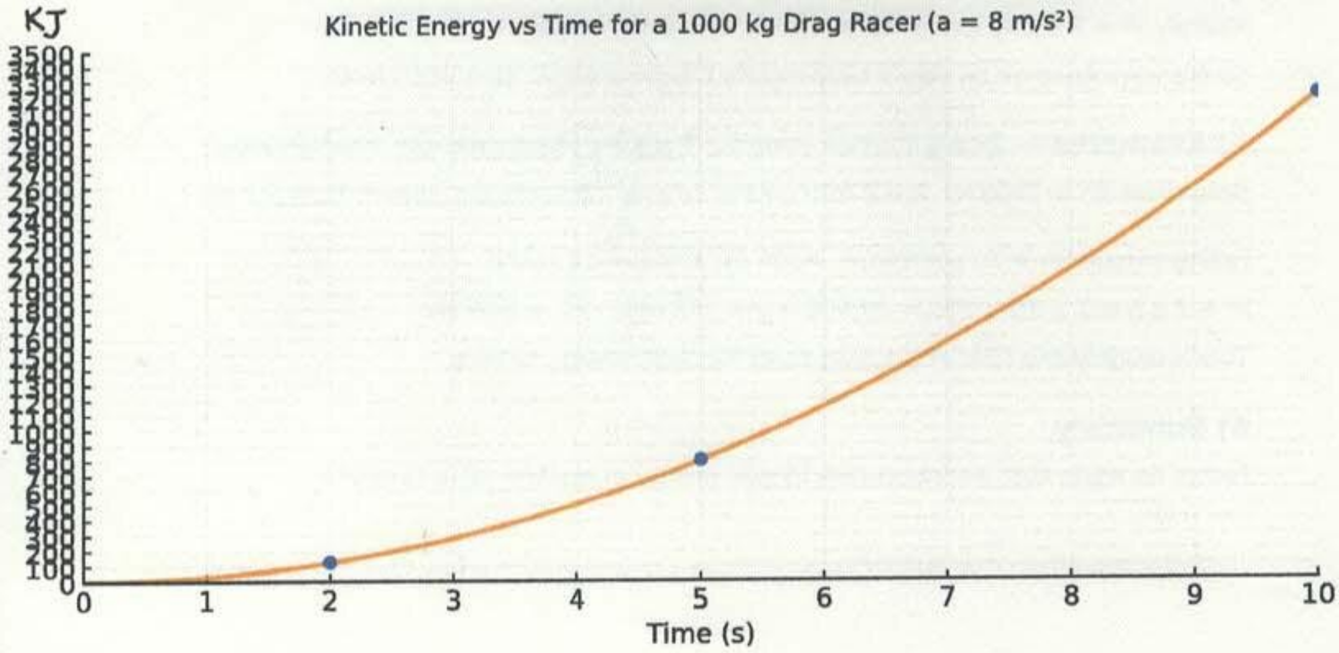
Use your dragster charts to read distance and velocity values as needed.

Time (s)	Velocity (m/s)	Distance (m)	KE (kJ) $= \frac{1}{2} m v^2$	Energy Comparison
2	16	16	128 kj	128,000 j
5	40	100	800 kj	800,000 j
10	80	400	3200 kj	3,200,000 j



🧑‍🏫 Adventure 6 – Area Under the Velocity Curve & Meaning of the Integral

1. OVERVIEW OF THE SESSION

Date: 11/05/2025

Duration: 1:45–2:00 (approx. Students present: Cyrus, Marc, Rooz

🧑‍🔬 MAIN OBJECTIVE

To understand integrals as area under the curve, and to connect 3Blue1Brown's visual explanation of integration with the distance–velocity–acceleration charts the kids have been studying.

🧩 SUMMARY OF STUDENT DISCOVERIES

They connected integrals with accumulated area under the velocity curve.

They recognized that negative acceleration meant braking.

They saw the chain: acceleration → velocity → distance.

Their histogram estimate (84 m) approached the true integral (85.33 m).

🎥 WARM-UP CONVERSATION / STORYTELLING / VIDEOS

We continued with 3Blue1Brown — Integration and the fundamental theorem of calculus | Chapter 8, Essence of calculus

We paused the video often to check understanding. The kids followed Grant's animations easily because of their previous practice with the three charts.

Before watching, we reviewed the full chart page and discussed how the functions relate through derivatives and integrals.

Afterward, the kids received printed charts, immediately noticing the negative acceleration section and identifying it as braking.

They then filled in the histogram table using the velocity graph. Rooz needed support; the others progressed steadily.

🧑‍🔬 ACTIVITIES COMPLETED

ACTIVITY #1 — WATCHING AND INTERPRETING VIDEO #8

Understood integration as accumulated area.

Connected Grant's geometric animations with the charts.

ACTIVITY #2 — UNDERSTANDING THE THREE LINKED CHARTS

Negative acceleration interpreted as braking.

Students reasoned correctly about the motion.

ACTIVITY #3 — HISTOGRAM TABLE (1-SECOND INTERVALS)

Read values from the graph.

Computed rectangle areas and summed them.

Estimated distance: 84 m (close to 85.33 m).

Marc first; Cyrus steady; Rooz needed help.

ACTIVITY #4 — APPROXIMATING DISTANCE FROM THE INTEGRAL

Filled the second table.

Understood why the integral is slightly larger.

⚖️ MINI-DISCUSSIONS AND SIDE TOPICS

Reading the velocity chart on the back of the page was confusing.

Clarified why negative acceleration does not imply negative velocity.

Kids recognized that they have already been applying the Fundamental Theorem of Calculus.

💡 WHAT WORKED WELL

Charts modeled on 3Blue1Brown were highly effective.

Histogram table grounded the idea of "adding rectangles."

Pausing the video improved comprehension.

Students enjoyed interpreting physical meaning from the graphs.

🚀 ADJUSTMENTS FOR NEXT TIME

Move the velocity chart to a separate page.

Clarify the table format so it cannot be misinterpreted.

Emphasize units and reading precision.

Provide more scaffolding for Rooz.

🧑‍🔬 STUDENT QUOTES

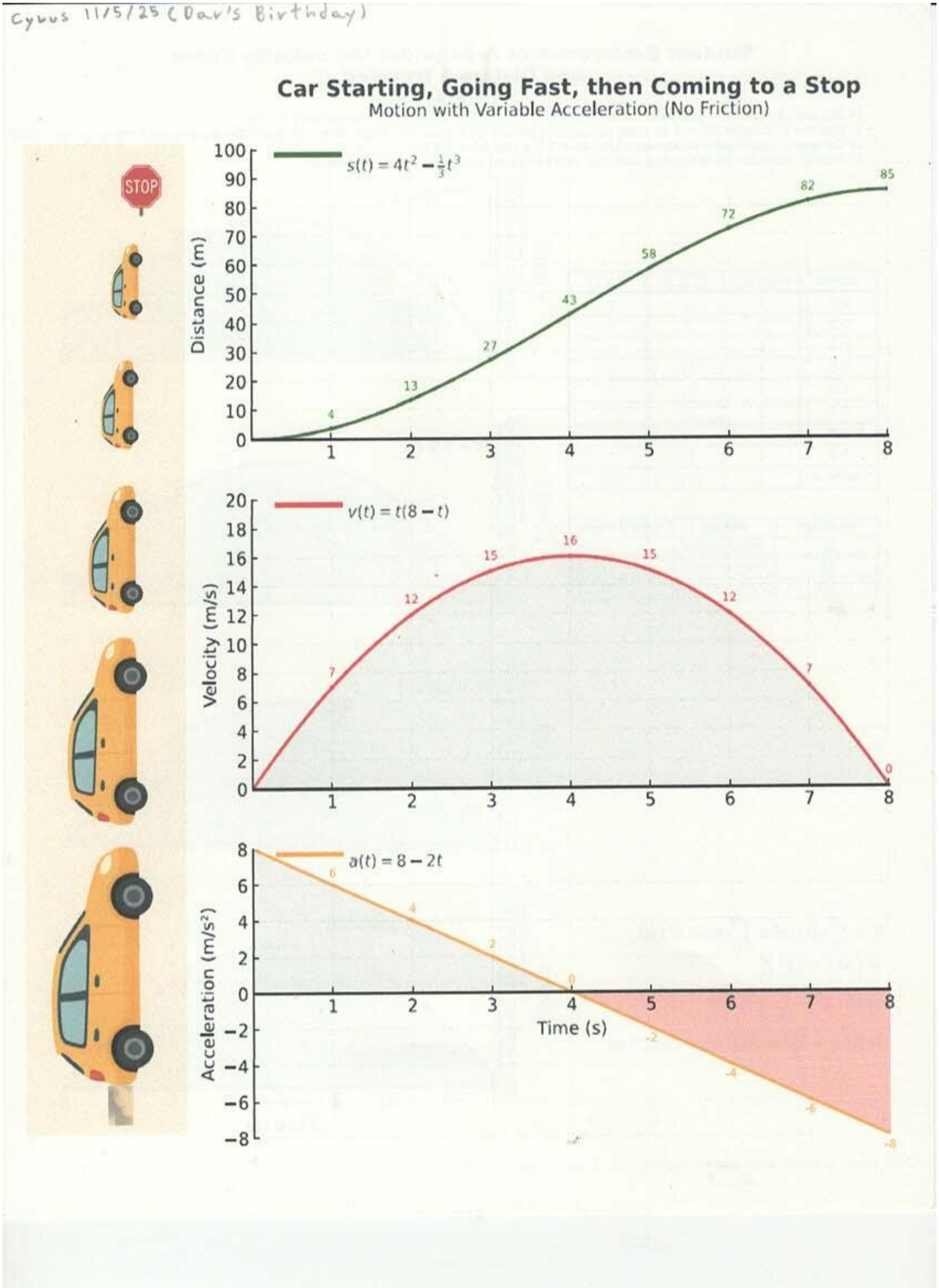
Marc: “I got 84! I beat you guys!”

Cyrus: “Ohhh — so that’s why it’s a little more.”

💡 CLOSING REFLECTION BY DR. SUPER

This session linked concrete calculations with the formal idea of integration. Using custom charts parallel to 3Blue1Brown helped reinforce the relationship between area, velocity, and distance. The histogram approximation showed real understanding. Next time, I will explicitly connect these ideas to the Fundamental Theorem of Calculus.

Completed Student Worksheets



Cyrus 11/5/25 (Dar's Birthday)

Car Starting, Going Fast, then Coming to a Stop

Motion with Variable Acceleration (No Friction)

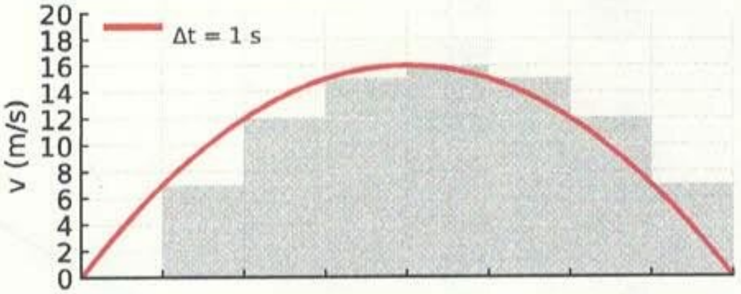
Student Exploration of Area under the Velocity Curve and Distance Traveled
(0-8 s)

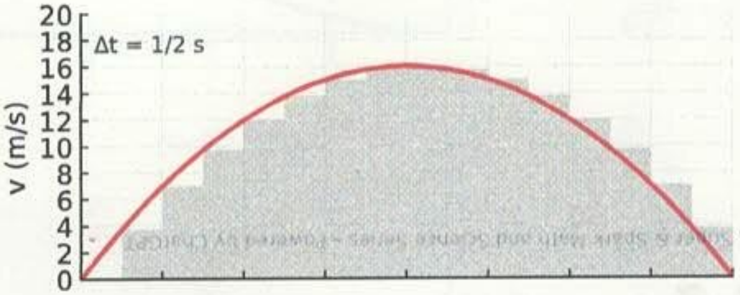
In this activity, you will estimate total distance by finding the area under the velocity curve.

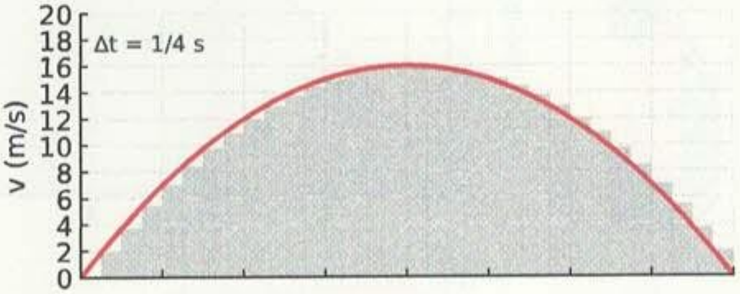
1) Use the first table (Δt = 1 s). Look up velocity values from your v(t) chart, then compute the area of each rectangle (v × Δt).
2) Compare your total with the sums for Δt = 1/2 s and Δt = 1/4 s.
3) Finally, compare all estimates with the exact integral value shown at the bottom.

Interval	Width (dt)	Velocity (lookup)	v × dt
0–1	1	0	0
1–2	1	7	7
2–3	1	12	12
3–4	1	15	15
4–5	1	16	16
5–6	1	15	15
6–7	1	12	12
7–8	1	7	7
Total (≈ m)	8	84	84

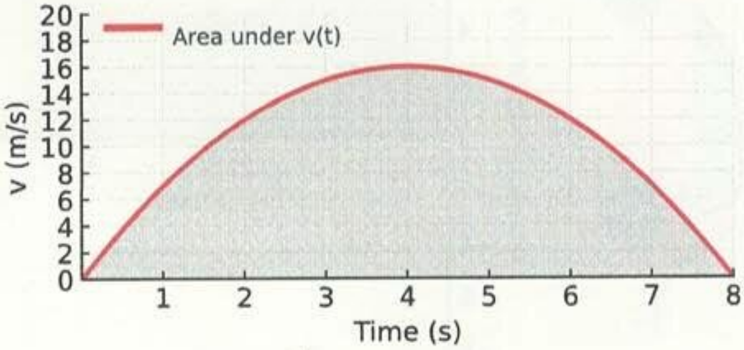
Width (dt)	Method	Sum of Areas
1	Your Estimate	84
1/2	Finer estimate	85.25
1/4	Very fine estimate	85.31







$$S = \int_0^8 v(t)\,dt = \int_0^8 (8t - t^2)\,dt$$
$$= [\,4t^2 - \tfrac{1}{3}t^3\,]_0^8$$
$$= (4 \cdot 8^2 - \tfrac{1}{3} \cdot 8^3) - 0$$
$$= 256 - \tfrac{512}{3} = 85\tfrac{1}{3}\text{ m} \approx 85.33\text{ m}$$



Cyrus 11/5/25 (Dar's Birthday)

Student Exploration of Area under the Velocity Curve and Distance Traveled Page 1
Variable acceleration increases then reverses, no friction from rest - Time ticks every 1 second

✳ Equations of Motion

- $x(t) = 4t^2 - t^3/3$
- $v(t) = t(8-t) = 8t - t^2$ **Antiderivative (integral) of 8t-t² = $4t^2 - t^3/3$**
- $a(t) = 8 - 2t$ **Antiderivative (integral) of 8-2t = $8t - t^2$**

Questions

1 How far, how fast?

Read the distance traveled at 2 seconds and distance traveled between 2 and 7 seconds from the distance vs time graph?

Distance traveled in 2 seconds: 13

Distance traveled from 2 to 7 seconds: 69

2.Area means distance

Evaluate the integral for the v(t) function to find the distance traveled in 2 seconds and between 2 and 7 seconds.

Distance traveled in 2 seconds:

$$\int_0^2 (8t - t^2)dt = [4t^2 - t^3/3]^2 - [4t^2 - t^3/3]^0 = 16 - \frac{8}{3} - 0$$

$$= 13\tfrac{1}{3}$$

Distance traveled between 2 and 7 seconds:

$$\int_2^7 (8t - t^2)dt = [4t^2 - t^3/3]^7 - [4t^2 - t^3/3]^2 = 81\tfrac{2}{3} - 13\tfrac{1}{3}$$

$$= 68\tfrac{1}{3}$$

Are these areas equal or close to what you found in part 1 of the exercise?

[✓ Yes] No

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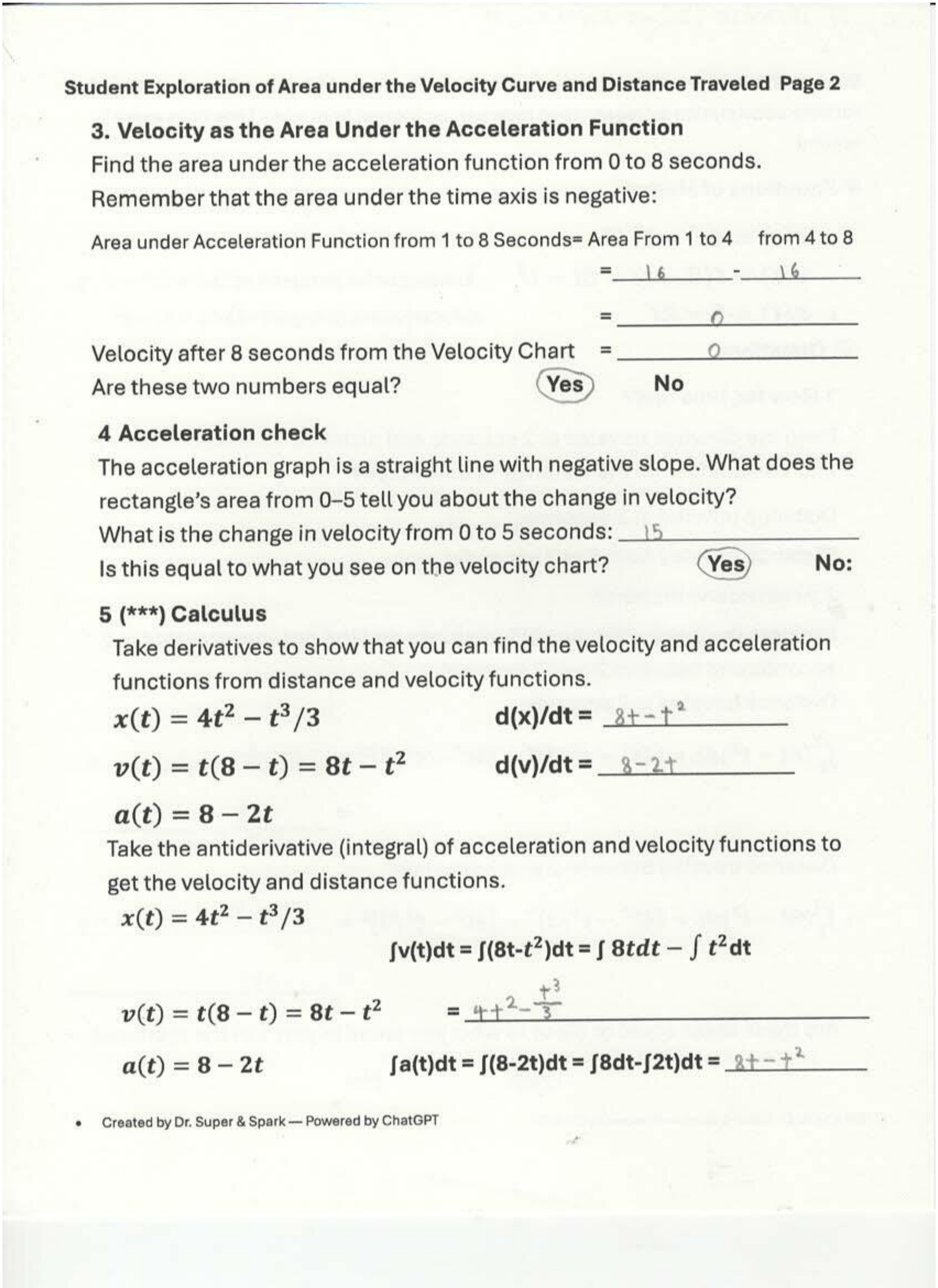

Student Exploration of Area under the Velocity Curve and Distance Traveled Page 2

3. Velocity as the Area Under the Acceleration Function

Find the area under the acceleration function from 0 to 8 seconds. Remember that the area under the time axis is negative:

Area under Acceleration Function from 1 to 8 Seconds= Area From 1 to 4 from 4 to 8

= 16 - 16

= 0

Velocity after 8 seconds from the Velocity Chart = 0

Are these two numbers equal? [✓ Yes] No

4 Acceleration check

The acceleration graph is a straight line with negative slope. What does the rectangle's area from 0–5 tell you about the change in velocity?

What is the change in velocity from 0 to 5 seconds: 15

Is this equal to what you see on the velocity chart? [✓ Yes] No:

5 (***) Calculus

Take derivatives to show that you can find the velocity and acceleration functions from distance and velocity functions.

$x(t) = 4t^2 - t^3/3$ d(x)/dt = $8t - t^2$

$v(t) = t(8-t) = 8t - t^2$ d(v)/dt = $8-2t$

$a(t) = 8 - 2t$

Take the antiderivative (integral) of acceleration and velocity functions to get the velocity and distance functions.

$x(t) = 4t^2 - t^3/3$

$\int v(t)dt = \int(8t-t^2)dt = \int 8t\,dt - \int t^2 dt$

$v(t) = t(8-t) = 8t - t^2$ $= 4t^2 - \frac{t^3}{3}$

$a(t) = 8 - 2t$ $\int a(t)dt = \int(8-2t)dt = \int 8dt - \int 2t)dt = 8t - t^2$

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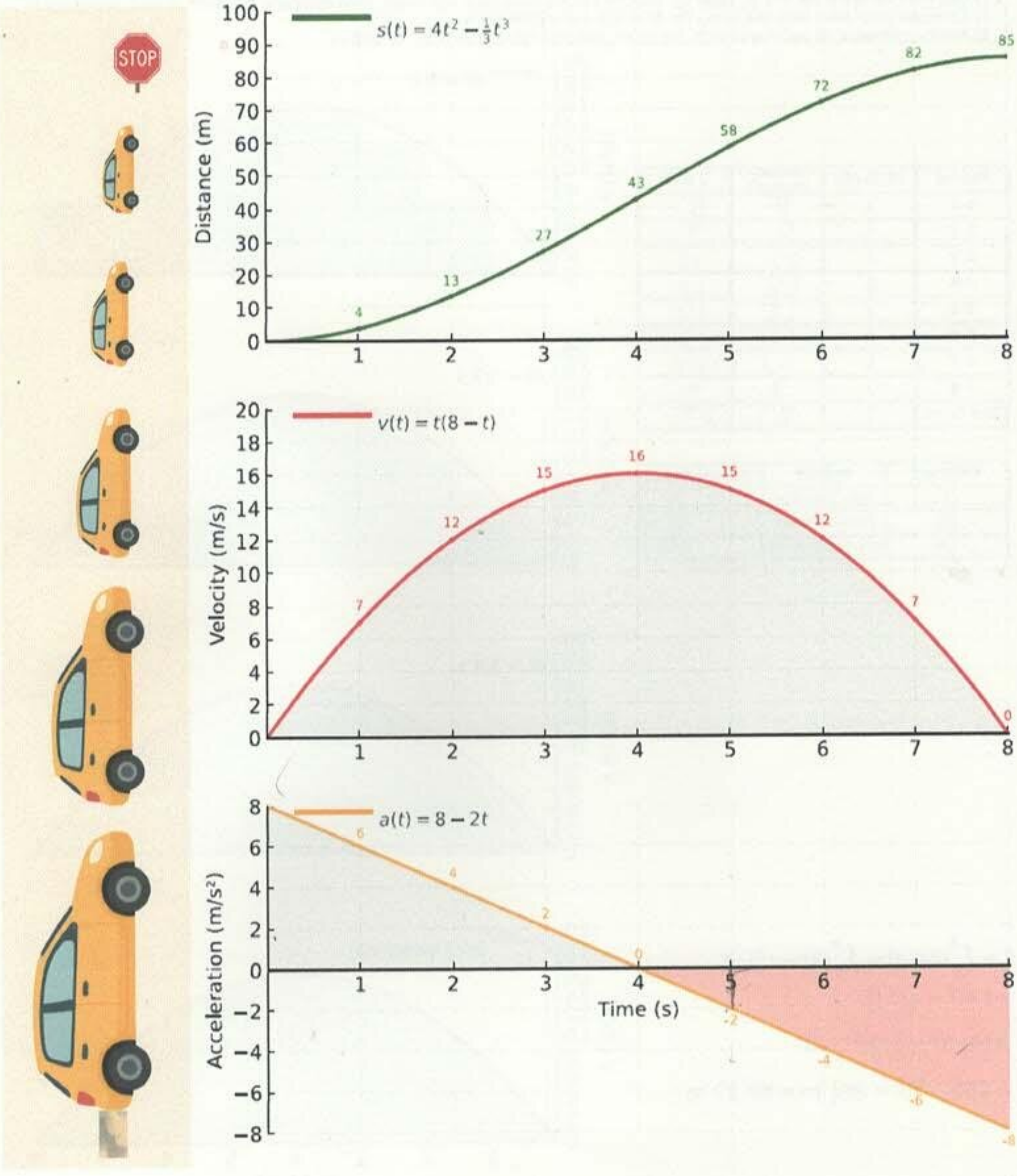
Rooz



Car Starting, Going Fast, then Coming to a Stop

Motion with Variable Acceleration (No Friction)

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Rooz

Student Exploration of Area under the Velocity Curve and Distance Traveled
(0-8 s)

In this activity, you will estimate total distance by finding the area under the velocity curve.

1) Use the first table ($\Delta t = 1$ s). Look up velocity values from your v(t) chart, then compute the area of each rectangle ($v \times \Delta t$).
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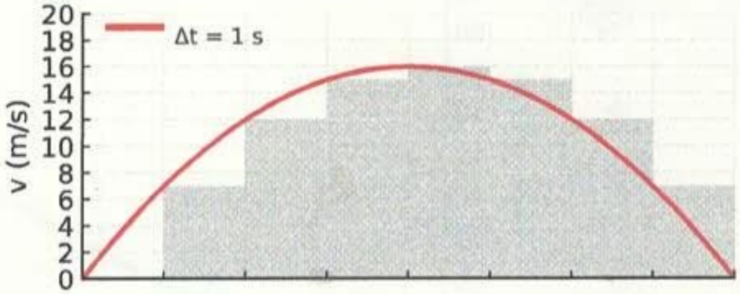
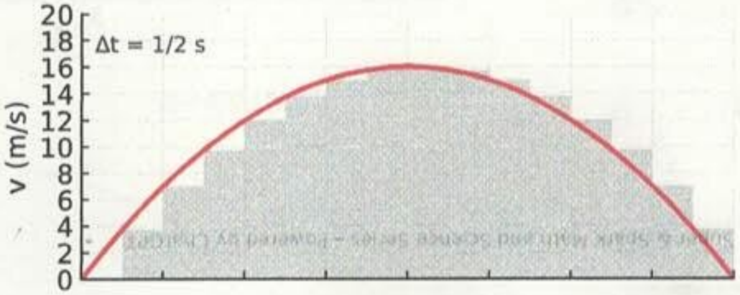
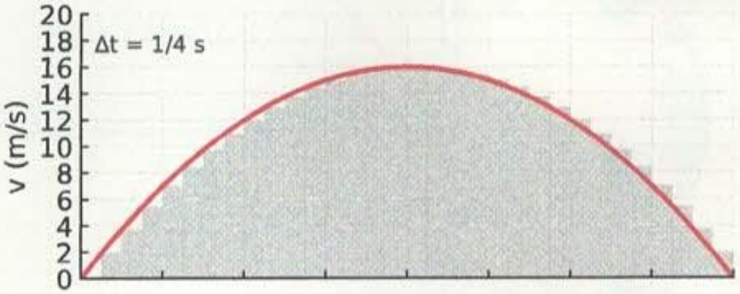




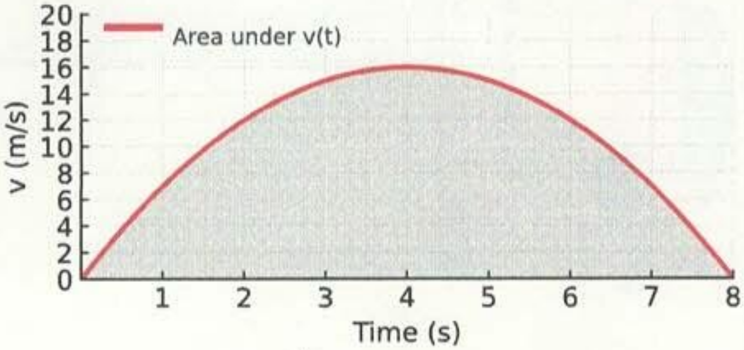


$$S = \int_0^8 v(t)\,dt = \int_0^8 (8t - t^2)\,dt$$

$$= [\,4t^2 - \tfrac{1}{3}t^3\,]_0^8$$

$$= (4 \cdot 8^2 - \tfrac{1}{3} \cdot 8^3) - 0$$

$$= 256 - \tfrac{512}{3} = 85\tfrac{1}{3}\text{ m} \approx 85.33\text{ m}$$



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Rooz

Student Exploration of Area under the Velocity Curve and Distance Traveled Page 1

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2.Area means distance

Evaluate the integral for the v(t) function to find the distance traveled in 2 seconds and between 2 and 7 seconds.

Distance traveled in 2 seconds:

$$\int_0^2 (8t - t^2)dt = [4t^2 - t^3/3]^{\overset{t=2}{2}} - [4t^2 - t^3/3]^{\overset{t=0}{0}} = 4\times4-\frac{8}{3}\div1-1\times4-8\div1 - 0$$

$$= 13\frac{1}{3}$$

Distance traveled between 2 and 7 seconds:

$$\int_2^7 (8t - t^2)dt = [4t^2 - t^3/3]^7 - [4t^2 - t^3/3]^2 = 4\times7^2-\frac{7^3}{3}-4\times49-\frac{7}{3}44 - 13\frac{1}{3}$$

$$= 68.(3)33\frac{7}{3})$$

Are these areas equal or close to what you found in part 1 of the exercise?

[✓ Yes] No

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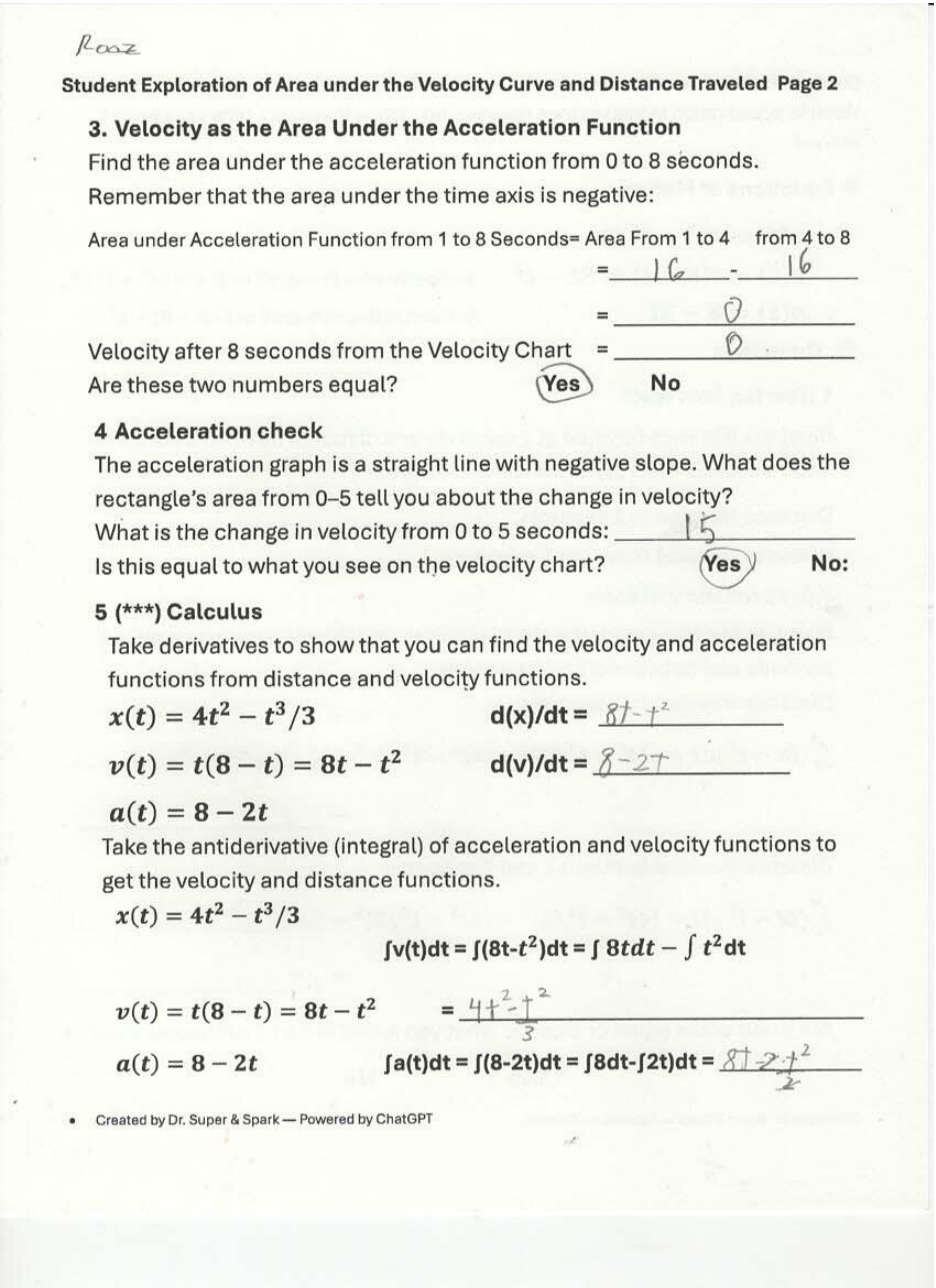

Raaz

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Velocity after 8 seconds from the Velocity Chart = 0

Are these two numbers equal? [✓ Yes] No

4 Acceleration check

The acceleration graph is a straight line with negative slope. What does the rectangle's area from 0–5 tell you about the change in velocity?

What is the change in velocity from 0 to 5 seconds: 15

Is this equal to what you see on the velocity chart? [✓ Yes] No:

5 (***) Calculus

Take derivatives to show that you can find the velocity and acceleration functions from distance and velocity functions.

$x(t) = 4t^2 - t^3/3$ d(x)/dt = $8t - t^2$

$v(t) = t(8 - t) = 8t - t^2$ d(v)/dt = $8-2t$

$a(t) = 8 - 2t$

Take the antiderivative (integral) of acceleration and velocity functions to get the velocity and distance functions.

$x(t) = 4t^2 - t^3/3$

$\int v(t)dt = \int (8t-t^2)dt = \int 8t\,dt - \int t^2 dt$

$v(t) = t(8 - t) = 8t - t^2$ $= 4t^2 - \frac{t^2}{3}$

$a(t) = 8 - 2t$ $\int a(t)dt = \int (8-2t)dt = \int 8dt - \int 2t)dt = 8t - \frac{2t^2}{2}$

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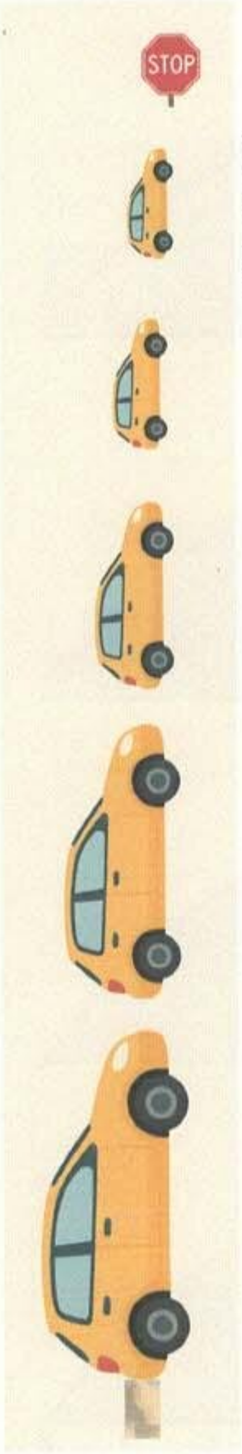
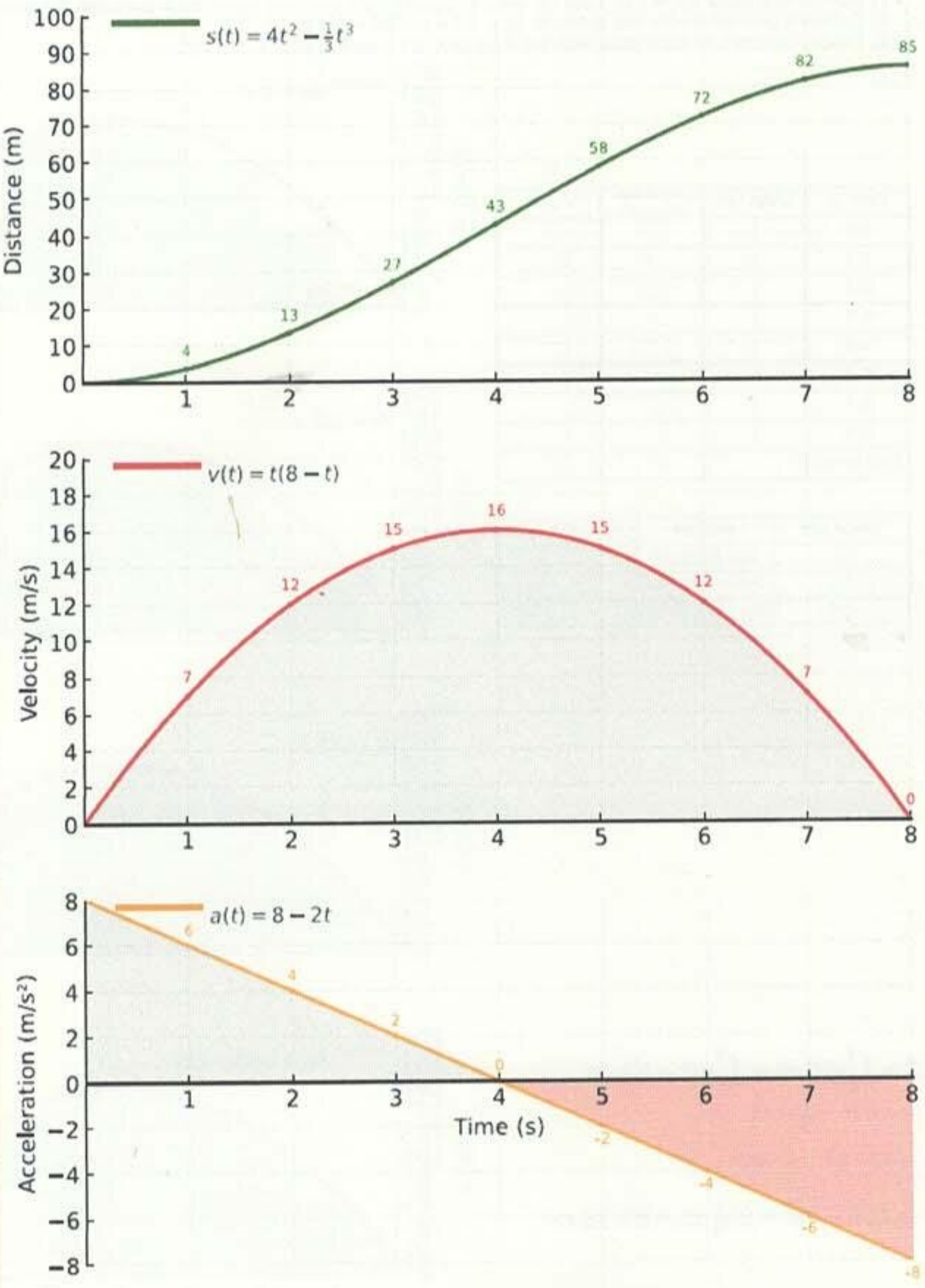
Marc 11/5/25

Car Starting, Going Fast, then Coming to a Stop

Motion with Variable Acceleration (No Friction)





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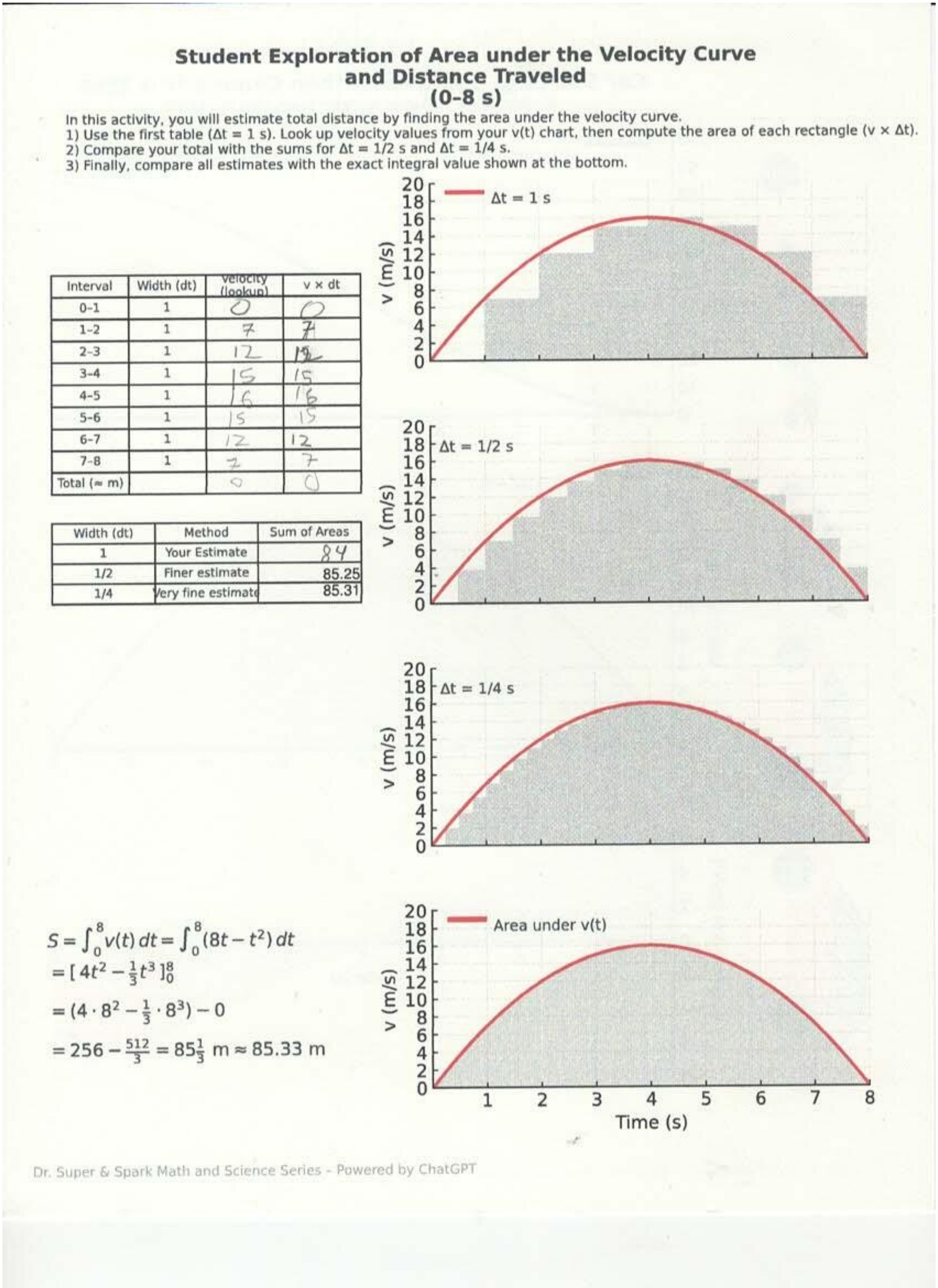

Student Exploration of Area under the Velocity Curve and Distance Traveled
(0–8 s)

In this activity, you will estimate total distance by finding the area under the velocity curve.

1) Use the first table (Δt = 1 s). Look up velocity values from your v(t) chart, then compute the area of each rectangle (v × Δt).
2) Compare your total with the sums for Δt = 1/2 s and Δt = 1/4 s.
3) Finally, compare all estimates with the exact integral value shown at the bottom.

Interval	Width (dt)	Velocity (lookup)	v × dt
0–1	1	0	0
1–2	1	7	7
2–3	1	12	12
3–4	1	15	15
4–5	1	16	16
5–6	1	15	15
6–7	1	12	12
7–8	1	7	7
Total (≈ m)		0	0

Width (dt)	Method	Sum of Areas
1	Your Estimate	84
1/2	Finer estimate	85.25
1/4	Very fine estimate	85.31

$$S=\int_0^8 v(t)\,dt=\int_0^8 (8t-t^2)\,dt$$

$$=[\,4t^2-\tfrac{1}{3}t^3\,]_0^8$$

$$=(4\cdot 8^2-\tfrac{1}{3}\cdot 8^3)-0$$

$$=256-\tfrac{512}{3}=85\tfrac{1}{3}\text{ m}\approx 85.33\text{ m}$$

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Marc 11/5/25

Student Exploration of Area under the Velocity Curve and Distance Traveled Page 1
Variable acceleration increases then reverses, no friction from rest - Time ticks every 1 second

✳ Equations of Motion

- $x(t) = 4t^2 - t^3/3$
- $v(t) = t(8 - t) = 8t - t^2$ **Antiderivative (integral) of 8t-t² = $4t^2 - t^3/3$**
- $a(t) = 8 - 2t$ **Antiderivative (integral) of 8-2t = $8t - t^2$**

Questions

1 How far, how fast?

Read the distance traveled at 2 seconds and distance traveled between 2 and 7 seconds from the distance vs time graph?

Distance traveled in 2 seconds: 13 m

Distance traveled from 2 to 7 seconds: 69m

2.Area means distance

Evaluate the integral for the v(t) function to find the distance traveled in 2 seconds and between 2 and 7 seconds.

Distance traveled in 2 seconds: $13\frac{1}{3}$ m

$$\int_0^2 (8t - t^2)dt = [4\underset{2}{t}^2 - \underset{2}{t}^3/3]^2 - [4t^2 - \overset{0}{t}^3/3]^0 = 16 - \frac{8}{3} - 0$$

$$= 13\frac{1}{3}$$

Distance traveled between 2 and 7 seconds:

$$\int_2^7 (8t - t^2)dt = [4\underset{7}{t}^2 - \underset{7}{t}^3/3]^7 - [4\underset{2}{t}^2 - \underset{2}{t}^3/3]^2 = 196 - \frac{343}{3} - \frac{40}{3}$$

$$= 68\frac{1}{3}$$

Are these areas equal or close to what you found in part 1 of the exercise?

[✓ Yes] No

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3. Velocity as the Area Under the Acceleration Function

Find the area under the acceleration function from 0 to 8 seconds.
Remember that the area under the time axis is negative:

Area under Acceleration Function from 1 to 8 Seconds= Area From 1 to 4 from 4 to 8

= 16 - 16

= 0

Velocity after 8 seconds from the Velocity Chart = ________

Are these two numbers equal? [✓ Yes] No

4 Acceleration check

The acceleration graph is a straight line with negative slope. What does the rectangle's area from 0–5 tell you about the change in velocity?

What is the change in velocity from 0 to 5 seconds: 15

Is this equal to what you see on the velocity chart? [✓ Yes] No:

5 (***) Calculus

Take derivatives to show that you can find the velocity and acceleration functions from distance and velocity functions.

$x(t) = 4t^2 - t^3/3$ d(x)/dt = $8t - t^2$

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$x(t) = 4t^2 - t^3/3$

$\int v(t)dt = \int(8t-t^2)dt = \int 8t\,dt - \int t^2 dt$

$v(t) = t(8 - t) = 8t - t^2$ = $4t^2 - \frac{1}{3}t^3$

$a(t) = 8 - 2t$ $\int a(t)dt = \int(8-2t)dt = \int 8dt - \int 2t)dt$ = $8t - t^2$

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🧑‍🏫 Adventure 7 – Momentum, Force & the Rocket Equation

🧩 1. Overview of the Session

Date: 11/19/2025
Duration: 1.5 hours (1:50–3:20 pm)
Students present: Cyrus, Rooz, Marc

Main objective:
To understand how momentum, mass, and velocity interact during a rocket launch, and to connect the **derivative of a product of functions** with the physics relationship

$$\boldsymbol{F = \frac{dp}{dt}}.$$

Summary of student discoveries:

- They identified **fuel** as the agent that accelerates the rocket forward.
- They recognized the **three main stages** of a launch: *Liftoff → Coasting → Escape*.
- They saw that **force is the derivative of momentum**, and that this generalizes Newton's 2nd Law when the mass is not constant.

🚀 2. Warm-Up Conversation / Storytelling / Videos

We began with the **Falcon Heavy launch** toward Europa. (Even little Ave, 5½ years old, joined us briefly.) This led naturally into a discussion about staging, boosters, and why rockets drop parts as they climb.

Next, we watched:
3Blue1Brown – Visualizing the Chain Rule and Product Rule (Chapter 4).
We paused several times, especially for the **product rule**, and connected it directly to the rocket physics via

$$F = \frac{dp}{dt}.$$

Each student then read a short paragraph from *Space Stargazing and Rockets*. I read the final one, which mentions Omar Khayyam and the Jalali Calendar. Cyrus proudly noted that Spark had added it on Dr. Super's recommendation, giving us a chance to highlight how little the West credits Persian scientific contributions.

I gave them the **two main charts** for the session. The new chart: **Mass, Momentum, and Force,** captured their attention. I explained that normally "moment" would appear at the top (mass × distance), but for these exercises we replaced it with mass since that was the relevant quantity for this activity.

📊 3. Activities Completed

Activity #1 — Reviewing the Rocket Mass Formulas in Each Stage

They examined the mass formulas and linked them to the mass chart for all stages.
We highlighted why **mass remains constant during Stage 2**: the engines are off and the rocket is coasting in Earth orbit, with centripetal effects balancing gravity.

I briefly described the spacecraft's path: orbit Earth → fire engines → travel toward Mars → orbit Mars → fire engines → proceed toward Uranus. This sequence appears near the end of the launch video.
They compared both sets of charts and tried to interpret them independently.

Activity #2 — Mass, Momentum, Force, and the Product Rule

I re-derived the expression for **force** by differentiating the momentum expression on their handout. We discussed how **Newton's Second Law,** $F = ma$, works only if mass is constant, and how $\frac{dp}{dt}$ is the correct generalization for rockets.

Notation remains challenging. Cyrus asked about the "d" in dm/dt, so I reiterated that derivatives are represented in several ways — sometimes even as m'.

They found it counterintuitive but interesting that **a rocket becomes easier to accelerate** as fuel burns and the mass decreases. Seeing this connection across the **mass**, **velocity**, and **force** charts deepened their understanding.

Activity #3 — Linking the Charts & Completing Tables to Determine Force

They read the velocity and acceleration charts fairly well. Slope calculations were harder, especially for Rooz.

All three correctly computed ma, but they missed the step of converting **velocity from km/min to km/sec**, which created discrepancies between their results and the Force chart.

There was a brief "panic moment," but once I recognized the unit mismatch, everything fell into place. They corrected their work and obtained the correct values at **5 minutes** and **58 minutes**.

They asked why 58 minutes was chosen — I explained it was simply to create a more challenging estimation point.

We did not review all completed tables due to time, so I will revisit them next session.

Activity #4 — Answering Conceptual Questions Using Both Charts

Despite nearing the 90-minute mark, they remained surprisingly focused.

- Cyrus worked slowly but very carefully.
- Marc sped through quickly.
- Rooz needed help writing but generated answers independently.

They noticed correctly that the **distance functions** in Stages 1 and 3 are **quadratic**, so linear approximations are imperfect.
For velocity, which is linear in these phases, slope estimates — and therefore acceleration — were exact.

🧮 4. Mini-Discussions & Side Topics

A major insight emerged when they saw that their force estimates didn't match the chart.
Discovering **why** (unit conversion) helped them appreciate the importance of consistency in scientific work.

🧑‍🏫 5. What Worked Well

- The combination of **video** + **story** + **charts** made the session coherent.
- The tables contained just enough information for them to succeed without being overwhelmed.
- The **Mass–Momentum–Force chart** fascinated them and sparked deeper questions.
- Repeated exposure to the product rule helped make a difficult concept more approachable. Grant's visuals were extremely effective.

🧠 6. Adjustments for Next Time

Items to refine:

- Add a **bold reminder** about converting velocity units.
- Clarify **units for** dm/dt in Stage 1.
- Include one or two **guiding questions** to keep them from drifting off-track.

- Reinforce the relationship between the three charts.

🪄 7. Student Quotes (Optional but Charming)

- After fixing the velocity-unit issue, Cyrus gave a little hug and said:
  “Great — now it works! I’m happy for you.”
- The “slingshot effect” became a running theme; we referred to it many times.

📹 8. Photos / Charts / Sketches

(To be added — you will attach the worksheets later.)

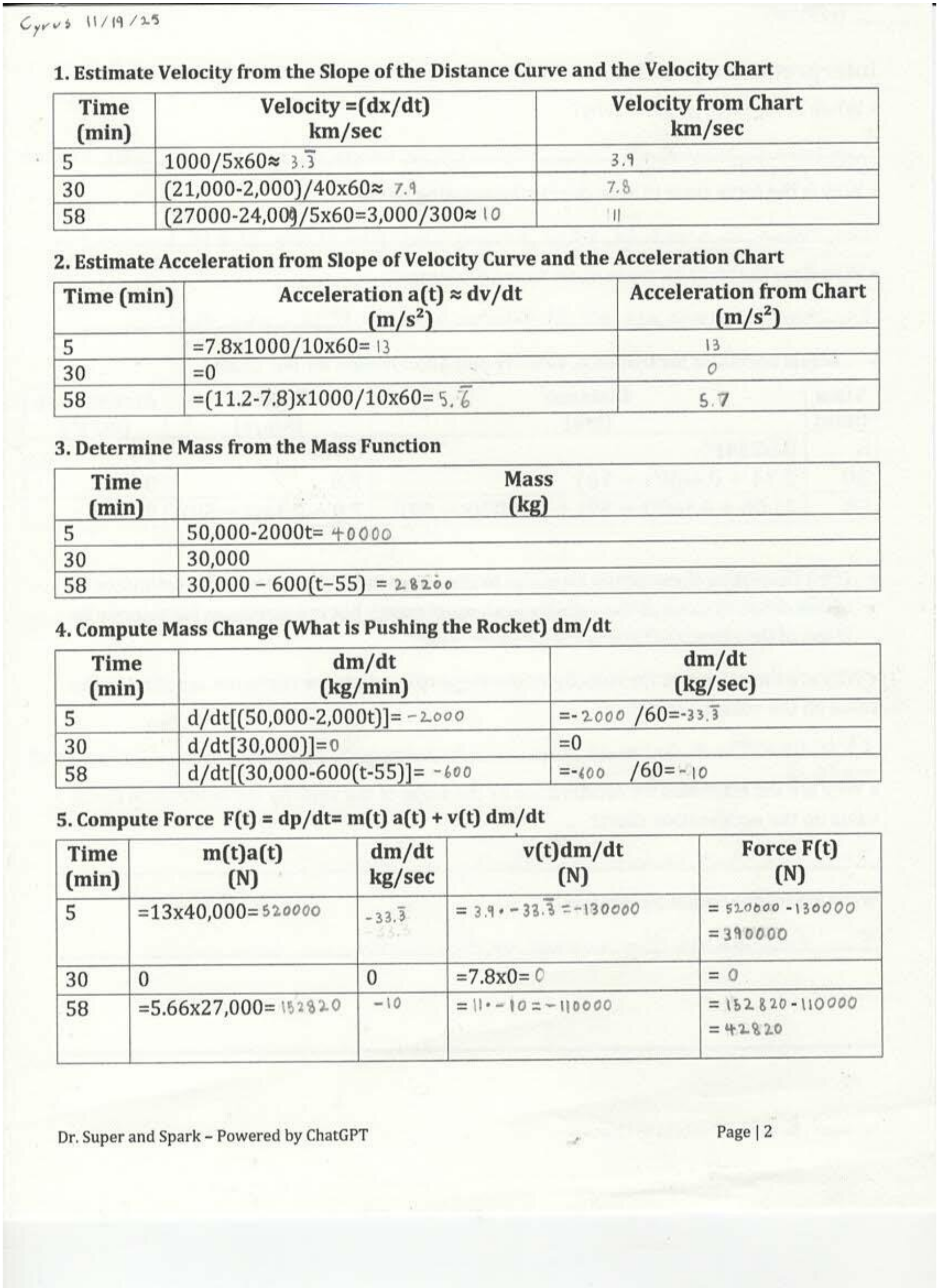
🧱 9. Closing Reflection by Dr. Super

This was the most time-intensive activity to prepare, and the difficulty level showed — but in a positive way. The kids lasted almost 90 minutes (well beyond our typical 60), and they stayed engaged throughout.

I was especially impressed that after the first hour — when he normally started drifting **Rooz’s focus increased**, and Marc stopped fidgeting. Cyrus remained meticulous as always.

The biggest success was their ability to **connect their calculations with the physics** shown in the charts: mass decreasing, velocity increasing, acceleration depending on both, and force emerging from the derivative of momentum. This session successfully linked calculus and mechanics in a way that felt real and meaningful to them. Going over their completed work, I see that there is still a gap here that has to be filled for connecting Calculus and Mechanics. Some of it was probably because of rushing through the questions on the last page.

📘 10. Student Completed Worksheets



Cyrus 11/19/25

1. Estimate Velocity from the Slope of the Distance Curve and the Velocity Chart

Time (min)	Velocity =(dx/dt) km/sec	Velocity from Chart km/sec
5	1000/5x60≈ $3.\overline{3}$	3.9
30	(21,000-2,000)/40x60≈ 7.9	7.8
58	(27000-24,000)/5x60=3,000/300≈ 10	11

2. Estimate Acceleration from Slope of Velocity Curve and the Acceleration Chart

Time (min)	Acceleration a(t) ≈ dv/dt (m/s^2)	Acceleration from Chart (m/s^2)
5	=7.8x1000/10x60= 13	13
30	=0	0
58	=(11.2-7.8)x1000/10x60= $5.\overline{6}$	5.7

3. Determine Mass from the Mass Function

Time (min)	Mass (kg)
5	50,000-2000t= 40000
30	30,000
58	30,000 – 600(t–55) = 28200

4. Compute Mass Change (What is Pushing the Rocket) dm/dt

Time (min)	dm/dt (kg/min)	dm/dt (kg/sec)
5	d/dt[(50,000-2,000t)]= -2000	=-2000 /60=$-33.\overline{3}$
30	d/dt[30,000)]=0	=0
58	d/dt[(30,000-600(t-55)]= -600	=-600 /60=-10

5. Compute Force F(t) = dp/dt= m(t) a(t) + v(t) dm/dt

Time (min)	m(t)a(t) (N)	dm/dt kg/sec	v(t)dm/dt (N)	Force F(t) (N)
5	=13x40,000=520000	$-33.\overline{3}$	= 3.9 · $-33.\overline{3}$ =-130000	= 520000 -130000 =390000
30	0	0	=7.8x0= 0	= 0
58	=5.66x27,000= 152820	-10	= 11 · -10 = -110000	= 152820 - 110000 = 42820

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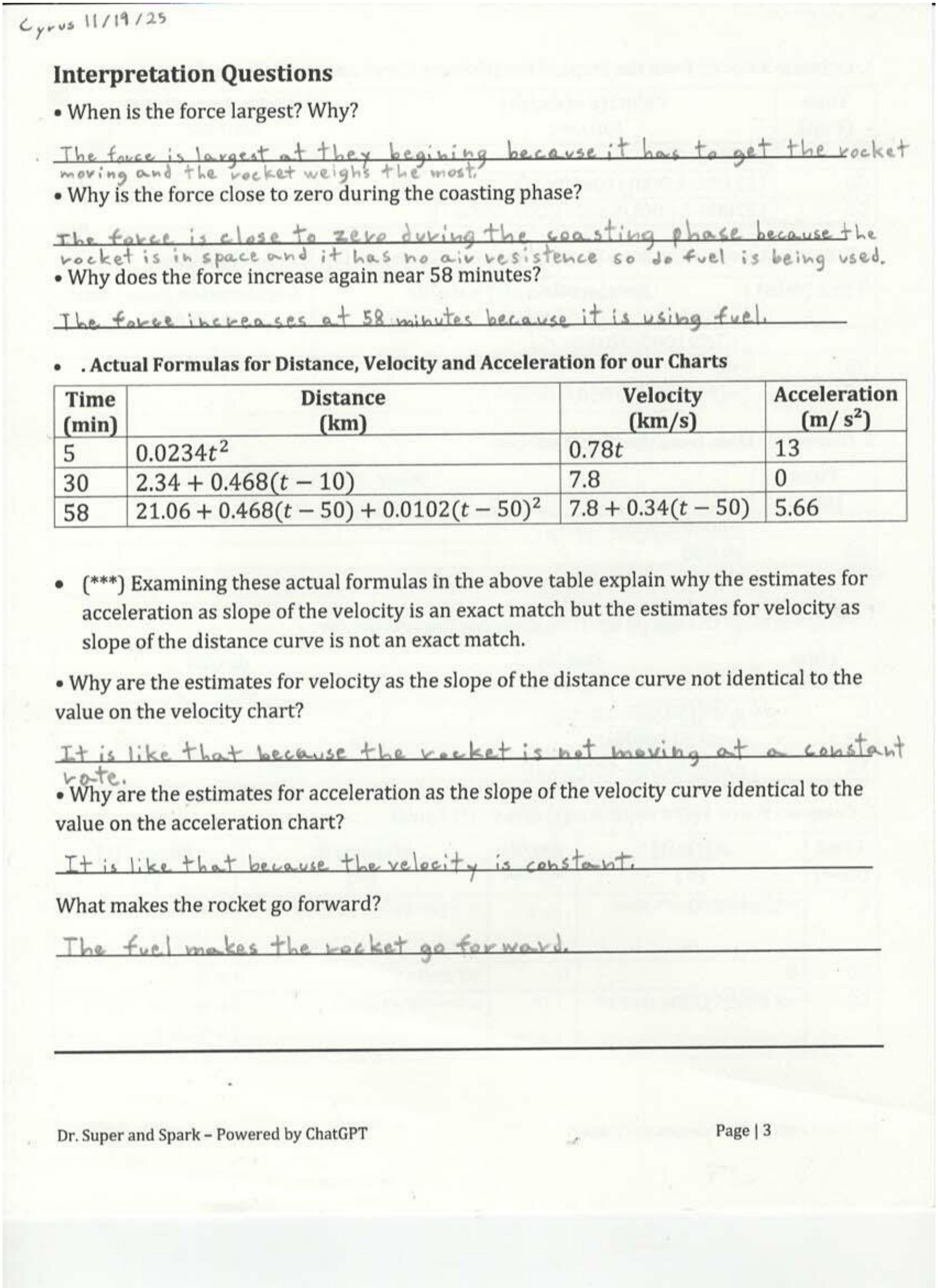

Cyrus 11/19/25

Interpretation Questions

- When is the force largest? Why?

The force is largest at they begining because it has to get the rocket moving and the rocket weighs the most.

- Why is the force close to zero during the coasting phase?

The force is close to zero during the coasting phase because the rocket is in space and it has no air resistence so do fuel is being used.

- Why does the force increase again near 58 minutes?

The force increases at 58 minutes because it is using fuel.

- **. Actual Formulas for Distance, Velocity and Acceleration for our Charts**

Time (min)	**Distance (km)**	**Velocity (km/s)**	**Acceleration (m/s^2)**
5	$0.0234t^2$	$0.78t$	13
30	$2.34 + 0.468(t - 10)$	7.8	0
58	$21.06 + 0.468(t - 50) + 0.0102(t - 50)^2$	$7.8 + 0.34(t - 50)$	5.66

- (***) Examining these actual formulas in the above table explain why the estimates for acceleration as slope of the velocity is an exact match but the estimates for velocity as slope of the distance curve is not an exact match.

- Why are the estimates for velocity as the slope of the distance curve not identical to the value on the velocity chart?

It is like that because the rocket is not moving at a constant rate.

- Why are the estimates for acceleration as the slope of the velocity curve identical to the value on the acceleration chart?

It is like that because the velocity is constant.

What makes the rocket go forward?

The fuel makes the rocket go forward.

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Rooz

1. Estimate Velocity from the Slope of the Distance Curve and the Velocity Chart

Time (min)	Velocity =(dx/dt) km/sec	Velocity from Chart km/sec
5	1000/5x60≈3.33	3.9
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Time (min)	Acceleration a(t) ≈ dv/dt (m/s^2)	Acceleration from Chart (m/s^2)
5	=7.8x1000/10x60= 13	13
30	=0	0
58	=(11.2-7.8)x1000/10x60= 5.7	5.7

3. Determine Mass from the Mass Function

Time (min)	Mass (kg)
5	50,000-2000t= 48000
30	30,000
58	30,000 – 600(t–55) = 28200

4. Compute Mass Change (What is Pushing the Rocket) dm/dt

Time (min)	dm/dt (kg/min)	dm/dt (kg/sec)
5	d/dt[(50,000-2,000t)]= -2000	=- 2000/60=33.$\overline{3}$
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5. Compute Force F(t) = dp/dt= m(t) a(t) + v(t) dm/dt

Time (min)	m(t)a(t) (N)	dm/dt kg/sec	v(t)dm/dt (N)	Force F(t) (N)
5	=13x40,000=520000	-33.$\overline{3}$	=-129.87x1000=13,000 =130,000	=52000-13,000 =39000
30	0	0	=7.8x0=0	= 0
58	=5.66x27,000=152820	-10	=-110,600	=152820-110,000 =42820

ROOZ

Interpretation Questions

- When is the force largest? Why?

At the start because it whighs most.

- Why is the force close to zero during the coasting phase?

Because it does not need fuel.

- Why does the force increase again near 58 minutes?

It increses the force to break the orbit.

- . **Actual Formulas for Distance, Velocity and Acceleration for our Charts**

Time (min)	Distance (km)	Velocity (km/s)	Acceleration (m/s^2)
5	$0.0234t^2$	$0.78t$	13
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- (***) Examining these actual formulas in the above table explain why the estimates for acceleration as slope of the velocity is an exact match but the estimates for velocity as slope of the distance curve is not an exact match.

- Why are the estimates for velocity as the slope of the distance curve not identical to the value on the velocity chart?

The distance curve beetween 0 and 10 and 50 and 60.

- Why are the estimates for acceleration as the slope of the velocity curve identical to the value on the acceleration chart?

It is acurate because it is made of lines.

What makes the rocket go forward?

fuel.

🧑‍🏫 Adventure 8 – Derivatives, Antiderivatives & the DiVA Chart

🧩 Purpose of This Math Circle Session

Adventure 8 builds on earlier DiVA work and moves students toward a deeper insight:

Rates of change control long-term behavior.

Students revisit derivatives and antiderivatives, but now the emphasis shifts from computation to interpretation:

- What does the derivative *mean* physically?
- How does accumulated change determine future motion?
- Why are slopes more revealing than raw values?

This session prepares students for more advanced modeling and ratio reasoning in later Adventures.

🌎 Story Connection — Reading Thin Layers

We began by revisiting the extinction story from earlier sections — the thin iridium layer that recorded a global catastrophe.

Just as that thin geological stripe encoded Earth's history, motion leaves behind its own informational layer:

Distance
Velocity
Acceleration

Stacked vertically, these form the **DiVA Chart** — the thin layer that records motion.

When students understand how to read this layer, motion stops being mysterious.

📊 Revisiting the DiVA Structure

We emphasized again:

Differentiate → move down the chart
Integrate → move up the chart

Students were asked repeatedly:

- "Which layer are we on?"
- "Are we simplifying or accumulating?"
- "Does the slope match your equation?"
- "Does the shaded area confirm your result?"

The goal was not just algebraic correctness, but coherence between:

Equation
Graph
Physical meaning

When all three agree, understanding is solid.

⚙️ From Constant to Changing Acceleration

Earlier sections focused heavily on constant acceleration.

In Adventure 8, we began exploring what happens when acceleration changes over time.

Students observed:

- Linear velocity can produce quadratic distance.
- Changing acceleration produces curved velocity graphs.
- Small differences in rate accumulate into large differences in outcome.

This is the first time students clearly see how behavior over long time spans depends more on rates than on initial values.

🧮 Derivatives as "Behavior Controllers"

A key discussion point:

The function tells you *where you are*.
The derivative tells you *where you are going*.

We examined:

- Positive vs. negative velocity
- Increasing vs. decreasing acceleration
- When motion speeds up, slows down, or reverses

Students were encouraged to predict graph shapes before computing.

Prediction before calculation strengthens conceptual control.

🔥 Energy Revisited

We briefly reconnected to kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Students were reminded:

- Work accumulates through force acting over distance.
- Acceleration changes velocity.
- Velocity determines kinetic energy.

Thus the DiVA structure is not isolated from energy — it feeds directly into it.

This reinforces the unification of calculus and mechanics.

🧠 Key Teaching Insight

Every layer of DiVA reveals a different aspect of the same motion.

Acceleration shows the cause.
Velocity shows the behavior.
Distance shows the history.

Students who can move comfortably between these three views are ready for more advanced modeling and long-time analysis.

🗣 Discussion Prompts Used in Session

- “If acceleration is zero, what must velocity look like?”
- “If velocity is constant, what does distance look like?”
- “Which graph reveals change most directly?”
- “Why are slopes often more informative than raw values?”

Students were asked to explain answers verbally, not just compute them.

🚀 Why Adventure 8 Matters

Adventure 8 is a transition point.

Students move from:

Procedural differentiation

to

Structural understanding of motion.

This prepares them for:

- Ratio reasoning in later sections
- Long-time behavior analysis
- Comparing growth rates
- Understanding why derivatives control everything that follows

✨ Reflection

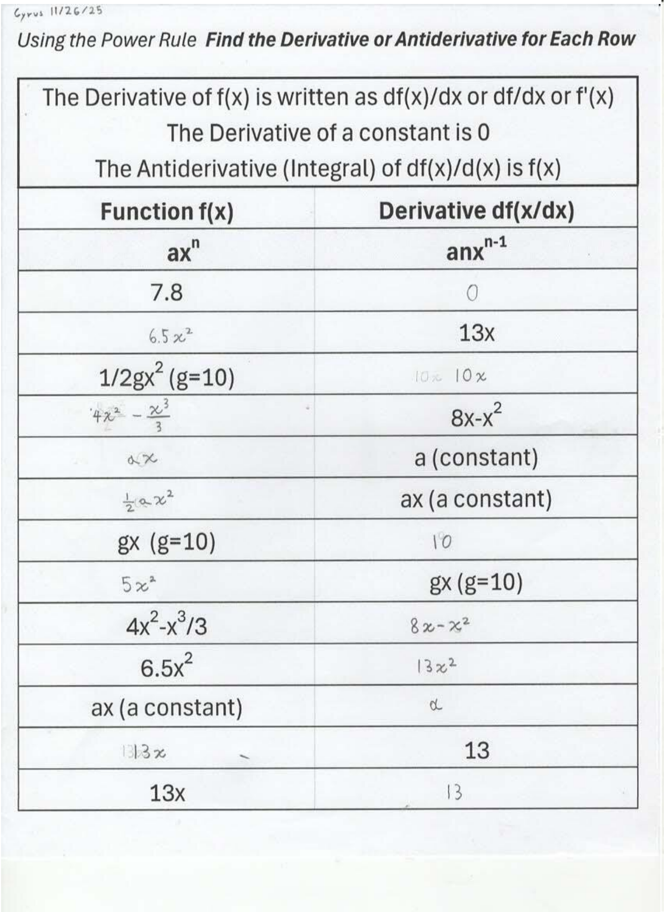
Motion is not chaos.

It only appears chaotic when we do not read its layer correctly.

The DiVA chart is the guiding light.

Adventure 8 deepens students’ ability to read that light.

📘 Completed Student Activity Sheets

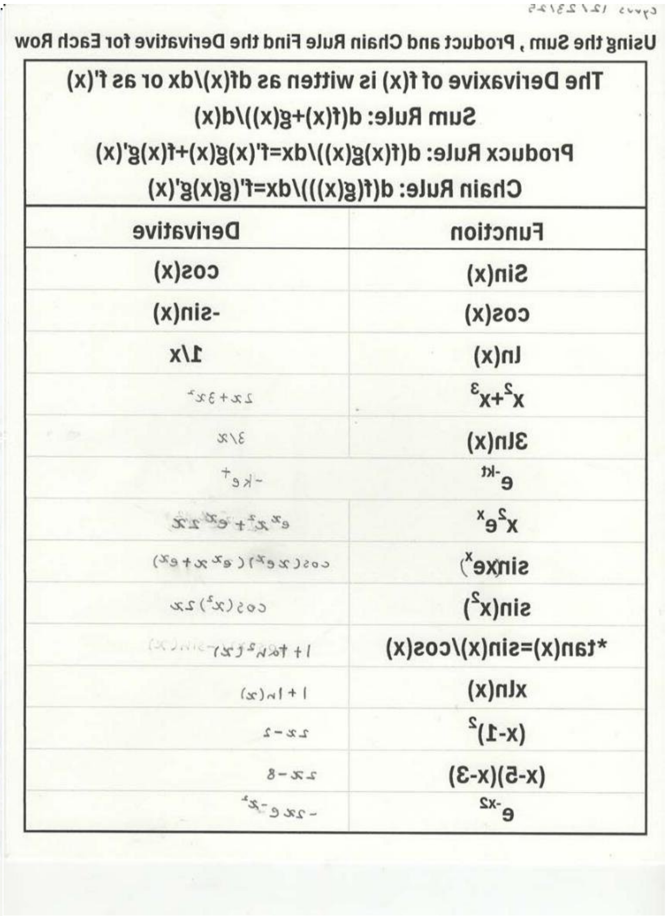


Cyrus 11/26/25

Using the Power Rule ***Find the Derivative or Antiderivative for Each Row***

The Derivative of f(x) is written as df(x)/dx or df/dx or f'(x)
The Derivative of a constant is 0
The Antiderivative (Integral) of df(x)/d(x) is f(x)

Function f(x)	**Derivative df(x/dx)**
ax^n	**anx^{n-1}**
7.8	0
$6.5x^2$	13x
$1/2gx^2$ (g=10)	~~10x~~ 10x
$4x^2 - \frac{x^3}{3}$	$8x-x^2$
ax	a (constant)
$\frac{1}{2}ax^2$	ax (a constant)
gx (g=10)	10
$5x^2$	gx (g=10)
$4x^2-x^3/3$	$8x-x^2$
$6.5x^2$	$13x^2$
ax (a constant)	a
~~13~~13x	13
13x	13

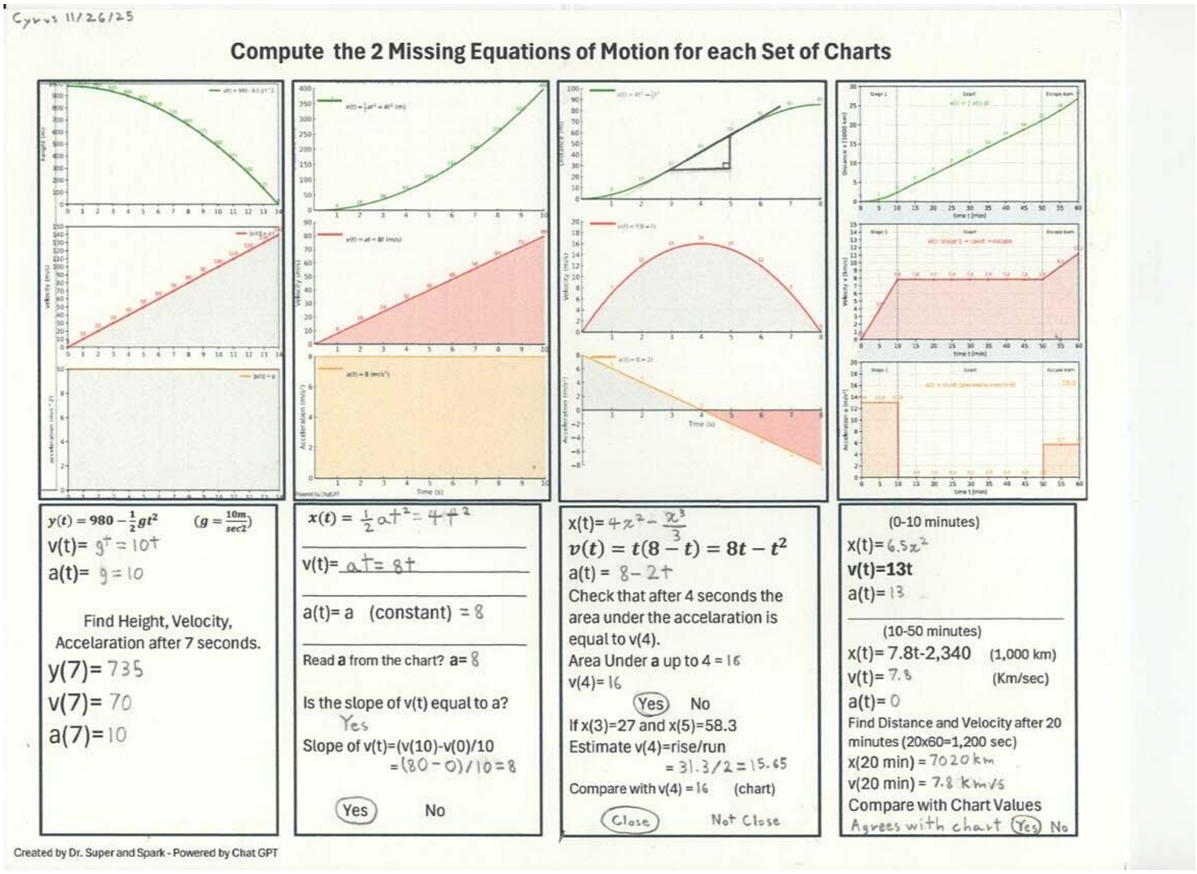


Cyrus 12/23/25

Using the Sum , Product and Chain Rule Find the Derivative for Each Row

The Derivaxive of f(x) is witten as df(x)/dx or as f'(x)
Sum Rule: d(f(x)+g(x))/d(x)
Producx Rule: d(f(x)g(x))/dx=f'(x)g(x)+f(x)g'(x)
Chain Rule: d(f(g(x)))/dx=f'(g(x))g'(x)

Function	**Derivative**
Sin(x)	**cos(x)**
cos(x)	**-sin(x)**
ln(x)	**1/x**
x^2+x^3	$2x+3x^2$
3ln(x)	3/x
e^{-kt}	$-ke^{t}$
x^2e^x	$e^x x^2 + e^x 2x$
$\sin(xe^x)$	$\cos(xe^x)(e^x x + e^x)$
$\sin(x^2)$	$\cos(x^2)2x$
***tan(x)=sin(x)/cos(x)**	$1+\tan^2(x)$
xln(x)	1+ln(x)
$(x-1)^2$	2x-2
(x-5)(x-3)	2x-8
e^{-x2}	$-2xe^{-x^2}$



Cyrus 11/26/25

Compute the 2 Missing Equations of Motion for each Set of Charts

$y(t) = 980 - \frac{1}{2}gt^2$ $(g = \frac{10m}{sec2})$
v(t)= gt = 10t
a(t)= g = 10

Find Height, Velocity, Accelaration after 7 seconds.
y(7)= 735
v(7)= 70
a(7)= 10

$x(t) = \frac{1}{2}at^2 = 4t^2$
v(t)= at = 8t
a(t)= a (constant) = 8
Read **a** from the chart? **a**= 8
Is the slope of v(t) equal to a? Yes
Slope of v(t)=(v(10)-v(0)/10 =(80-0)/10=8
[✓ Yes] No

x(t)= $4x^2 - \frac{x^3}{3}$
$v(t) = t(8-t) = 8t - t^2$
a(t) = 8-2t
Check that after 4 seconds the area under the accelaration is equal to v(4).
Area Under a up to 4 = 16
v(4)= 16
[✓ Yes] No
If x(3)=27 and x(5)=58.3
Estimate v(4)=rise/run = 31.3/2 = 15.65
Compare with v(4) = 16 (chart)
[✓ Close] Not Close

(0-10 minutes)
x(t)= $6.5x^2$
v(t)=13t
a(t)= 13

(10-50 minutes)
x(t)= 7.8t-2,340 (1,000 km)
v(t)= 7.8 (Km/sec)
a(t)= 0
Find Distance and Velocity after 20 minutes (20x60=1,200 sec)
x(20 min) = 7020 km
v(20 min) = 7.8 km/s
Compare with Chart Values
Agrees with chart [✓ Yes] No

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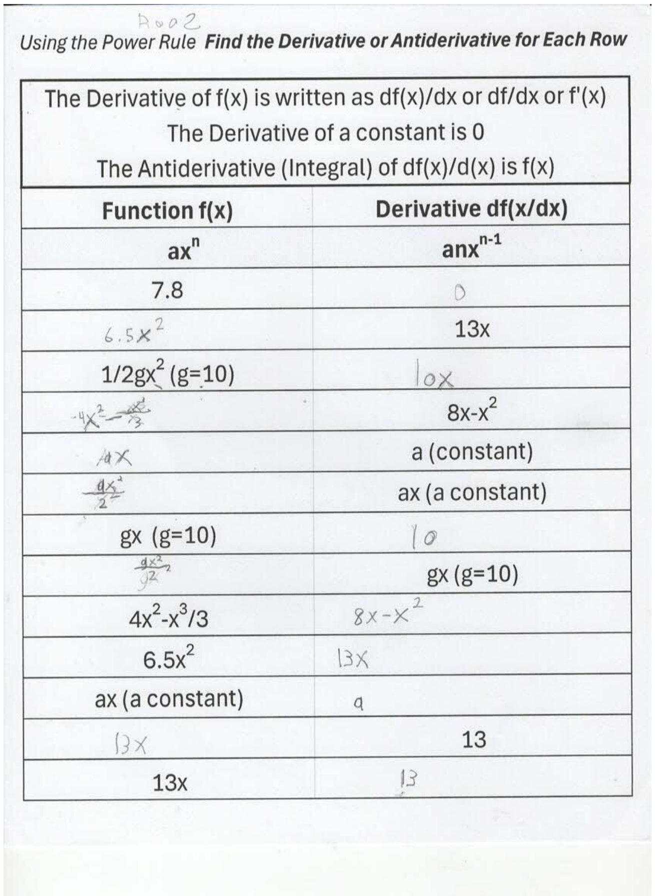
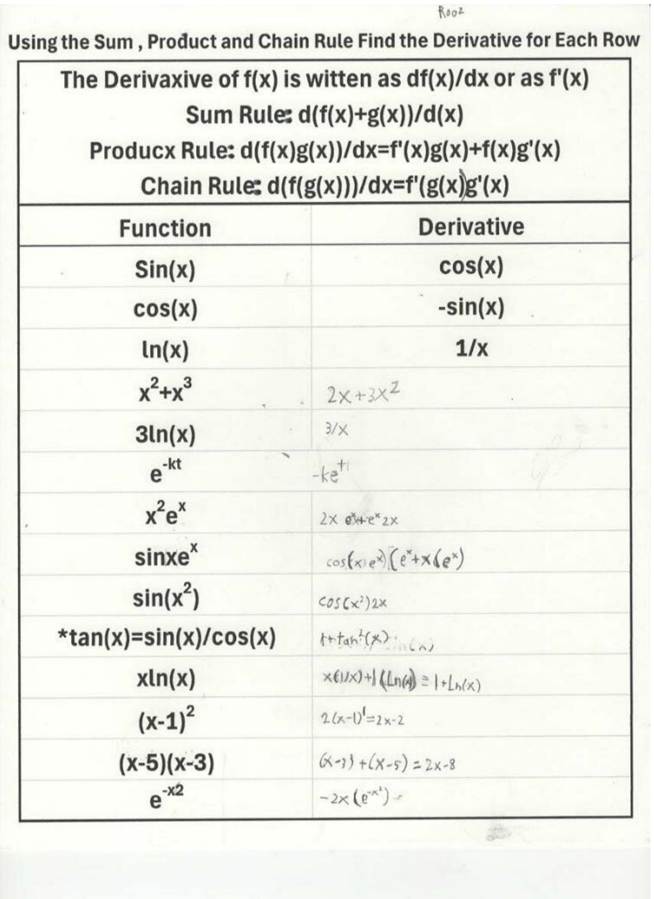


R002

Using the Power Rule ***Find the Derivative or Antiderivative for Each Row***

The Derivative of f(x) is written as df(x)/dx or df/dx or f'(x)
The Derivative of a constant is 0
The Antiderivative (Integral) of df(x)/d(x) is f(x)

Function f(x)	**Derivative df(x/dx)**
ax^n	anx^{n-1}
7.8	0
$6.5x^2$	13x
$1/2gx^2$ (g=10)	10x
$-4x^2-\frac{x^3}{3}$	$8x-x^2$
ax	a (constant)
$\frac{ax^2}{2}$	ax (a constant)
gx (g=10)	10
$\frac{gx^2}{2}$	gx (g=10)
$4x^2-x^3/3$	$8x-x^2$
$6.5x^2$	13x
ax (a constant)	a
13x	13
13x	13

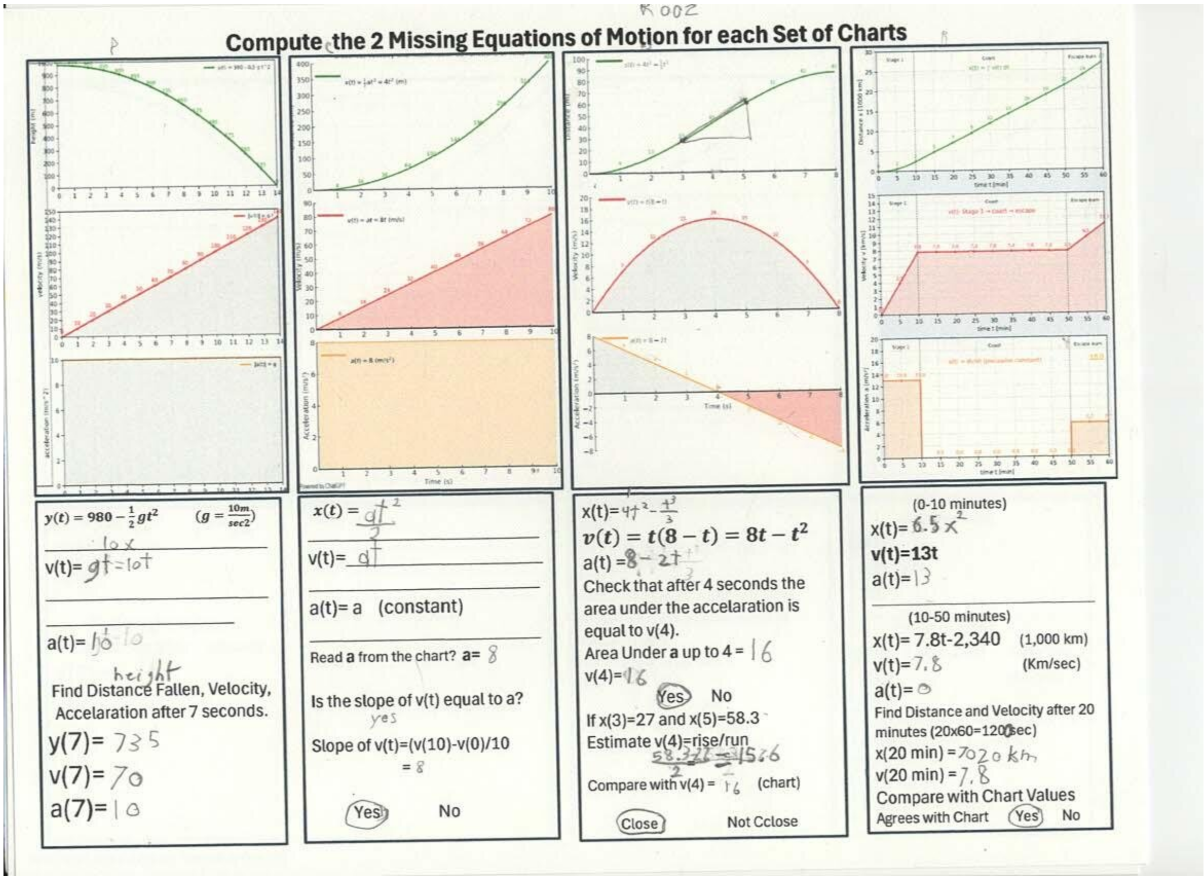


R002

Using the Sum , Product and Chain Rule Find the Derivative for Each Row

The Derivaxive of f(x) is witten as df(x)/dx or as f'(x)
Sum Rule: d(f(x)+g(x))/d(x)
Producx Rule: d(f(x)g(x))/dx=f'(x)g(x)+f(x)g'(x)
Chain Rule: d(f(g(x)))/dx=f'(g(x))g'(x)

Function	**Derivative**
Sin(x)	**cos(x)**
cos(x)	**-sin(x)**
ln(x)	**1/x**
x^2+x^3	$2x+3x^2$
3ln(x)	3/x
e^{-kt}	$-ke^{t}$
x^2e^x	$2x\, e^x+e^x 2x$
$\sin xe^x$	$\cos(x e^x)(e^x+x(e^x))$
$\sin(x^2)$	$\cos(x^2)2x$
***tan(x)=sin(x)/cos(x)**	$1+\tan^2(x)$
xln(x)	x(1/x)+1(Ln(x)) = 1+Ln(x)
$(x-1)^2$	$2(x-1)'=2x-2$
(x-5)(x-3)	(x-3)+(x-5) = 2x-8
e^{-x2}	$-2x(e^{-x^2})$



R002

Compute the 2 Missing Equations of Motion for each Set of Charts

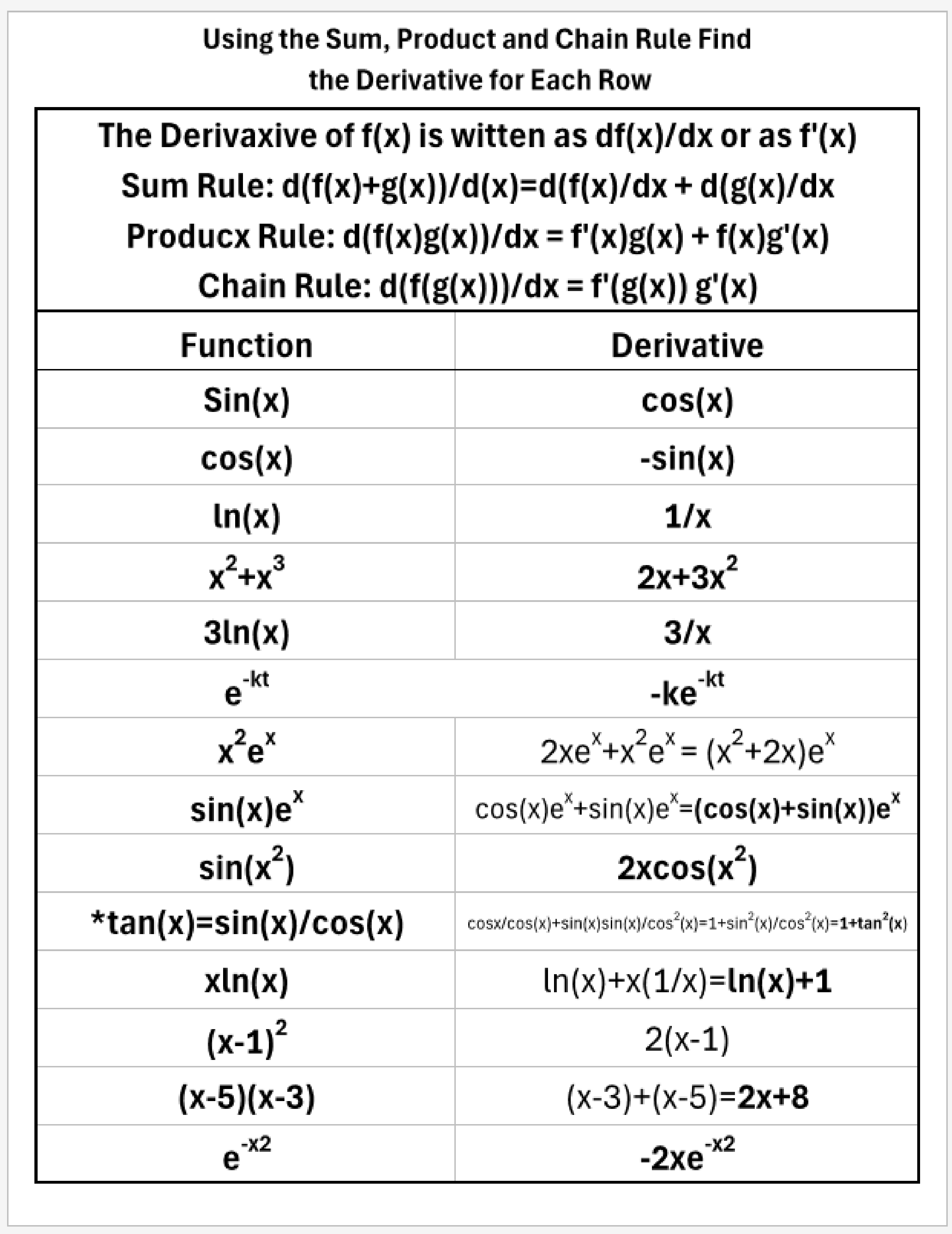
$y(t) = 980 - \frac{1}{2}gt^2$ $(g = \frac{10m}{sec2})$
10x
v(t)= gt=10t
a(t)= 10
height
Find Distance Fallen, Velocity, Accelaration after 7 seconds.
y(7)= 735
v(7)= 70
a(7)= 10

$x(t) = \frac{at^2}{2}$
v(t)= at
a(t)= a (constant)
Read **a** from the chart? **a**= 8
Is the slope of v(t) equal to a?
yes
Slope of v(t)=(v(10)-v(0)/10
= 8
[✓ Yes] No

$x(t)=4t^2-\frac{t^3}{3}$
$v(t) = t(8-t) = 8t - t^2$
a(t) =8-2t
Check that after 4 seconds the area under the accelaration is equal to v(4).
Area Under **a** up to 4 = 16
v(4)=16
[✓ Yes] No
If x(3)=27 and x(5)=58.3
Estimate v(4)=rise/run
$\frac{58.3-27}{2} = 15.6$
Compare with v(4) = 16 (chart)
[✓ Close] Not Cclose

(0-10 minutes)
$x(t)= 6.5x^2$
v(t)=13t
a(t)= 13
(10-50 minutes)
x(t)= 7.8t-2,340 (1,000 km)
v(t)=7.8 (Km/sec)
a(t)= 0
Find Distance and Velocity after 20 minutes (20x60=1200sec)
x(20 min) = 7020 km
v(20 min) = 7.8
Compare with Chart Values
Agrees with Chart [✓ Yes] No

Student Activity Sheets Solutions



Using the Sum, Product and Chain Rule Find the Derivative for Each Row

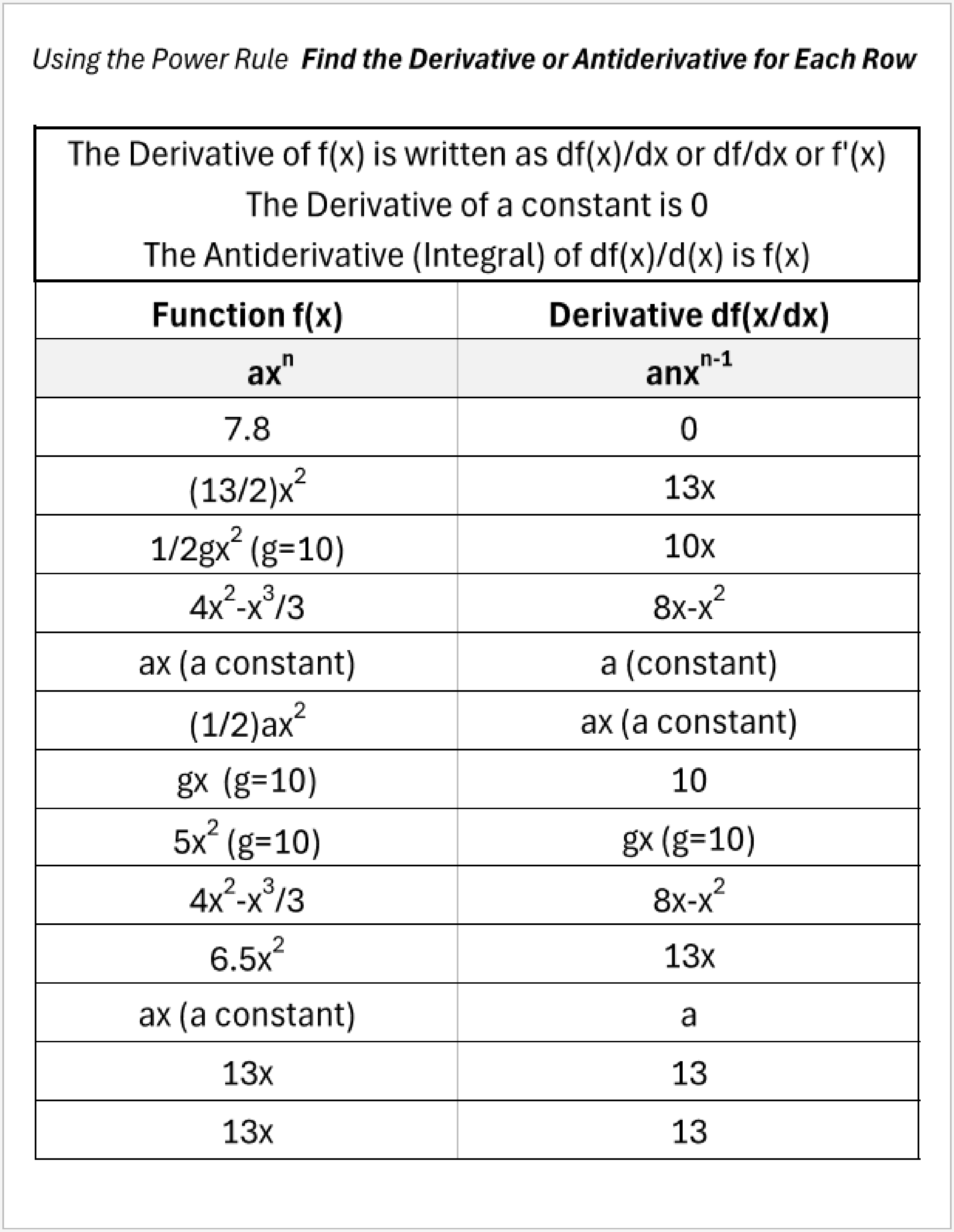
The Derivaxive of f(x) is witten as df(x)/dx or as f'(x)

Sum Rule: d(f(x)+g(x))/d(x)=d(f(x)/dx + d(g(x)/dx

Producx Rule: d(f(x)g(x))/dx = f'(x)g(x) + f(x)g'(x)

Chain Rule: d(f(g(x)))/dx = f'(g(x)) g'(x)

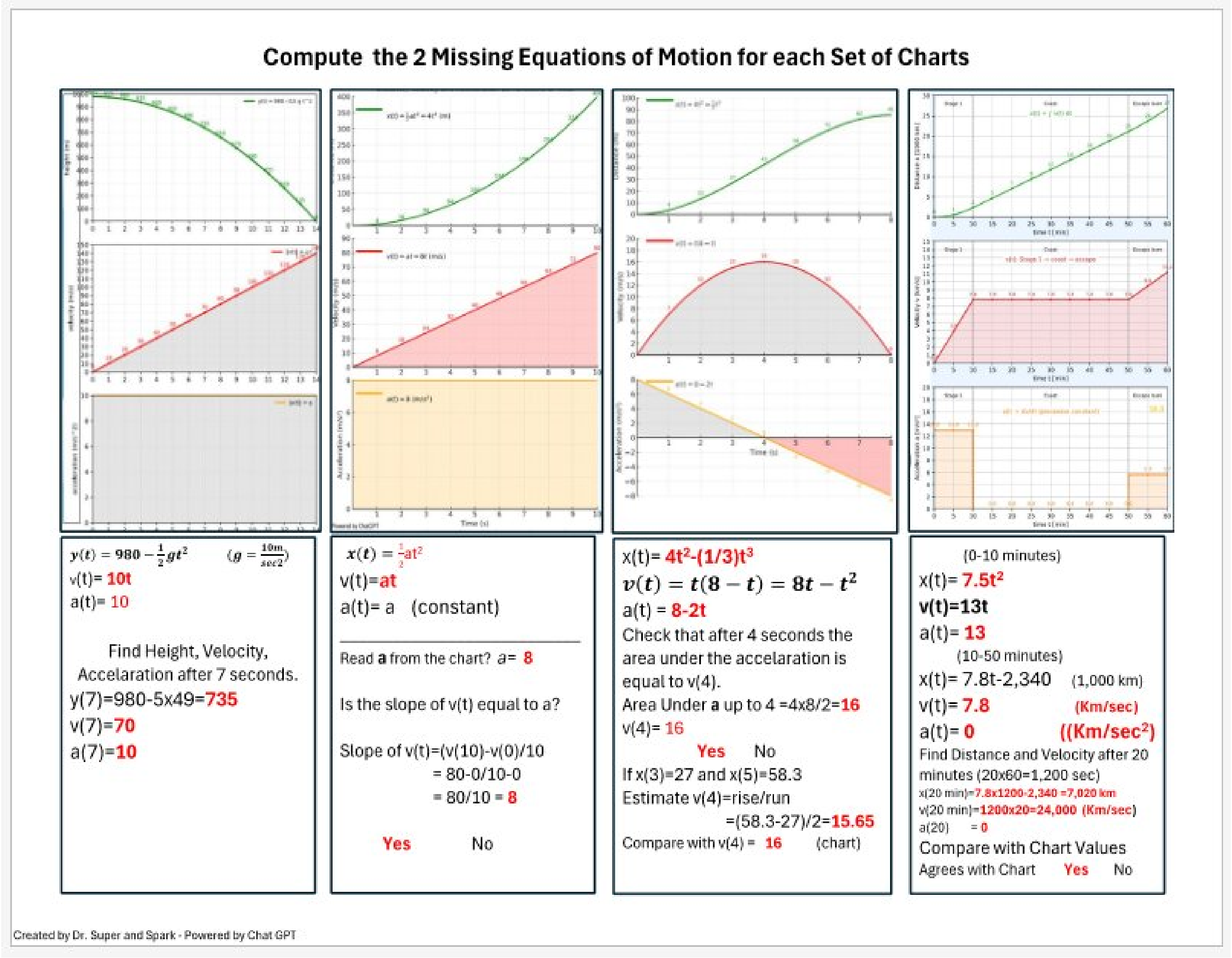
Function	Derivative
Sin(x)	cos(x)
cos(x)	-sin(x)
ln(x)	1/x
x^2+x^3	$2x+3x^2$
3ln(x)	3/x
e^{-kt}	$-ke^{-kt}$
x^2e^x	$2xe^x+x^2e^x = (x^2+2x)e^x$
$\sin(x)e^x$	$\cos(x)e^x+\sin(x)e^x=(\cos(x)+\sin(x))e^x$
$\sin(x^2)$	$2x\cos(x^2)$
*tan(x)=sin(x)/cos(x)	$\cos x/\cos(x)+\sin(x)\sin(x)/\cos^2(x)=1+\sin^2(x)/\cos^2(x)=1+\tan^2(x)$
xln(x)	ln(x)+x(1/x)=ln(x)+1
$(x-1)^2$	2(x-1)
(x-5)(x-3)	(x-3)+(x-5)=2x+8
e^{-x2}	$-2xe^{-x2}$



Using the Power Rule ***Find the Derivative or Antiderivative for Each Row***

The Derivative of f(x) is written as df(x)/dx or df/dx or f'(x)
The Derivative of a constant is 0
The Antiderivative (Integral) of df(x)/d(x) is f(x)

Function f(x)	**Derivative df(x/dx)**
ax^n	**anx^{n-1}**
7.8	0
$(13/2)x^2$	13x
$1/2gx^2$ (g=10)	10x
$4x^2-x^3/3$	$8x-x^2$
ax (a constant)	a (constant)
$(1/2)ax^2$	ax (a constant)
gx (g=10)	10
$5x^2$ (g=10)	gx (g=10)
$4x^2-x^3/3$	$8x-x^2$
$6.5x^2$	13x
ax (a constant)	a
13x	13
13x	13



Compute the 2 Missing Equations of Motion for each Set of Charts

$y(t) = 980 - \frac{1}{2}gt^2$ $\quad (g = \frac{10m}{sec2})$

v(t)= 10t

a(t)= 10

Find Height, Velocity, Accelaration after 7 seconds.

y(7)=980-5x49=735

v(7)=70

a(7)=10

$x(t) = \frac{1}{2}at^2$

v(t)=at

a(t)= a (constant)

Read **a** from the chart? a= 8

Is the slope of v(t) equal to a?

Slope of v(t)=(v(10)-v(0)/10
= 80-0/10-0
= 80/10 = 8

[✓ Yes] No

---

x(t)= $4t^2$-(1/3)t^3

$v(t) = t(8-t) = 8t - t^2$

a(t) = 8-2t

Check that after 4 seconds the area under the accelaration is equal to v(4).

Area Under **a** up to 4 =4x8/2=16

v(4)= 16

[✓ Yes] No

If x(3)=27 and x(5)=58.3

Estimate v(4)=rise/run
=(58.3-27)/2=15.65

Compare with v(4) = 16 (chart)

(0-10 minutes)

x(t)= 7.5t^2

v(t)=13t

a(t)= 13

(10-50 minutes)

x(t)= 7.8t-2,340 (1,000 km)

v(t)= 7.8 (Km/sec)

a(t)= 0 ((Km/sec^2)

Find Distance and Velocity after 20 minutes (20x60=1,200 sec)

x(20 min)=7.8x1200-2,340 =7,020 km

v(20 min)=1200x20=24,000 (Km/sec)

a(20) = 0

Compare with Chart Values

Agrees with Chart [✓ Yes] No

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Adventure 9 – The Age of the Earth — Clair Patterson & Exponential Decay

(Clair Patterson and the Age of Earth – U-235 Exponential)

- Date: 12/03/2025
- Duration: 1.5 Hours 1:50-3:20 pm
- Students present: Cyrus, Rooz, Marc
- Main objective of the session: Study the decay of Uranium-235 to Lead and redo the calculations of Clair Patterson to estimate the age of the earth using the half-life for U-235.
- Summary of what the students discovered:
  - They discovered that the derivative of e^x is e^x.
  - They became familiar with the decay formula for U-235 and that speed and acceleration for this process have similar shape.
  - Estimated the age of Earth both from graphs and charts
  - Using integrals and charts to estimate the mass of U-235 remaining and Lead produced.

---

2. Warm-Up Conversation / Storytelling/Videos

- I told them that Spark and I had spent a lot of time on preparing this activity for them and it will be long but if they focused, they will learn and connect many interesting ideas from Calculus to Mechanics and estimate the age of Earth themselves.
- Then we watched the video: What's so special about Euler's number e? | Chapter 5, Essence of calculus from 3Blue1Brown Chanelle on You Tube. As usual we did stop it a few times and discussed it. Cyrus and Marc are more familiar with Euler's number and exponentials but for Rooz this was relatively new. I wanted to make sure that they all were comfortable with how the derivatives of exponentials were calculated.
- Next each one of us read a paragraph of the story: ***The Age of Earth, Clair Patterson, and the Planet Covered in Lead.*** *I gave them* story sheet first and went to get them some drinks so they would have time to read it for themselves first.

- They liked the story (Cyrus and Rooz had heard it before). This is my favorite science story as it brings so many different aspects of math, physics and science history together.
- The charts for the activity were on the back of the Story sheets, and I had them examine it and talked a little about U-235 turning to lead and how we start with 1024 grams of U-235. I told them that we were calling these charts DiVA charts and asked them if they knew why. But did not get and answer so I explained that from Distance-Velocity-Acceleration we made the acronym DiVA. Reception was Lukewarm. Of course, Distance can be replaced by other quantities in these charts, here it is Mass of U-235 remaining and Lead produced.
- I also told them that we should try every problem together and discuss it before they move on. This will make the Math Circle much more interactive, and this is the purpose of the Math Circle. Usually, Marc rushes forward Cyrus will meticulously do his work slowly and I have to drag Rooz along.

📊 3. Activities Completed

Part 1 — The derivative of e^x is e^x

- They needed a little push, and I also started this on the board but they got the concept. I helped Rooz more than Marc and Cyrus.
- We went over the Chain rule for the derivative of $\mathbf{e}^{-x}$ and they did it and I went over it a few times even when we were watching the video.

Part 2 — Finding the Decay Constant from the Half-Life of U-235

- At the beginning, I did not discuss enough how the formula for the decay matched with the chart for M(t) but did mention that they had to find k. Cyrus said that there was no k on the charts and I had them come up with:”k would turn out to be 1”.
- Even though I had tried to simplify the steps it was not clear exactly what they had to do at each step. I had to help them so they would find the value of k in the formula. Then it was easy for them to agree that it was close to 1.

Part 3 — Completing the Half-Life Table

- This activity went well.
- All 3 quickly filled the half-life chart. Rooz was first as he loves his powers of 2.
- They could all see and recorded that the Mass was cut in half after each half-life.
- There was a lot of discussion about what to use for the final half-live. Cyrus and Marc went with 6.5, but Rooz kept going from 6.4 to finally 6.35. This was great and I did mention that to correctly estimate we need to see that as we go on less mass remains.
- This could have been a good place to talk about linear and nonlinear extrapolations.
- Reading the charts was not easy to find where 18 grams will remain. They each went with their own numbers.

Part 4 — Reading the Charts

- I explained the relation of L(t) and M(t) and how the derivative of L(t) was v(t).
- Cyrus asked about the derivative of M(t) and that was what I had lost a night of sleep on, So I explained that that the values will be negative and it is v(t) that is the derivative of L(t).
- They had a good time going back and forth estimating the values from the charts.

Part 5 — Checking the Mass at $t = 1$Using an Integral

- This was not that hard for them to do, and I was surprised as I had struggled with tit myself. But they all could see how the values were calculated. I asked about the integral of v(t) and what that was showing geometrically. Eventually both Cyrus and Rooz said that it was the area under v(t) and Cyrus even showed the area.

🧑‍🏫 4. What Worked Well/Or Did Not

- They really like the videos that are well made and explain everything well.
- They liked the story very much and the trick of letting them read it for themselves before we read it together worked well. I must try it again.
- Finding the age of the Earth went well as they like powers of 2.
- The reading of the charts finding and doing the integrals also went well.
- The estimation of k was hard for them, so the steps have to be simplified.

🧠 5. Adjustments for Next Time

- Initially the decay formula should be clearly connected to the chart.
- Activity really needs more time if the students are to do it by themselves.

🪄 6. Student Quotes

- Cyrus was quick to see that v(t) was not really the derivative of M(t).
- Rooz was great on the Half-life estimation and more creative with his estimations.
- Marc took his time more and did not rush forward and participated well.

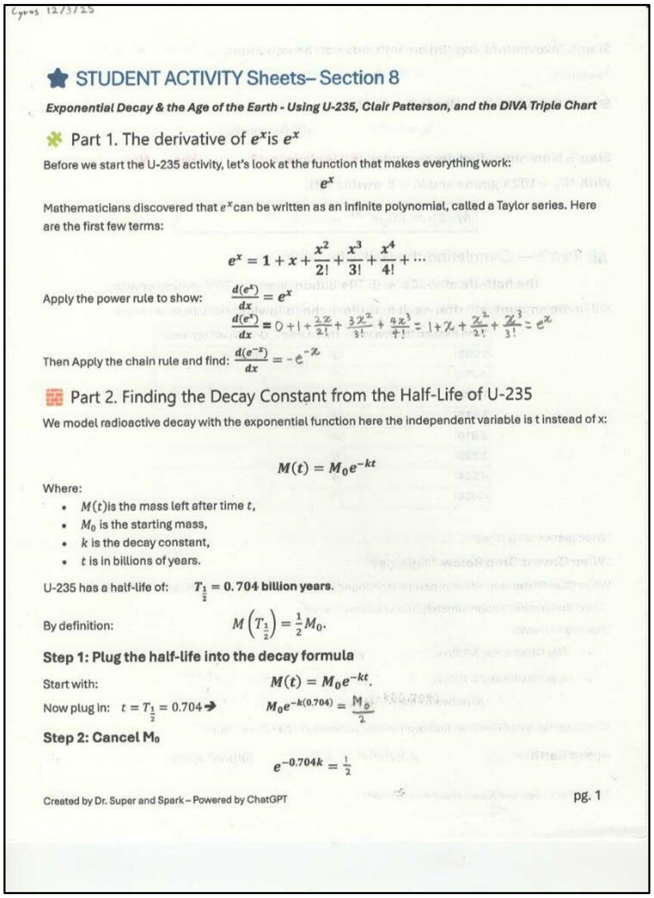
🧱 8. Closing Reflection by Dr. Super

Spark and I spent even more time preparing and testing this activity than the last one. I had to bribe them with "Honest Kids" drinks and promise of ice cream afterwards, but they lasted for an hour and thirty minutes and were focused almost all the time. Rooz did so much better and as soon as we got to the estimation of the age of earth he became the front runner. I am so glad that this activity went so well as it has a lot of math, physics, and science history and it is truly an inspiring activity for the kids to estimate the age of Earth.

📽 .7. Photos / Charts / Sketches

(From Left to Right Rooz, Cyrus and Marc)

8. Student Completed Worksheets



Cyrus 12/3/25

STUDENT ACTIVITY Sheets– Section 8

Exponential Decay & the Age of the Earth - Using U-235, Clair Patterson, and the DIVA Triple Chart

Part 1. The derivative of e^x is e^x

Before we start the U-235 activity, let's look at the function that makes everything work:

$$e^x$$

Mathematicians discovered that e^x can be written as an infinite polynomial, called a Taylor series. Here are the first few terms:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Apply the power rule to show: $\frac{d(e^x)}{dx} = e^x$

$$\frac{d(e^x)}{dx} = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} = e^x$$

Then Apply the chain rule and find: $\frac{d(e^{-x})}{dx} = -e^{-x}$

Part 2. Finding the Decay Constant from the Half-Life of U-235

We model radioactive decay with the exponential function here the independent variable is t instead of x:

$$M(t) = M_0 e^{-kt}$$

Where:

- $M(t)$ is the mass left after time t,
- M_0 is the starting mass,
- k is the decay constant,
- t is in billions of years.

U-235 has a half-life of: $T_{\frac{1}{2}} = 0.704$ **billion years.**

By definition: $M\left(T_{\frac{1}{2}}\right) = \frac{1}{2} M_0.$

Step 1: Plug the half-life into the decay formula

Start with: $M(t) = M_0 e^{-kt}.$

Now plug in: $t = T_{\frac{1}{2}} = 0.704$ ➔ $M_0 e^{-k(0.704)} = \frac{M_0}{2}$

Step 2: Cancel M₀

$$e^{-0.704k} = \frac{1}{2}$$

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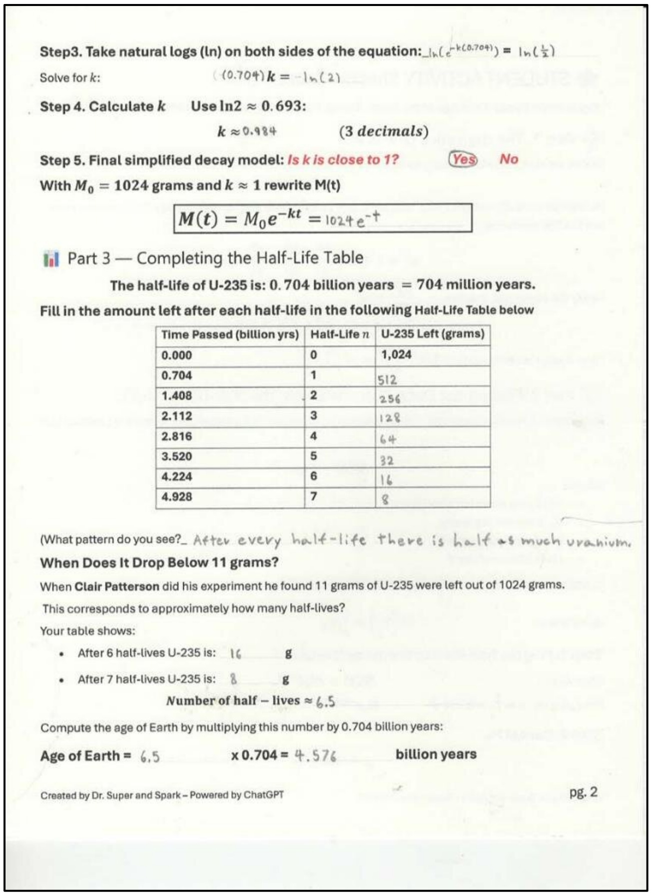
pg. 1



Step3. Take natural logs (ln) on both sides of the equation: $\ln(e^{-k(0.704)}) = \ln(\frac{1}{2})$

Solve for k: $(0.704)k = -\ln(2)$

Step 4. Calculate k Use $\ln 2 \approx 0.693$:

$k \approx 0.984$ $(3\ decimals)$

Step 5. Final simplified decay model: ***Is k is close to 1?*** [✓ Yes] No

With $M_0 = 1024$ grams and $k \approx 1$ rewrite M(t)

$$\boxed{M(t) = M_0 e^{-kt} = 1024 e^{-t}}$$

Part 3 — Completing the Half-Life Table

The half-life of U-235 is: 0.704 billion years = 704 million years.

Fill in the amount left after each half-life in the following Half-Life Table below

Time Passed (billion yrs)	Half-Life n	U-235 Left (grams)
0.000	0	1,024
0.704	1	512
1.408	2	256
2.112	3	128
2.816	4	64
3.520	5	32
4.224	6	16
4.928	7	8

(What pattern do you see?_ After every half-life there is half as much uranium.

When Does It Drop Below 11 grams?

When **Clair Patterson** did his experiment he found 11 grams of U-235 were left out of 1024 grams.

This corresponds to approximately how many half-lives?

Your table shows:

- After 6 half-lives U-235 is: 16 **g**
- After 7 half-lives U-235 is: 8 **g**

$$\mathit{Number\ of\ half-lives} \approx 6.5$$

Compute the age of Earth by multiplying this number by 0.704 billion years:

Age of Earth = 6.5 **x 0.704 =** 4.576 **billion years**

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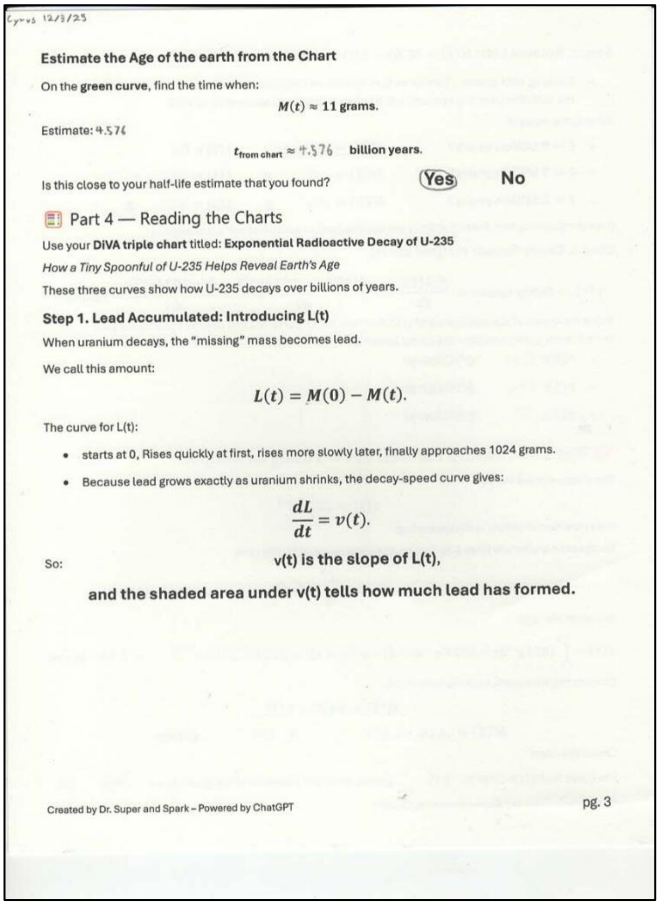
pg. 2



Cyrus 12/3/25

Estimate the Age of the earth from the Chart

On the **green curve**, find the time when:

$$M(t) \approx 11 \text{ grams.}$$

Estimate: 4.576

$t_{\text{from chart}} \approx 4.576$ **billion years.**

Is this close to your half-life estimate that you found? [✓ Yes] No

Part 4 — Reading the Charts

Use your **DiVA triple chart** titled: **Exponential Radioactive Decay of U-235**

How a Tiny Spoonful of U-235 Helps Reveal Earth's Age

These three curves show how U-235 decays over billions of years.

Step 1. Lead Accumulated: Introducing L(t)

When uranium decays, the "missing" mass becomes lead.

We call this amount:

$$L(t) = M(0) - M(t).$$

The curve for L(t):

- starts at 0, Rises quickly at first, rises more slowly later, finally approaches 1024 grams.
- Because lead grows exactly as uranium shrinks, the decay-speed curve gives:

$$\frac{dL}{dt} = v(t).$$

So: **v(t) is the slope of L(t),**

and the shaded area under v(t) tells how much lead has formed.

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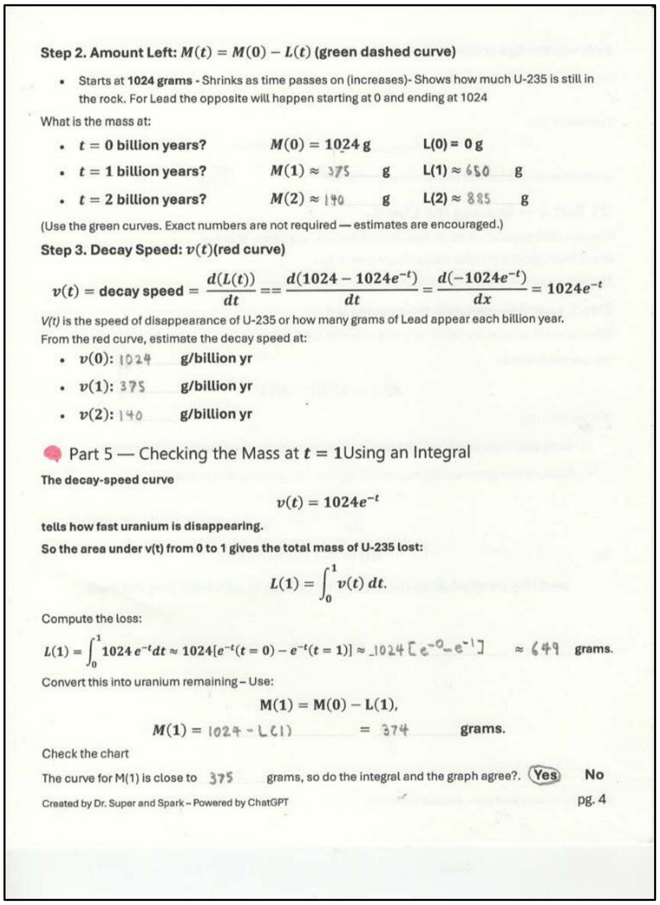
pg. 3



Step 2. Amount Left: $M(t) = M(0) - L(t)$ (green dashed curve)

- Starts at **1024 grams** - Shrinks as time passes on (increases)- Shows how much U-235 is still in the rock. For Lead the opposite will happen starting at 0 and ending at 1024

What is the mass at:

- $t = 0$ **billion years?** $M(0) = 1024$ g L(0) = 0 g
- $t = 1$ **billion years?** $M(1) \approx 375$ g L(1) ≈ 650 g
- $t = 2$ **billion years?** $M(2) \approx 140$ g L(2) ≈ 885 g

(Use the green curves. Exact numbers are not required — estimates are encouraged.)

Step 3. Decay Speed: $v(t)$(red curve)

$$v(t) = \text{decay speed} = \frac{d(L(t))}{dt} == \frac{d(1024 - 1024e^{-t})}{dt} = \frac{d(-1024e^{-t})}{dx} = 1024e^{-t}$$

V(t) is the speed of disappearance of U-235 or how many grams of Lead appear each billion year.

From the red curve, estimate the decay speed at:

- $v(0)$: 1024 **g/billion yr**
- $v(1)$: 375 **g/billion yr**
- $v(2)$: 140 **g/billion yr**

Part 5 — Checking the Mass at $t = 1$ Using an Integral

The decay-speed curve

$$v(t) = 1024e^{-t}$$

tells how fast uranium is disappearing.

So the area under v(t) from 0 to 1 gives the total mass of U-235 lost:

$$L(1) = \int_0^1 v(t)\, dt.$$

Compute the loss:

$$L(1) = \int_0^1 1024 e^{-t} dt \approx 1024[e^{-t}(t=0) - e^{-t}(t=1)] \approx 1024[e^{-0} - e^{-1}] \approx 649 \text{ grams.}$$

Convert this into uranium remaining – Use:

$$\mathrm{M(1) = M(0) - L(1),}$$

$M(1) = 1024 - L(1)$ = 374 **grams.**

Check the chart

The curve for M(1) is close to 375 grams, so do the integral and the graph agree?. [✓ Yes] No

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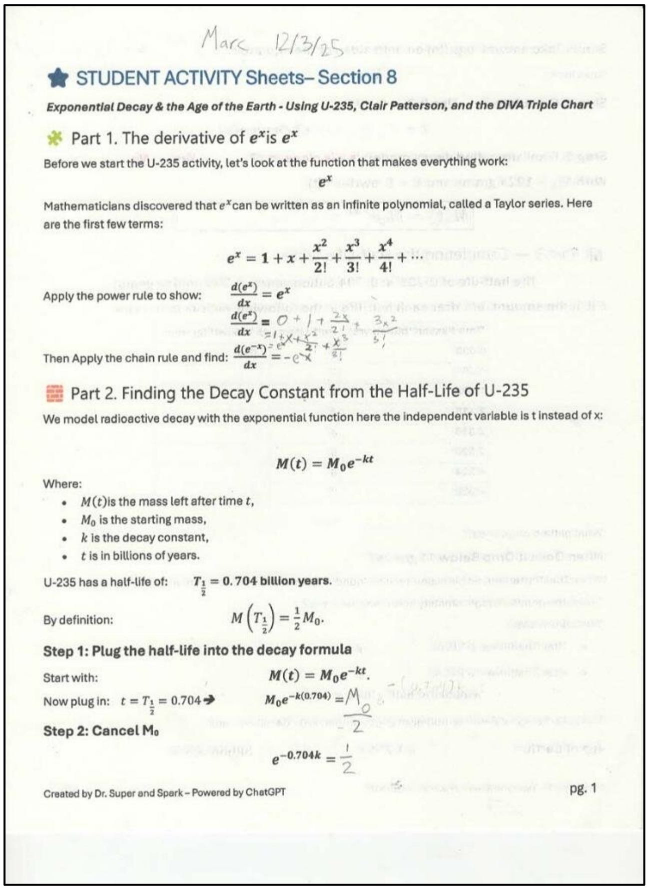
pg. 4



Marc 12/3/25

STUDENT ACTIVITY Sheets– Section 8

Exponential Decay & the Age of the Earth - Using U-235, Clair Patterson, and the DIVA Triple Chart

Part 1. The derivative of e^x is e^x

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$$e^x$$

Mathematicians discovered that e^x can be written as an infinite polynomial, called a Taylor series. Here are the first few terms:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Apply the power rule to show: $\frac{d(e^x)}{dx} = e^x$

$$\frac{d(e^x)}{dx} = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{5!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} = e^x$$

Then Apply the chain rule and find: $\frac{d(e^{-x})}{dx} = -e^{-x}$

Part 2. Finding the Decay Constant from the Half-Life of U-235

We model radioactive decay with the exponential function here the independent variable is t instead of x:

$$M(t) = M_0 e^{-kt}$$

Where:

- $M(t)$ is the mass left after time t,
- M_0 is the starting mass,
- k is the decay constant,
- t is in billions of years.

U-235 has a half-life of: $T_{\frac{1}{2}} = 0.704$ **billion years.**

By definition: $M\left(T_{\frac{1}{2}}\right) = \frac{1}{2} M_0.$

Step 1: Plug the half-life into the decay formula

Start with: $M(t) = M_0 e^{-kt}.$

Now plug in: $t = T_{\frac{1}{2}} = 0.704$ ➔ $M_0 e^{-k(0.704)} = \frac{M_0}{2}$

Step 2: Cancel M₀

$$e^{-0.704k} = \frac{1}{2}$$

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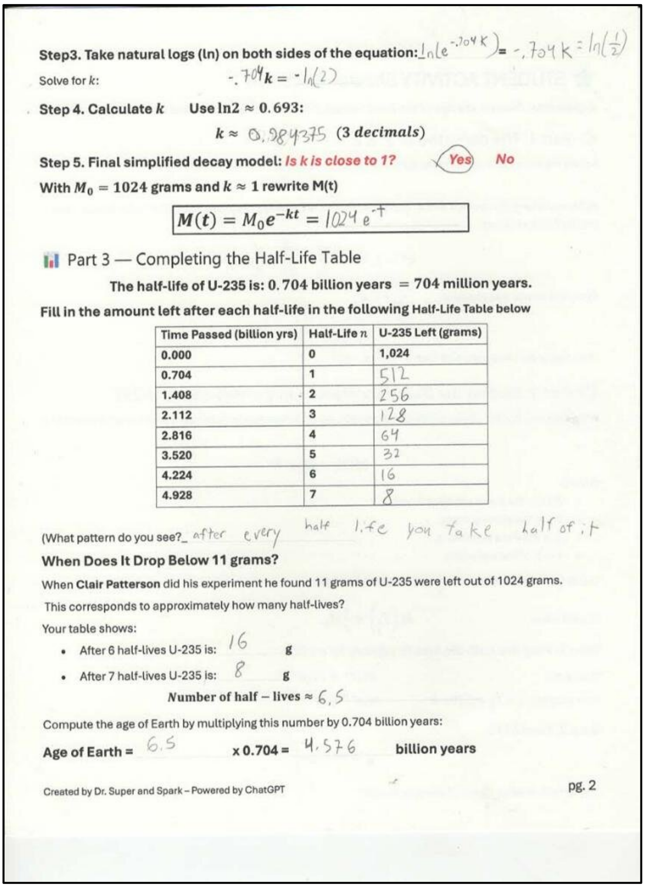

Step3. Take natural logs (ln) on both sides of the equation: $\ln(e^{-.704k}) = -.704k = \ln(\frac{1}{2})$

Solve for k: $-.704k = -\ln(2)$

Step 4. Calculate k Use $\ln 2 \approx 0.693$:

$k \approx 0.984375$ ***(3 decimals)***

Step 5. Final simplified decay model: ***Is k is close to 1?*** [✓ Yes] No

With $M_0 = 1024$ grams and $k \approx 1$ rewrite M(t)

$$\boxed{M(t) = M_0 e^{-kt} = 1024e^{-t}}$$

Part 3 — Completing the Half-Life Table

The half-life of U-235 is: 0.704 billion years = 704 million years.

Fill in the amount left after each half-life in the following Half-Life Table below

Time Passed (billion yrs)	Half-Life n	U-235 Left (grams)
0.000	0	1,024
0.704	1	512
1.408	2	256
2.112	3	128
2.816	4	64
3.520	5	32
4.224	6	16
4.928	7	8

(What pattern do you see?_ after every half life you take half of it

When Does It Drop Below 11 grams?

When **Clair Patterson** did his experiment he found 11 grams of U-235 were left out of 1024 grams.

This corresponds to approximately how many half-lives?

Your table shows:

- After 6 half-lives U-235 is: 16 **g**
- After 7 half-lives U-235 is: 8 **g**

Number of half – lives $\approx$ 6.5

Compute the age of Earth by multiplying this number by 0.704 billion years:

Age of Earth = 6.5 **x 0.704 =** 4.576 **billion years**

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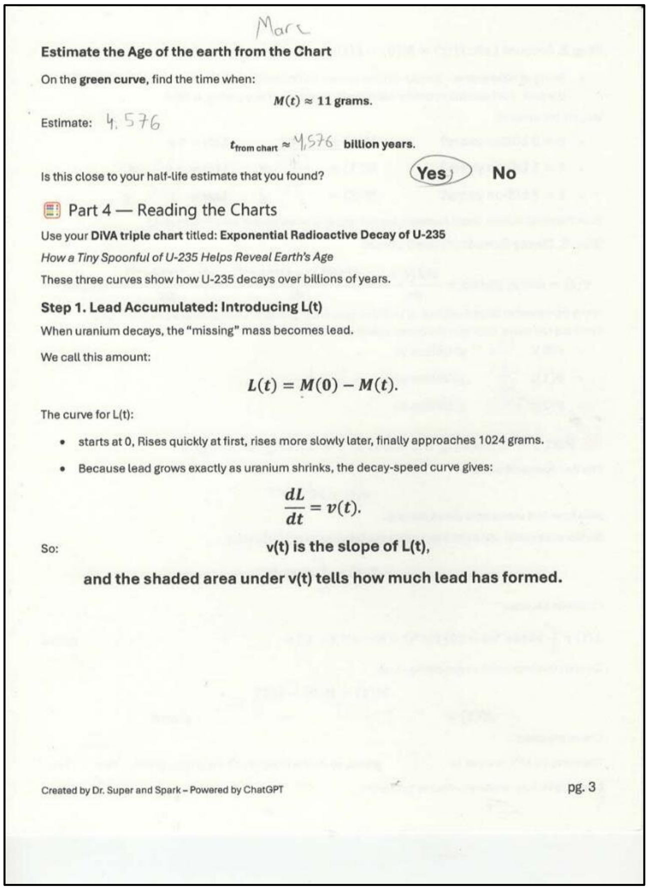

Marc

Estimate the Age of the earth from the Chart

On the **green curve,** find the time when:

$$M(t) \approx 11 \text{ grams.}$$

Estimate: 4.576

$t_{\text{from chart}} \approx 4.576$ **billion years.**

Is this close to your half-life estimate that you found? [✓ Yes] No

Part 4 — Reading the Charts

Use your **DIVA triple chart** titled: **Exponential Radioactive Decay of U-235**

How a Tiny Spoonful of U-235 Helps Reveal Earth's Age

These three curves show how U-235 decays over billions of years.

Step 1. Lead Accumulated: Introducing L(t)

When uranium decays, the "missing" mass becomes lead.

We call this amount:

$$L(t) = M(0) - M(t).$$

The curve for L(t):

- starts at 0, Rises quickly at first, rises more slowly later, finally approaches 1024 grams.
- Because lead grows exactly as uranium shrinks, the decay-speed curve gives:

$$\frac{dL}{dt} = v(t).$$

So: **v(t) is the slope of L(t),**

and the shaded area under v(t) tells how much lead has formed.

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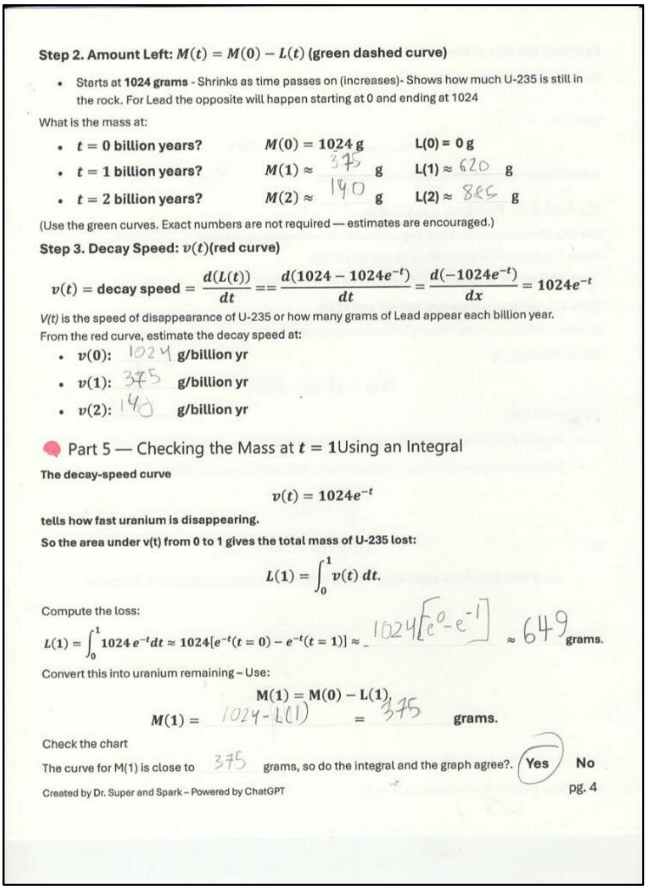

Step 2. Amount Left: $M(t) = M(0) - L(t)$ (green dashed curve)

- Starts at **1024 grams** - Shrinks as time passes on (increases)- Shows how much U-235 is still in the rock. For Lead the opposite will happen starting at 0 and ending at 1024

What is the mass at:

- **$t = 0$ billion years?** $M(0) = 1024$ **g** L(0) = 0 g
- **$t = 1$ billion years?** $M(1) \approx$ 375 **g** L(1) ≈ 620 g
- **$t = 2$ billion years?** $M(2) \approx$ 140 **g** L(2) ≈ 885 g

(Use the green curves. Exact numbers are not required — estimates are encouraged.)

Step 3. Decay Speed: $v(t)$(red curve)

$$v(t) = \text{decay speed} = \frac{d(L(t))}{dt} == \frac{d(1024 - 1024e^{-t})}{dt} = \frac{d(-1024e^{-t})}{dx} = 1024e^{-t}$$

V(t) is the speed of disappearance of U-235 or how many grams of Lead appear each billion year.

From the red curve, estimate the decay speed at:

- $v(0)$: 1024 **g/billion yr**
- $v(1)$: 375 **g/billion yr**
- $v(2)$: 140 **g/billion yr**

Part 5 — Checking the Mass at $t = 1$Using an Integral

The decay-speed curve

$$v(t) = 1024e^{-t}$$

tells how fast uranium is disappearing.

So the area under v(t) from 0 to 1 gives the total mass of U-235 lost:

$$L(1) = \int_0^1 v(t)\,dt.$$

Compute the loss:

$L(1) = \int_0^1 1024\,e^{-t}dt \approx 1024[e^{-t}(t=0) - e^{-t}(t=1)] \approx$ $1024[e^{0} - e^{-1}]$ ≈ 649 **grams.**

Convert this into uranium remaining – Use:

$$\mathbf{M(1) = M(0) - L(1),}$$

$M(1) =$ 1024 - L(1) = 375 **grams.**

Check the chart

The curve for M(1) is close to 375 grams, so do the integral and the graph agree?. [✓ **Yes**] **No**

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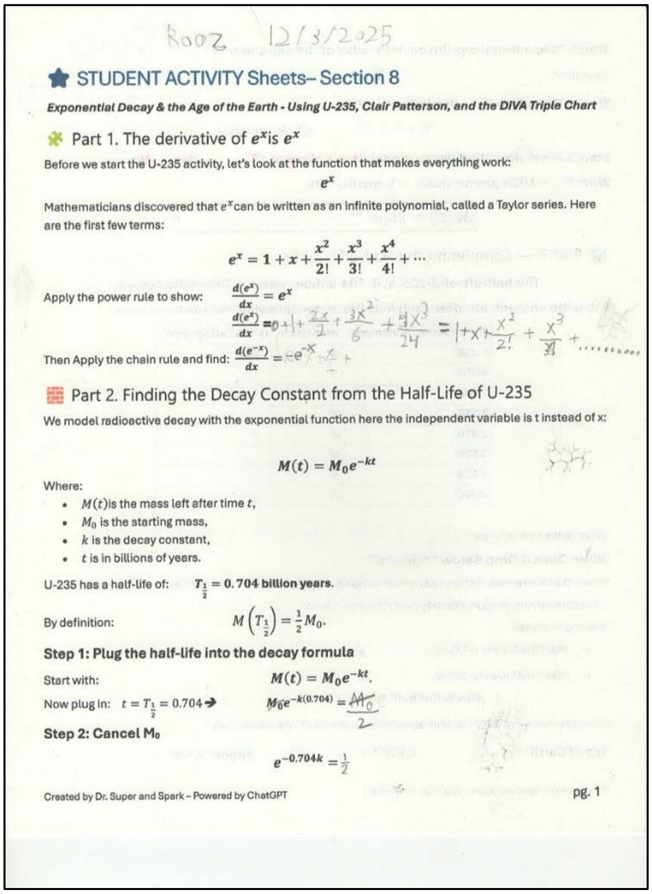

Rooz 12/3/2025

STUDENT ACTIVITY Sheets– Section 8

Exponential Decay & the Age of the Earth - Using U-235, Clair Patterson, and the DIVA Triple Chart

Part 1. The derivative of e^x is e^x

Before we start the U-235 activity, let's look at the function that makes everything work:

$$e^x$$

Mathematicians discovered that e^x can be written as an infinite polynomial, called a Taylor series. Here are the first few terms:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Apply the power rule to show: $\frac{d(e^x)}{dx} = e^x$

$$\frac{d(e^x)}{dx} = 0 + 1 + \frac{2x}{2} + \frac{3x^2}{6} + \frac{4x^3}{24} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

Then Apply the chain rule and find: $\frac{d(e^{-x})}{dx} = -e^{-x} + \frac{x}{1} +$

Part 2. Finding the Decay Constant from the Half-Life of U-235

We model radioactive decay with the exponential function here the independent variable is t instead of x:

$$M(t) = M_0 e^{-kt}$$

Where:

- $M(t)$ is the mass left after time t,
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U-235 has a half-life of: $T_{\frac{1}{2}} = 0.704$ **billion years.**

By definition: $M\left(T_{\frac{1}{2}}\right) = \frac{1}{2} M_0$.

Step 1: Plug the half-life into the decay formula

Start with: $M(t) = M_0 e^{-kt}$.

Now plug in: $t = T_{\frac{1}{2}} = 0.704$ → $M_0 e^{-k(0.704)} = \frac{M_0}{2}$

Step 2: Cancel M₀

$$e^{-0.704k} = \frac{1}{2}$$

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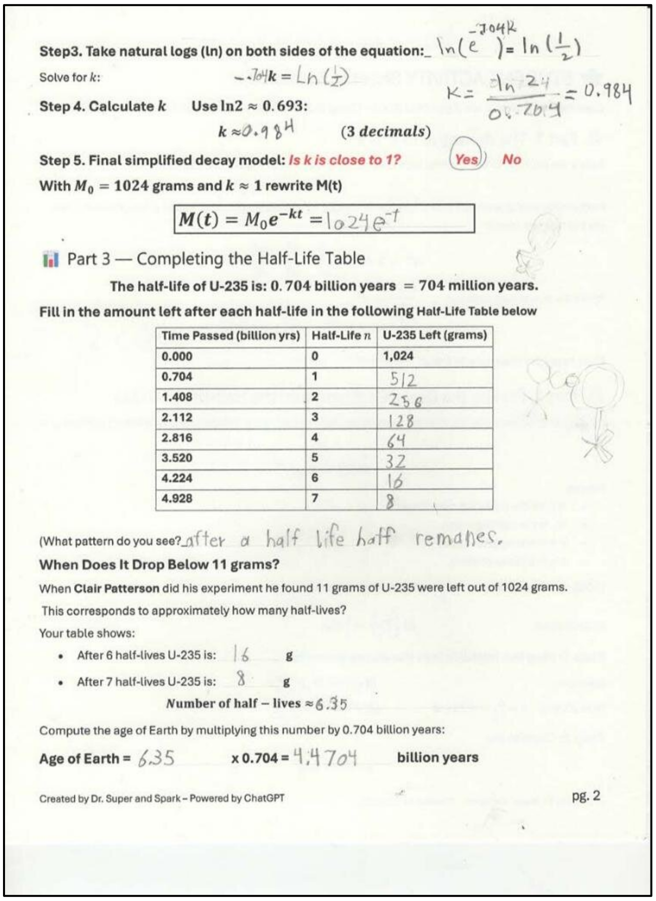
pg. 1



Step3. Take natural logs (ln) on both sides of the equation: $\ln(e^{-.704k}) = \ln(\frac{1}{2})$

Solve for k: $-.704k = \ln(\frac{1}{2})$

$$K = \frac{-\ln 2}{0.704} = 0.984$$

Step 4. Calculate k Use $\ln 2 \approx 0.693$:

$k \approx 0.984$ **(3 decimals)**

Step 5. Final simplified decay model: ***Is k is close to 1?*** [✓ Yes] No

With $M_0 = 1024$ grams and $k \approx 1$ rewrite M(t)

$$\boxed{M(t) = M_0 e^{-kt} = 1024e^{-t}}$$

Part 3 — Completing the Half-Life Table

The half-life of U-235 is: 0.704 billion years = 704 million years.

Fill in the amount left after each half-life in the following Half-Life Table below

Time Passed (billion yrs)	Half-Life n	U-235 Left (grams)
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2.112	3	128
2.816	4	64
3.520	5	32
4.224	6	16
4.928	7	8

(What pattern do you see? after a half life half remanes.

When Does It Drop Below 11 grams?

When **Clair Patterson** did his experiment he found 11 grams of U-235 were left out of 1024 grams.

This corresponds to approximately how many half-lives?

Your table shows:

- After 6 half-lives U-235 is: 16 g
- After 7 half-lives U-235 is: 8 g

Number of half – lives ≈ 6.35

Compute the age of Earth by multiplying this number by 0.704 billion years:

Age of Earth = 6.35 **x 0.704 =** 4.4704 **billion years**

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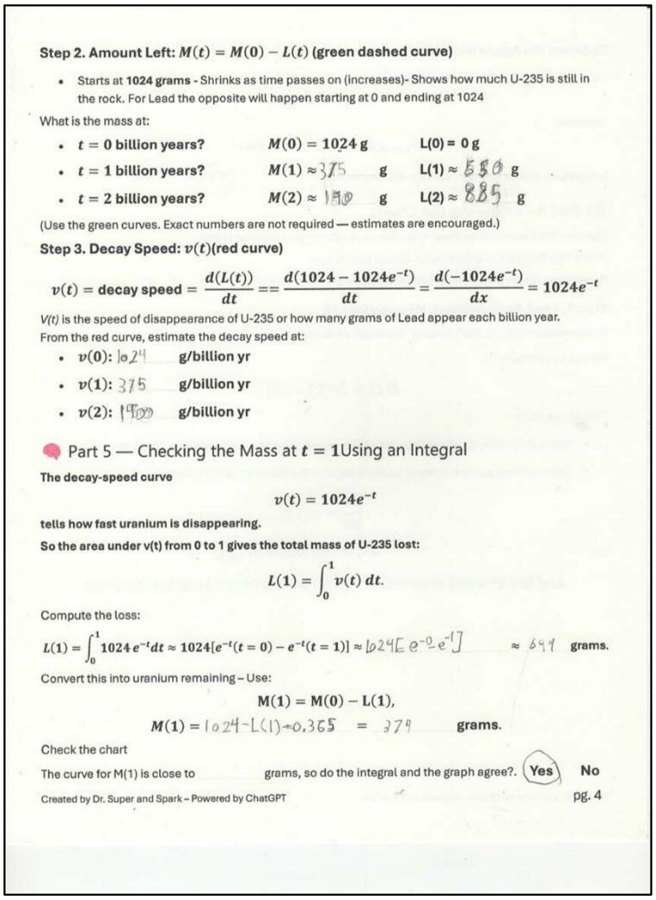
pg. 2



Step 2. Amount Left: $M(t) = M(0) - L(t)$ (green dashed curve)

- Starts at **1024 grams** - Shrinks as time passes on (increases)- Shows how much U-235 is still in the rock. For Lead the opposite will happen starting at 0 and ending at 1024

What is the mass at:

- **$t = 0$ billion years?** $M(0) = 1024$ g L(0) = 0 g
- **$t = 1$ billion years?** $M(1) \approx 375$ g L(1) ≈ 550 g
- **$t = 2$ billion years?** $M(2) \approx 190$ g L(2) ≈ 885 g

(Use the green curves. Exact numbers are not required — estimates are encouraged.)

Step 3. Decay Speed: $v(t)$(red curve)

$$v(t) = \text{decay speed} = \frac{d(L(t))}{dt} == \frac{d(1024 - 1024e^{-t})}{dt} = \frac{d(-1024e^{-t})}{dx} = 1024e^{-t}$$

V(t) is the speed of disappearance of U-235 or how many grams of Lead appear each billion year.

From the red curve, estimate the decay speed at:

- $v(0)$: 1024 g/billion yr
- $v(1)$: 375 g/billion yr
- $v(2)$: 190 g/billion yr

Part 5 — Checking the Mass at $t = 1$Using an Integral

The decay-speed curve

$$v(t) = 1024e^{-t}$$

tells how fast uranium is disappearing.

So the area under v(t) from 0 to 1 gives the total mass of U-235 lost:

$$L(1) = \int_0^1 v(t)\,dt.$$

Compute the loss:

$$L(1) = \int_0^1 1024\,e^{-t}dt \approx 1024[e^{-t}(t=0) - e^{-t}(t=1)] \approx 1024[e^{-0} - e^{-1}] \approx 649 \text{ grams.}$$

Convert this into uranium remaining – Use:

$$\mathbf{M(1) = M(0) - L(1),}$$

$$M(1) = 1024 - L(1) - 0.365 = 379 \text{ grams.}$$

Check the chart

The curve for M(1) is close to ______ grams, so do the integral and the graph agree?. [✓ Yes] No

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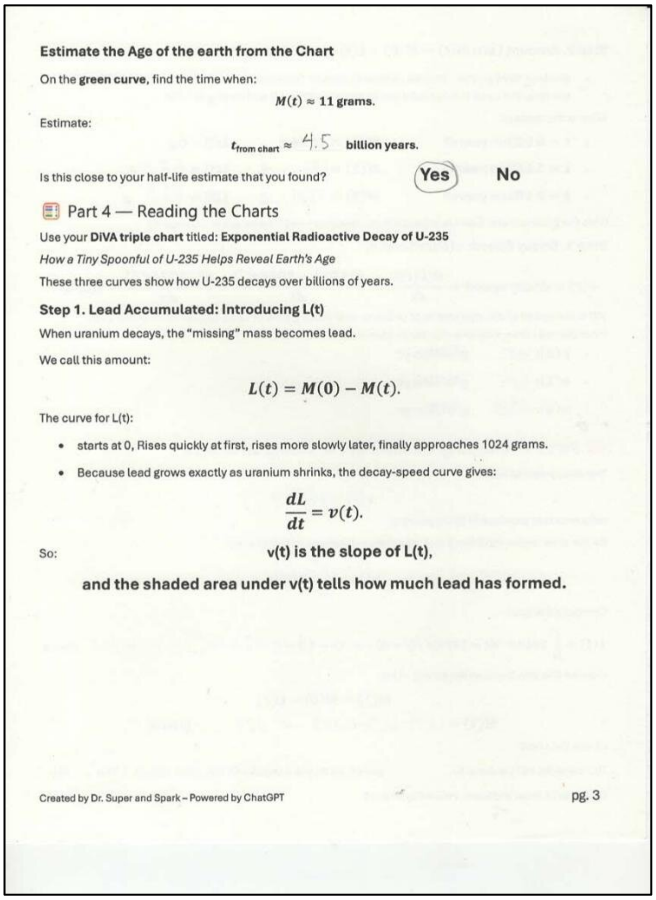
pg. 4



Estimate the Age of the earth from the Chart

On the **green curve**, find the time when:

$$M(t) \approx 11 \text{ grams.}$$

Estimate:

$t_{\text{from chart}} \approx$ 4.5 **billion years.**

Is this close to your half-life estimate that you found? [✓ Yes] No

Part 4 — Reading the Charts

Use your **DIVA triple chart** titled: **Exponential Radioactive Decay of U-235**

How a Tiny Spoonful of U-235 Helps Reveal Earth's Age

These three curves show how U-235 decays over billions of years.

Step 1. Lead Accumulated: Introducing L(t)

When uranium decays, the "missing" mass becomes lead.

We call this amount:

$$L(t) = M(0) - M(t).$$

The curve for L(t):

- starts at 0, Rises quickly at first, rises more slowly later, finally approaches 1024 grams.
- Because lead grows exactly as uranium shrinks, the decay-speed curve gives:

$$\frac{dL}{dt} = v(t).$$

So: **v(t) is the slope of L(t),**

and the shaded area under v(t) tells how much lead has formed.

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pg. 3

Adventure 10– L'Hôpital's Rule, Ratios, & Feynman–Gamow Space Mission

1. Overview of the Session

- Date: 12/10/2025
- Duration: 1:40 Hours 1:50-3:30 pm
- Students present: Cyrus, Rooz, Marc
- Summary of what the students discovered:
    - How to use integrals to calculate probabilities: Buffon's Needle Problem
    - Ratios that produce $0/0$and $\frac{\infty}{\infty}$forms
    - The idea of comparing *rates* rather than *values*
    - The DiVA structure (Distance → Velocity → Acceleration) as a natural realization of L'Hôpital's Rule
    - Long-time behavior of functions and ratios
    - L'Hôpital's Rule in a conceptual explanation ("which derivative changes faster?")

2. Warm-Up Conversation / Storytelling/Videos

- I repeated for them that Spark and I had spent even more time on preparing this activity for them and it will be long but if they focused, they will learn and connect many interesting ideas from Calculus to Mechanics and discover how DiVA charts and L'Hôpital's Rule can help them find limiting values for undefined Ratios 0/0 and infinity/infinity.
- We did the Buffon's Needle problem, and I showed the solution on the same newsprint.
- Then we watched the video: from 3Blue1Brown Chanelle on You Tube Limits, L'Hôpital's rule, and epsilon delta definitions | Chapter 7, Essence of calculus. As usual we did stop it a few times and discussed it. We did not get into the epsilon delta definition too much but focused more on the l'Hôpital's' Rule.
- Next, I had to run out but I gave them the story sheet they told me that they read the story***: The Feynman–Gamow Deep-Space Mission*.**
- The charts for the activity were on the back of the Story sheets, and I had them examine it and see how the DiVA chart was showing the derivatives of f and g and we

were calling f'(t), g'(t), f''(t), g''(t) represented respectively the velocity and acceleration for f 9t) and g(t).

- We continued to try to solve the problems together, but Marc was going faster and I helped Rooz to keep up.

📊 3. Activities Completed

Buffon's Needle Problem

I had prepared a newsprint (see figure) with lines that were 20 cm apart and a pencil of the same length. Each of us dropped the pencil 5 times with closed eyes and in all we had the pencil cross a line 13 times out of 20 nearly 65%. Then I did the math for them with integrals on the same newsprint and found the answer to be 2/Л which is about 64% this was remarkable, but I told them that was probably lucky but if we had done it a 100 times we would probably see a number this close. I think they sort of followed the calculations and I will go over it next time also.

Part 1 — Finding f', g', f'', g''

We discussed the probes a bit and what they wanted to find out, and they had no trouble finding f', g', f'', g'' using the power rule that they have seen now several times.

Part 2 — Distance Table

They also had no trouble drawing in the curve for the distance ratio and putting the values on the chart from the table. We discussed that these values were going down but still not close to 2.

Part 3 — Velocity Table

They filled in the velocity table using f' and g' and calculated the ratio then they charted these and put the values on the curves.

Part 4 — Acceleration table

The acceleration table was straightforward; they observed the ratios were approaching 2 and completed the task quickly.

Part 5 — Longtime Behavior

They found the rations for distance and velocity and noticed that they were both getting close to 2 and of course the ratio of f'' to g'' was equal to 2. We need to go over the questions again in the next session.

Challenge Question – L'Hôpital's Rule in Action

This was left for the next session we ran out of time.

4. What Worked Well/Or Did Not

- They had no trouble with filling in the tables and charting and enjoyed it.
- We ran out of time to discuss the result of the acceleration being 2.

5. Adjustments for Next Time

- The Buffon's Needle problem took a long time so we ran out of time. Probably more time had to be given to the probe activity itself.

6. Student Quotes

- Both Rooz and Cyrus were asking when we can stop at the end.

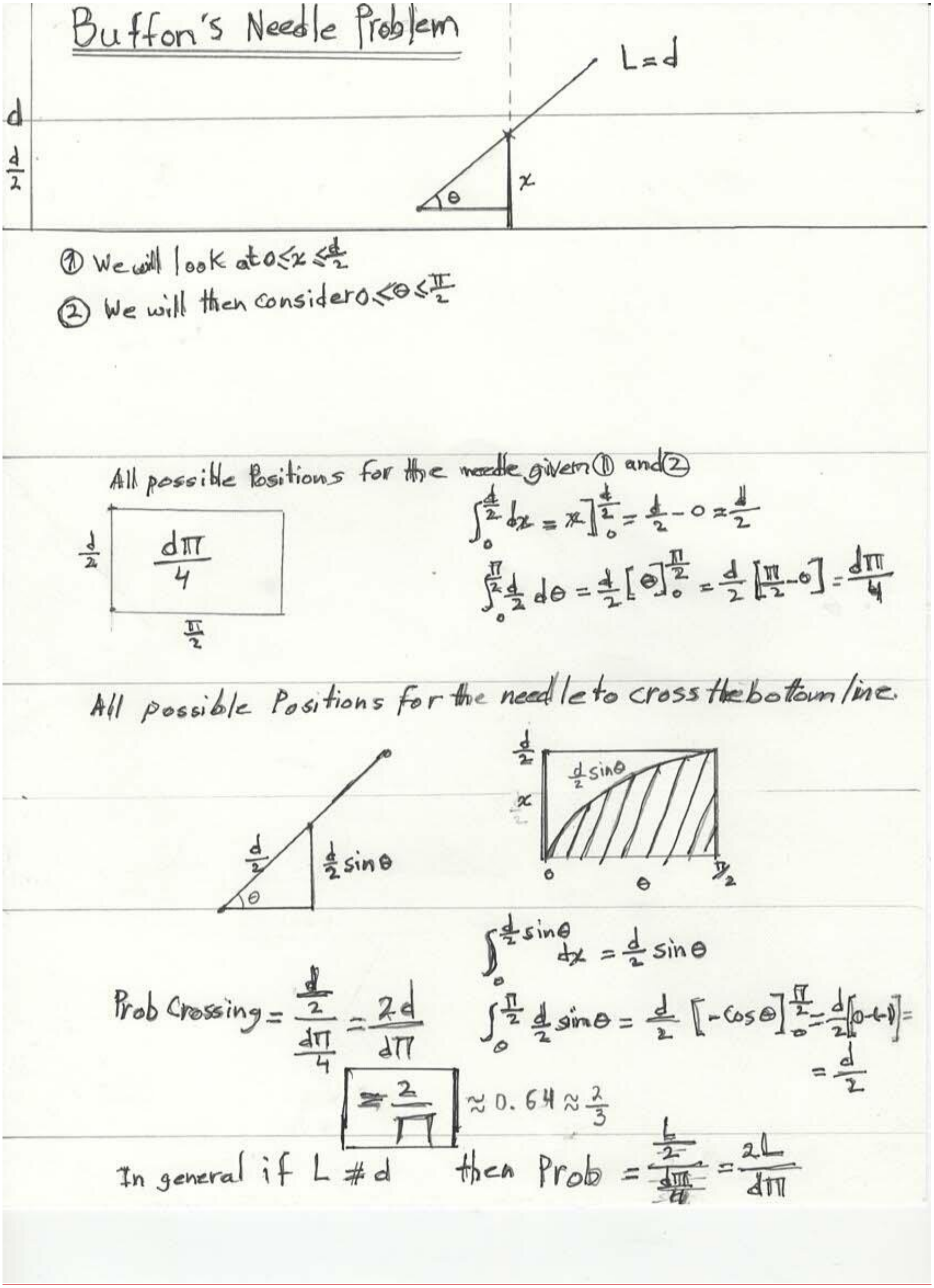
7. Closing Reflection by Dr. Super

This was also a very good session. They enjoyed Buffon's Needle Problem, but we ran out of time at the end on the main activity, but I will go over it next time. Rooz is keeping up.

.8. Photos / Charts / Sketches

(From Left to Right Rooz, Cyrus and Marc)

📘 9. Buffon's Needle Problem Solutions



Buffon's Needle Problem

① We will look at $0 \le x \le \frac{d}{2}$

② We will then consider $0 \le \theta \le \frac{\pi}{2}$

All possible Positions for the needle given ① and ②

$$\int_0^{\frac{d}{2}} dx = x\Big]_0^{\frac{d}{2}} = \frac{d}{2} - 0 = \frac{d}{2}$$

$$\int_0^{\frac{\pi}{2}} \frac{d}{2}\, d\theta = \frac{d}{2}\left[\theta\right]_0^{\frac{\pi}{2}} = \frac{d}{2}\left[\frac{\pi}{2} - 0\right] = \frac{d\pi}{4}$$

All possible Positions for the needle to cross the bottom line.

$$\int_0^{\frac{d}{2}\sin\theta} dx = \frac{d}{2}\sin\theta$$

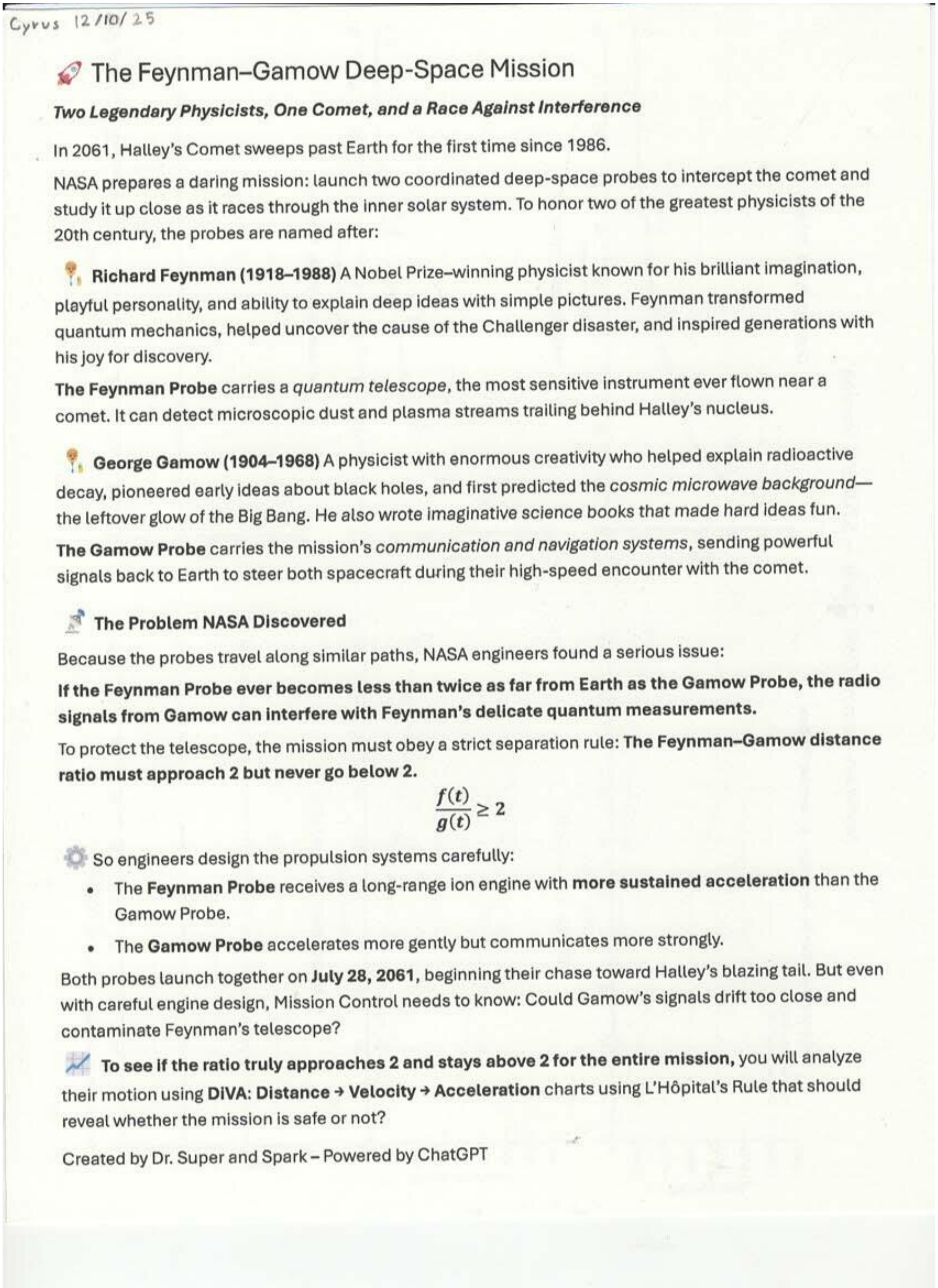
$$\text{Prob Crossing} = \frac{\frac{d}{2}}{\frac{d\pi}{4}} = \frac{2d}{d\pi}$$

$$\boxed{= \frac{2}{\pi}} \approx 0.64 \approx \frac{2}{3}$$

$$\int_0^{\frac{\pi}{2}} \frac{d}{2}\sin\theta = \frac{d}{2}\left[-\cos\theta\right]_0^{\frac{\pi}{2}} = \frac{d}{2}\left[0 - (-1)\right] = \frac{d}{2}$$

In general if $L \neq d$ then $\text{Prob} = \frac{\frac{L}{2}}{\frac{d\pi}{4}} = \frac{2L}{d\pi}$

10. Student Completed Worksheets



Cyrus 12/10/25

The Feynman–Gamow Deep-Space Mission

Two Legendary Physicists, One Comet, and a Race Against Interference

In 2061, Halley's Comet sweeps past Earth for the first time since 1986.

NASA prepares a daring mission: launch two coordinated deep-space probes to intercept the comet and study it up close as it races through the inner solar system. To honor two of the greatest physicists of the 20th century, the probes are named after:

Richard Feynman (1918–1988) A Nobel Prize–winning physicist known for his brilliant imagination, playful personality, and ability to explain deep ideas with simple pictures. Feynman transformed quantum mechanics, helped uncover the cause of the Challenger disaster, and inspired generations with his joy for discovery.

The Feynman Probe carries a *quantum telescope*, the most sensitive instrument ever flown near a comet. It can detect microscopic dust and plasma streams trailing behind Halley's nucleus.

George Gamow (1904–1968) A physicist with enormous creativity who helped explain radioactive decay, pioneered early ideas about black holes, and first predicted the *cosmic microwave background*—the leftover glow of the Big Bang. He also wrote imaginative science books that made hard ideas fun.

The Gamow Probe carries the mission's *communication and navigation systems*, sending powerful signals back to Earth to steer both spacecraft during their high-speed encounter with the comet.

The Problem NASA Discovered

Because the probes travel along similar paths, NASA engineers found a serious issue:

If the Feynman Probe ever becomes less than twice as far from Earth as the Gamow Probe, the radio signals from Gamow can interfere with Feynman's delicate quantum measurements.

To protect the telescope, the mission must obey a strict separation rule: **The Feynman–Gamow distance ratio must approach 2 but never go below 2.**

$$\frac{f(t)}{g(t)} \geq 2$$

So engineers design the propulsion systems carefully:

- The **Feynman Probe** receives a long-range ion engine with **more sustained acceleration** than the Gamow Probe.
- The **Gamow Probe** accelerates more gently but communicates more strongly.

Both probes launch together on **July 28, 2061**, beginning their chase toward Halley's blazing tail. But even with careful engine design, Mission Control needs to know: Could Gamow's signals drift too close and contaminate Feynman's telescope?

To see if the ratio truly approaches 2 and stays above 2 for the entire mission, you will analyze their motion using **DiVA: Distance → Velocity → Acceleration** charts using L'Hôpital's Rule that should reveal whether the mission is safe or not?

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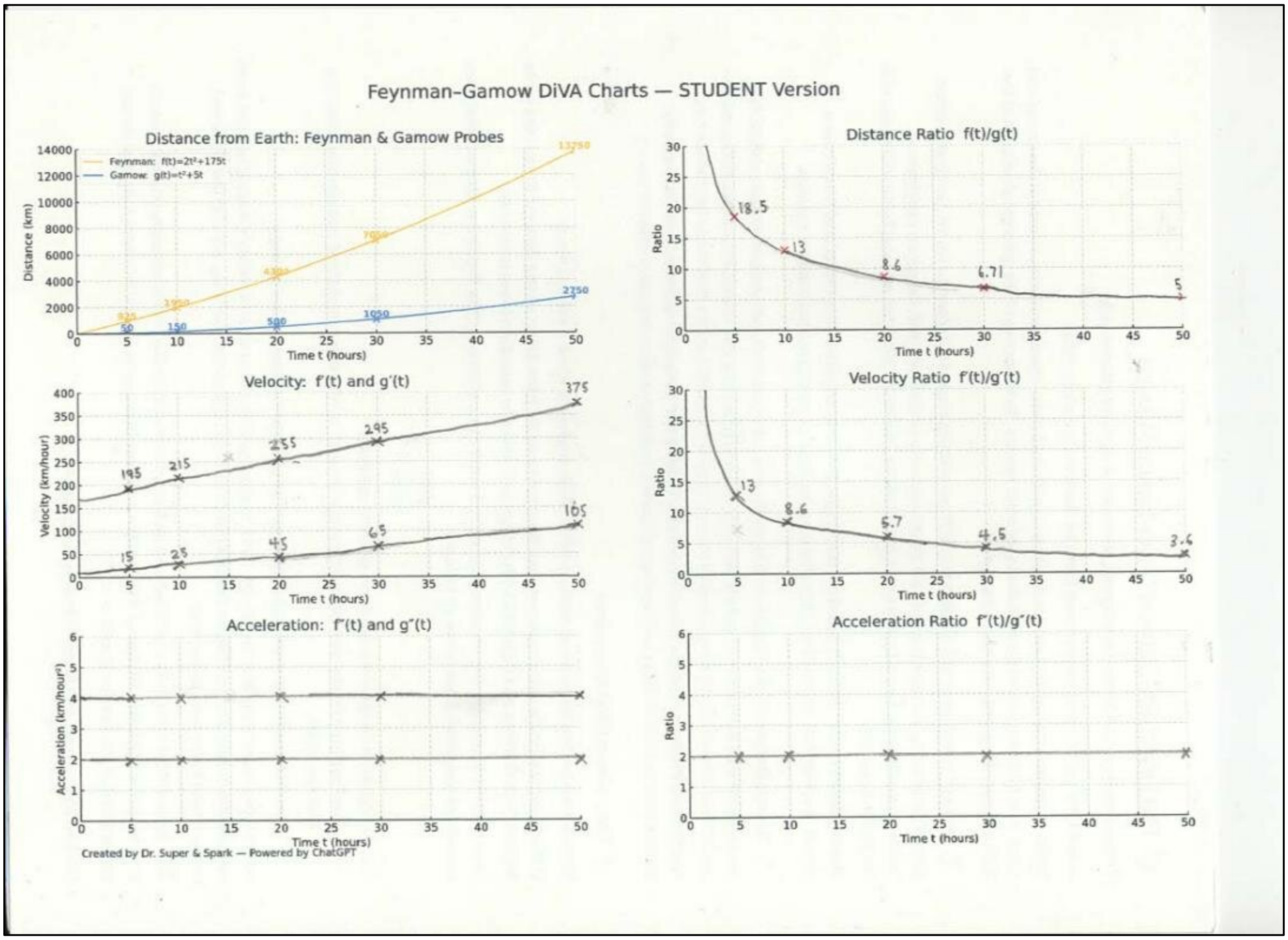
Feynman–Gamow DiVA Charts — STUDENT Version
Distance from Earth: Feynman & Gamow Probes
Feynman: f(t)=2t²+175t
Gamow: g(t)=t²+5t
Distance (km)
14000
12000
10000
8000
6000
4000
2000
0
925
1950
4300
7050
13750
50
150
500
1050
2750
Time t (hours)
0
5
10
15
20
25
30
35
40
45
50
Distance Ratio f(t)/g(t)
Ratio
30
25
20
15
10
5
0
18.5
13
8.6
6.71
5
Time t (hours)
Velocity: f′(t) and g′(t)
Velocity (km/hour)
400
350
300
250
200
150
100
50
0
195
215
255
295
375
15
25
45
65
105
Time t (hours)
Velocity Ratio f′(t)/g′(t)
Ratio
13
8.6
5.7
4.5
3.6
Time t (hours)
Acceleration: f″(t) and g″(t)
Acceleration (km/hour²)
6
5
4
3
2
1
0
Time t (hours)
Acceleration Ratio f″(t)/g″(t)
Ratio
Time t (hours)
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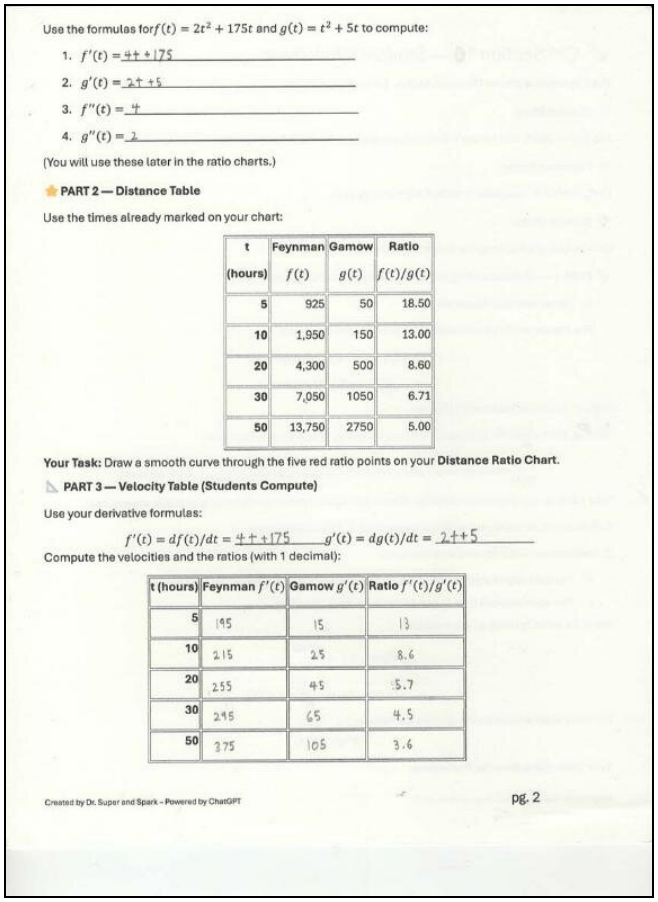

Use the formulas for $f(t) = 2t^2 + 175t$ and $g(t) = t^2 + 5t$ to compute:

1. $f'(t) =$ 4t+175
2. $g'(t) =$ 2t+5
3. $f''(t) =$ 4
4. $g''(t) =$ 2

(You will use these later in the ratio charts.)

PART 2 — Distance Table

Use the times already marked on your chart:

t (hours)	Feynman $f(t)$	Gamow $g(t)$	Ratio $f(t)/g(t)$
5	925	50	18.50
10	1,950	150	13.00
20	4,300	500	8.60
30	7,050	1050	6.71
50	13,750	2750	5.00

Your Task: Draw a smooth curve through the five red ratio points on your **Distance Ratio Chart.**

PART 3 — Velocity Table (Students Compute)

Use your derivative formulas:

$f'(t) = df(t)/dt =$ 4t+175 $g'(t) = dg(t)/dt =$ 2t+5

Compute the velocities and the ratios (with 1 decimal):

t (hours)	Feynman $f'(t)$	Gamow $g'(t)$	Ratio $f'(t)/g'(t)$
5	195	15	13
10	215	25	8.6
20	255	45	5.7
30	295	65	4.5
50	375	105	3.6

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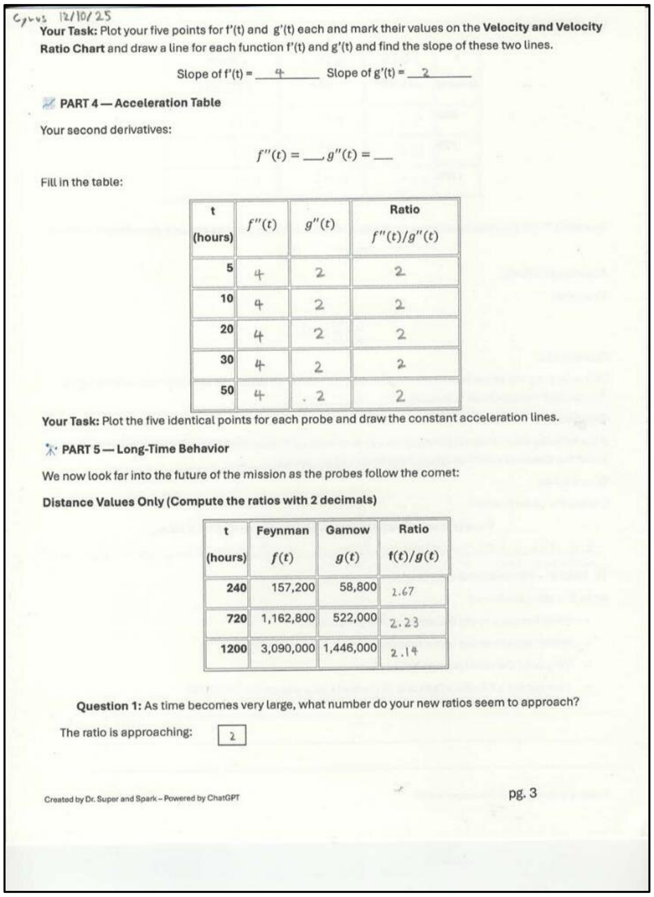
pg. 2



Cyrus 12/10/25

Your Task: Plot your five points for f'(t) and g'(t) each and mark their values on the **Velocity and Velocity Ratio Chart** and draw a line for each function f'(t) and g'(t) and find the slope of these two lines.

Slope of f'(t) = 4 Slope of g'(t) = 2

PART 4 — Acceleration Table

Your second derivatives:

$$f''(t) = \_\_,\ g''(t) = \_\_$$

Fill in the table:

t (hours)	$f''(t)$	$g''(t)$	Ratio $f''(t)/g''(t)$
5	4	2	2
10	4	2	2
20	4	2	2
30	4	2	2
50	4	. 2	2

Your Task: Plot the five identical points for each probe and draw the constant acceleration lines.

PART 5 — Long-Time Behavior

We now look far into the future of the mission as the probes follow the comet:

Distance Values Only (Compute the ratios with 2 decimals)

t (hours)	Feynman $f(t)$	Gamow $g(t)$	Ratio $f(t)/g(t)$
240	157,200	58,800	1.67
720	1,162,800	522,000	2.23
1200	3,090,000	1,446,000	2.14

Question 1: As time becomes very large, what number do your new ratios seem to approach?

The ratio is approaching: $\boxed{2}$

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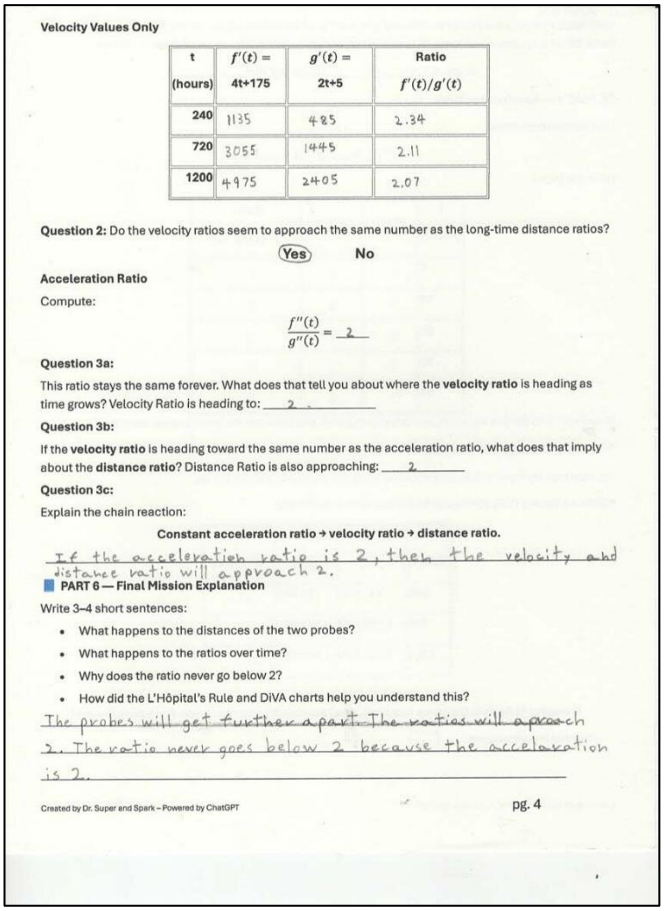

Velocity Values Only

t (hours)	$f'(t) =$ 4t+175	$g'(t) =$ 2t+5	Ratio $f'(t)/g'(t)$
240	1135	485	2.34
720	3055	1445	2.11
1200	4975	2405	2.07

Question 2: Do the velocity ratios seem to approach the same number as the long-time distance ratios?

[✓ Yes] No

Acceleration Ratio

Compute:

$$\frac{f''(t)}{g''(t)} = 2$$

Question 3a:

This ratio stays the same forever. What does that tell you about where the **velocity ratio** is heading as time grows? Velocity Ratio is heading to: 2 .

Question 3b:

If the **velocity ratio** is heading toward the same number as the acceleration ratio, what does that imply about the **distance ratio**? Distance Ratio is also approaching: 2

Question 3c:

Explain the chain reaction:

Constant acceleration ratio → velocity ratio → distance ratio.

If the acceleration ratio is 2, then the velocity and distance ratio will approach 2.

PART 6 — Final Mission Explanation

Write 3–4 short sentences:

- What happens to the distances of the two probes?
- What happens to the ratios over time?
- Why does the ratio never go below 2?
- How did the L'Hôpital's Rule and DiVA charts help you understand this?

The probes will get further apart. The ratios will aproach 2. The ratio never goes below 2 because the accelaration is 2.

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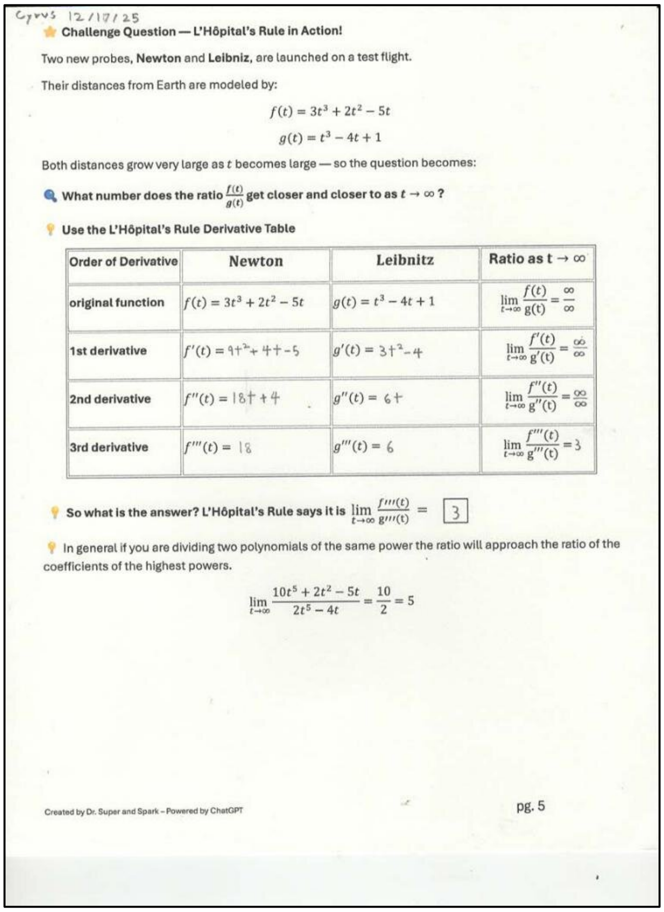
pg. 4



Cyrus 12/17/25

Challenge Question — L'Hôpital's Rule in Action!

Two new probes, **Newton** and **Leibniz**, are launched on a test flight.

Their distances from Earth are modeled by:

$$f(t) = 3t^3 + 2t^2 - 5t$$
$$g(t) = t^3 - 4t + 1$$

Both distances grow very large as t becomes large — so the question becomes:

What number does the ratio $\frac{f(t)}{g(t)}$ get closer and closer to as $t \to \infty$?

Use the L'Hôpital's Rule Derivative Table

Order of Derivative	Newton	Leibniz	Ratio as t → ∞
original function	$f(t) = 3t^3 + 2t^2 - 5t$	$g(t) = t^3 - 4t + 1$	$\lim_{t\to\infty} \frac{f(t)}{g(t)} = \frac{\infty}{\infty}$
1st derivative	$f'(t) = 9t^2 + 4t - 5$	$g'(t) = 3t^2 - 4$	$\lim_{t\to\infty} \frac{f'(t)}{g'(t)} = \frac{\infty}{\infty}$
2nd derivative	$f''(t) = 18t + 4$	$g''(t) = 6t$	$\lim_{t\to\infty} \frac{f''(t)}{g''(t)} = \frac{\infty}{\infty}$
3rd derivative	$f'''(t) = 18$	$g'''(t) = 6$	$\lim_{t\to\infty} \frac{f'''(t)}{g'''(t)} = 3$

So what is the answer? L'Hôpital's Rule says it is $\lim_{t\to\infty} \frac{f'''(t)}{g'''(t)} = \boxed{3}$

In general if you are dividing two polynomials of the same power the ratio will approach the ratio of the coefficients of the highest powers.

$$\lim_{t\to\infty} \frac{10t^5 + 2t^2 - 5t}{2t^5 - 4t} = \frac{10}{2} = 5$$

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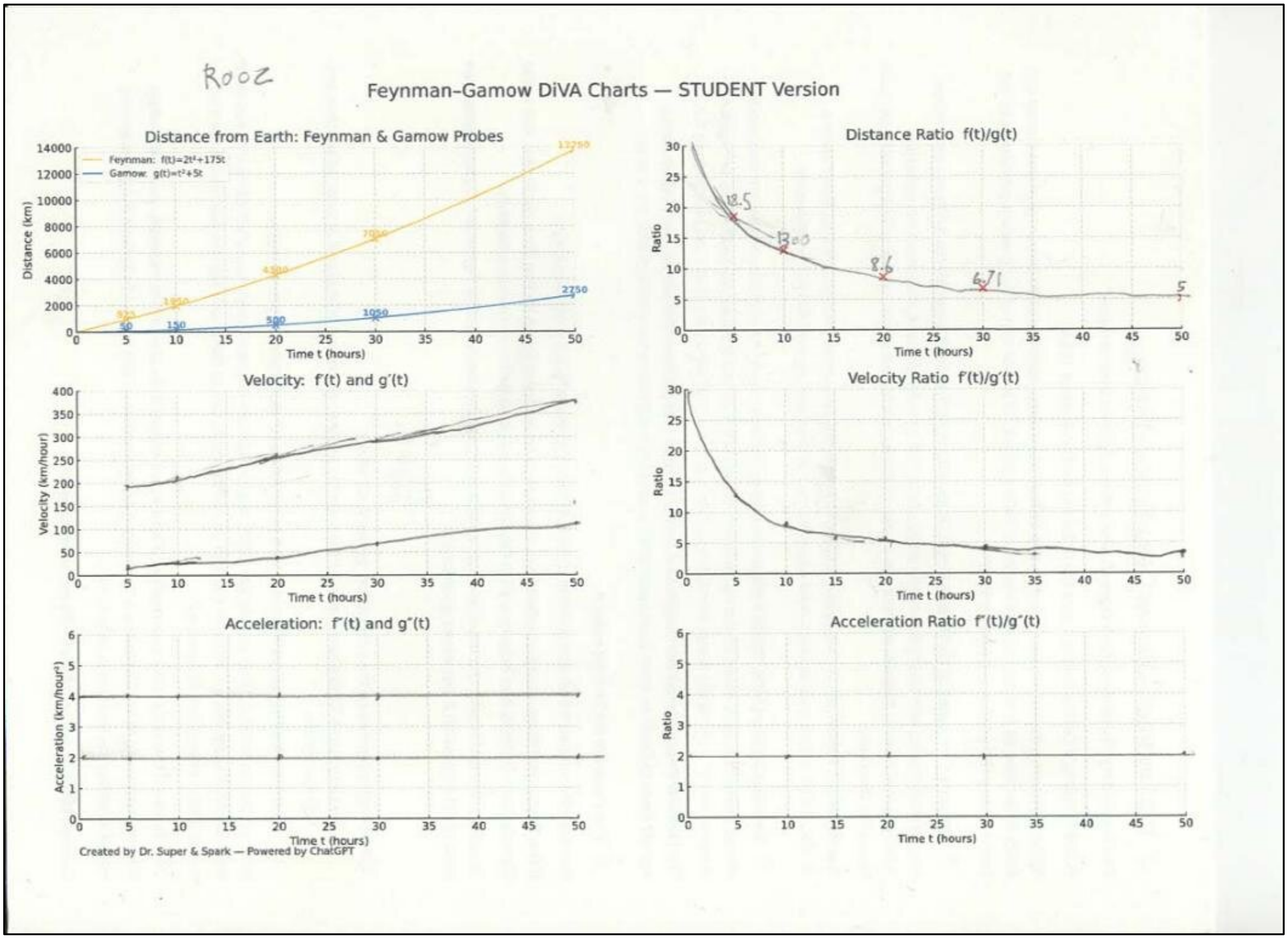
pg. 5



Rooz

Feynman–Gamow DiVA Charts — STUDENT Version

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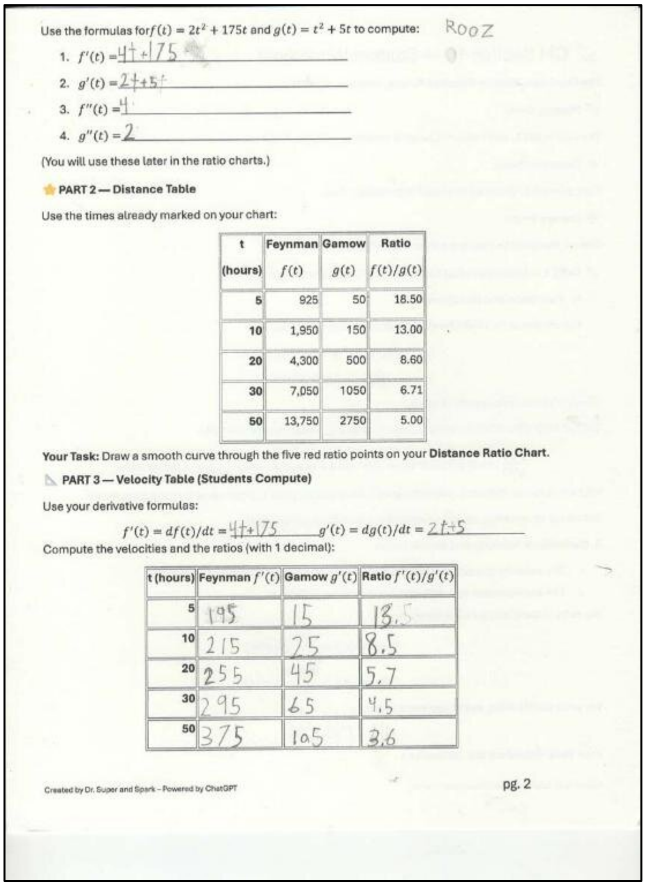

Use the formulas for $f(t) = 2t^2 + 175t$ and $g(t) = t^2 + 5t$ to compute: ROOZ

1. $f'(t) =$ 4t+175
2. $g'(t) =$ 2t+5
3. $f''(t) =$ 4
4. $g''(t) =$ 2

(You will use these later in the ratio charts.)

PART 2 — Distance Table

Use the times already marked on your chart:

t	Feynman	Gamow	Ratio
(hours)	$f(t)$	$g(t)$	$f(t)/g(t)$
5	925	50	18.50
10	1,950	150	13.00
20	4,300	500	8.60
30	7,050	1050	6.71
50	13,750	2750	5.00

Your Task: Draw a smooth curve through the five red ratio points on your **Distance Ratio Chart.**

PART 3 — Velocity Table (Students Compute)

Use your derivative formulas:

$f'(t) = df(t)/dt =$ 4t+175 $g'(t) = dg(t)/dt =$ 2t+5

Compute the velocities and the ratios (with 1 decimal):

t (hours)	Feynman $f'(t)$	Gamow $g'(t)$	Ratio $f'(t)/g'(t)$
5	195	15	13.5
10	215	25	8.5
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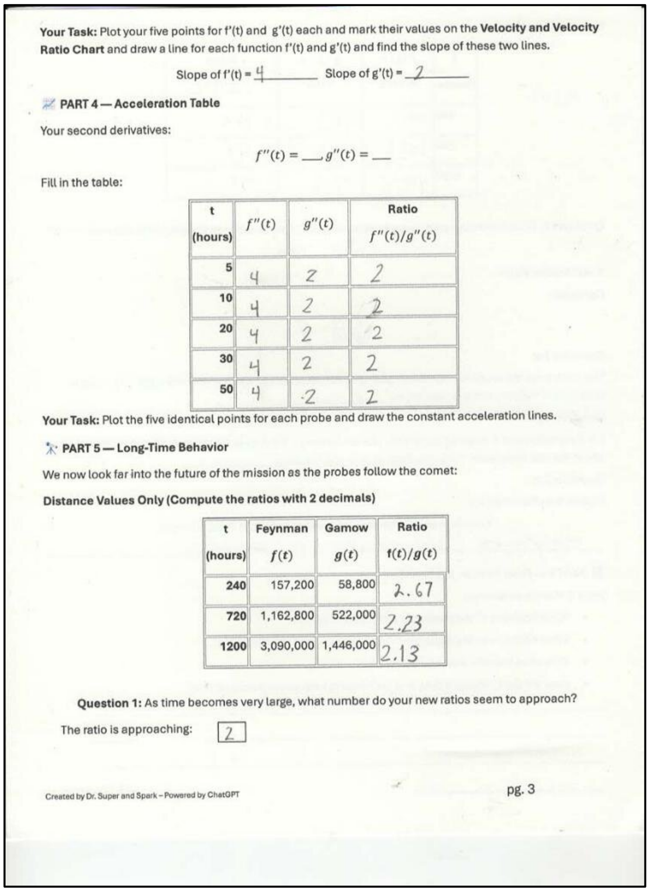

Your Task: Plot your five points for f'(t) and g'(t) each and mark their values on the **Velocity and Velocity Ratio Chart** and draw a line for each function f'(t) and g'(t) and find the slope of these two lines.

Slope of f'(t) = 4 Slope of g'(t) = 2

PART 4 — Acceleration Table

Your second derivatives:

$f''(t) =$ __, $g''(t) =$ __

Fill in the table:

t	$f''(t)$	$g''(t)$	Ratio
(hours)			$f''(t)/g''(t)$
5	4	2	2
10	4	2	2
20	4	2	2
30	4	2	2
50	4	2	2

Your Task: Plot the five identical points for each probe and draw the constant acceleration lines.

PART 5 — Long-Time Behavior

We now look far into the future of the mission as the probes follow the comet:

Distance Values Only (Compute the ratios with 2 decimals)

t	Feynman	Gamow	Ratio
(hours)	$f(t)$	$g(t)$	$f(t)/g(t)$
240	157,200	58,800	2.67
720	1,162,800	522,000	2.23
1200	3,090,000	1,446,000	2.13

Question 1: As time becomes very large, what number do your new ratios seem to approach?

The ratio is approaching: $\boxed{2}$

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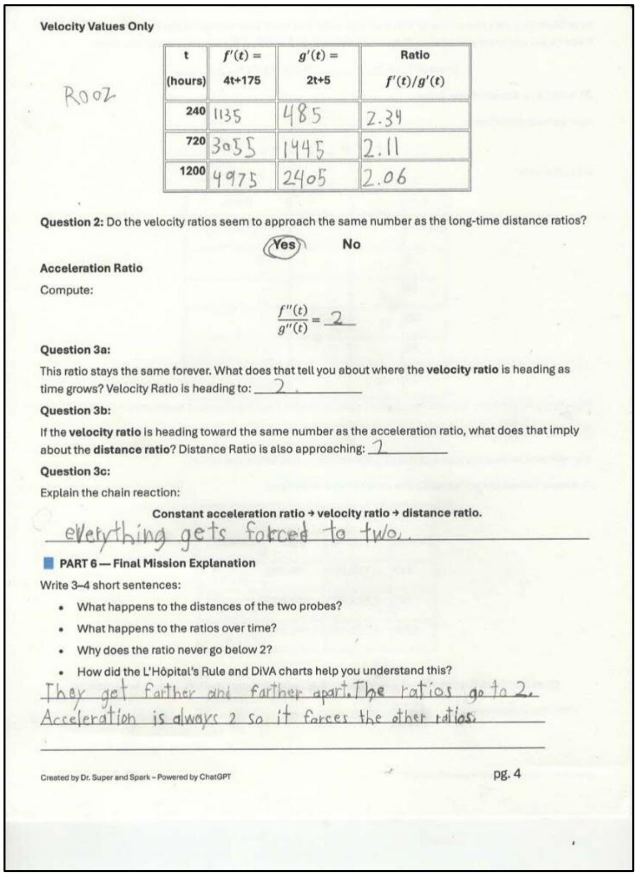

Velocity Values Only

ROOZ

t	$f'(t) =$	$g'(t) =$	Ratio
(hours)	4t+175	2t+5	$f'(t)/g'(t)$
240	1135	485	2.34
720	3055	1445	2.11
1200	4975	2405	2.06

Question 2: Do the velocity ratios seem to approach the same number as the long-time distance ratios?

[✓ Yes] No

Acceleration Ratio

Compute:

$$\frac{f''(t)}{g''(t)} = 2$$

Question 3a:

This ratio stays the same forever. What does that tell you about where the **velocity ratio** is heading as time grows? Velocity Ratio is heading to: 2.

Question 3b:

If the **velocity ratio** is heading toward the same number as the acceleration ratio, what does that imply about the **distance ratio**? Distance Ratio is also approaching: 2

Question 3c:

Explain the chain reaction:

Constant acceleration ratio → velocity ratio → distance ratio.

everything gets forced to two.

PART 6 — Final Mission Explanation

Write 3–4 short sentences:

- What happens to the distances of the two probes?
- What happens to the ratios over time?
- Why does the ratio never go below 2?
- How did the L'Hôpital's Rule and DiVA charts help you understand this?

They get farther and farther apart. The ratios go to 2.
Acceleration is always 2 so it forces the other ratios.

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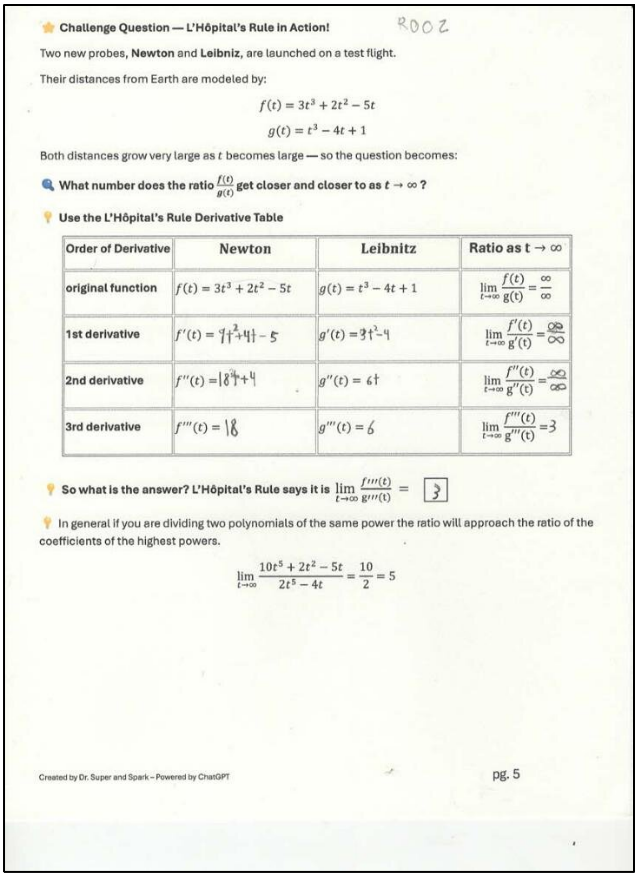

Challenge Question — L'Hôpital's Rule in Action!

ROOZ

Two new probes, **Newton** and **Leibniz**, are launched on a test flight.

Their distances from Earth are modeled by:

$$f(t) = 3t^3 + 2t^2 - 5t$$

$$g(t) = t^3 - 4t + 1$$

Both distances grow very large as t becomes large — so the question becomes:

What number does the ratio $\frac{f(t)}{g(t)}$ get closer and closer to as $t \to \infty$?

Use the L'Hôpital's Rule Derivative Table

Order of Derivative	Newton	Leibniz	Ratio as t → ∞
original function	$f(t) = 3t^3 + 2t^2 - 5t$	$g(t) = t^3 - 4t + 1$	$\lim_{t\to\infty} \frac{f(t)}{g(t)} = \frac{\infty}{\infty}$
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2nd derivative	$f''(t) = 18t+4$	$g''(t) = 6t$	$\lim_{t\to\infty} \frac{f''(t)}{g''(t)} = \frac{\infty}{\infty}$
3rd derivative	$f'''(t) = 18$	$g'''(t) = 6$	$\lim_{t\to\infty} \frac{f'''(t)}{g'''(t)} = 3$

So what is the answer? L'Hôpital's Rule says it is $\lim_{t\to\infty} \frac{f'''(t)}{g'''(t)} = \boxed{3}$

In general if you are dividing two polynomials of the same power the ratio will approach the ratio of the coefficients of the highest powers.

$$\lim_{t\to\infty} \frac{10t^5 + 2t^2 - 5t}{2t^5 - 4t} = \frac{10}{2} = 5$$

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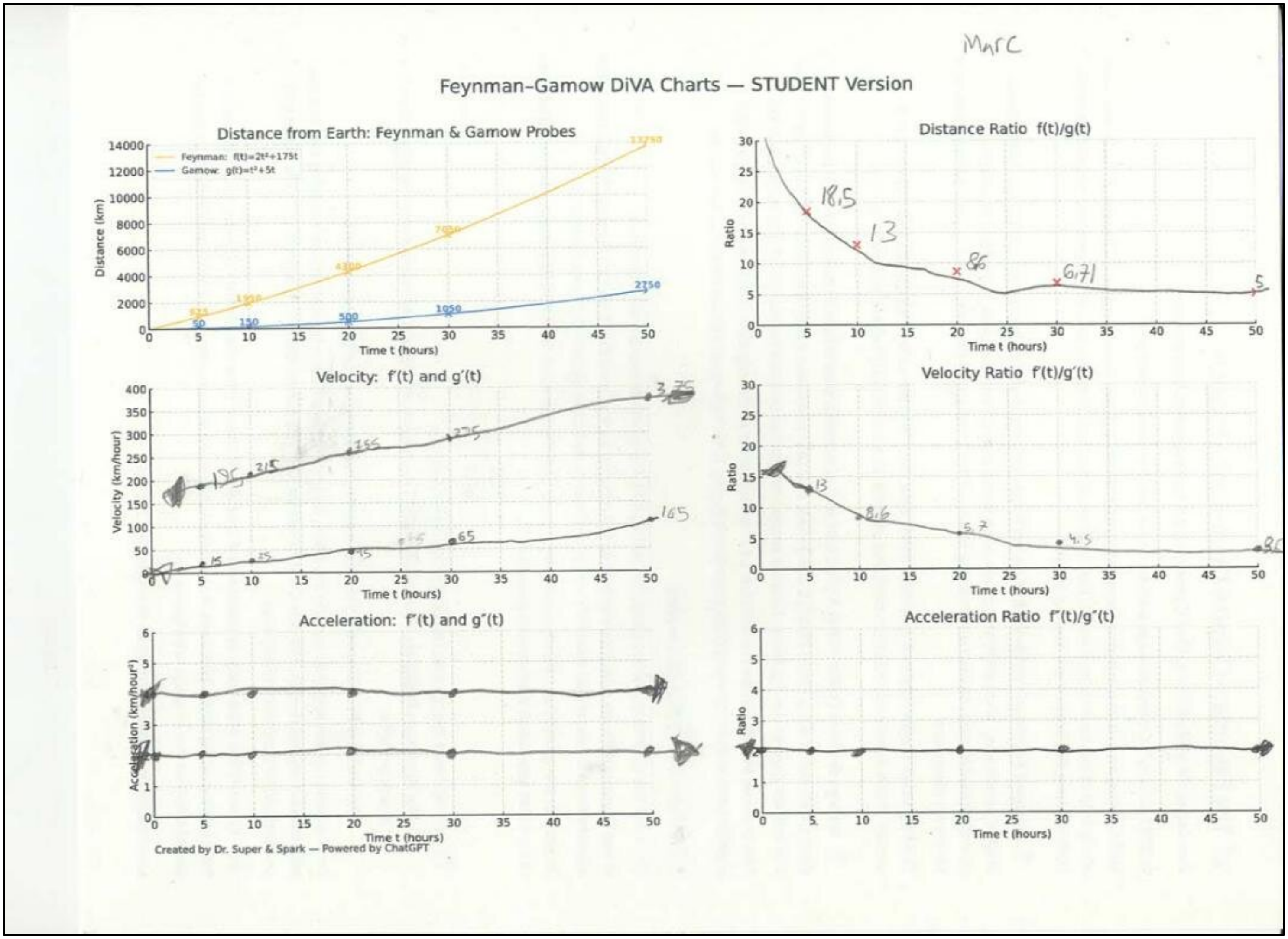
pg. 5



Marc

Feynman–Gamow DiVA Charts — STUDENT Version

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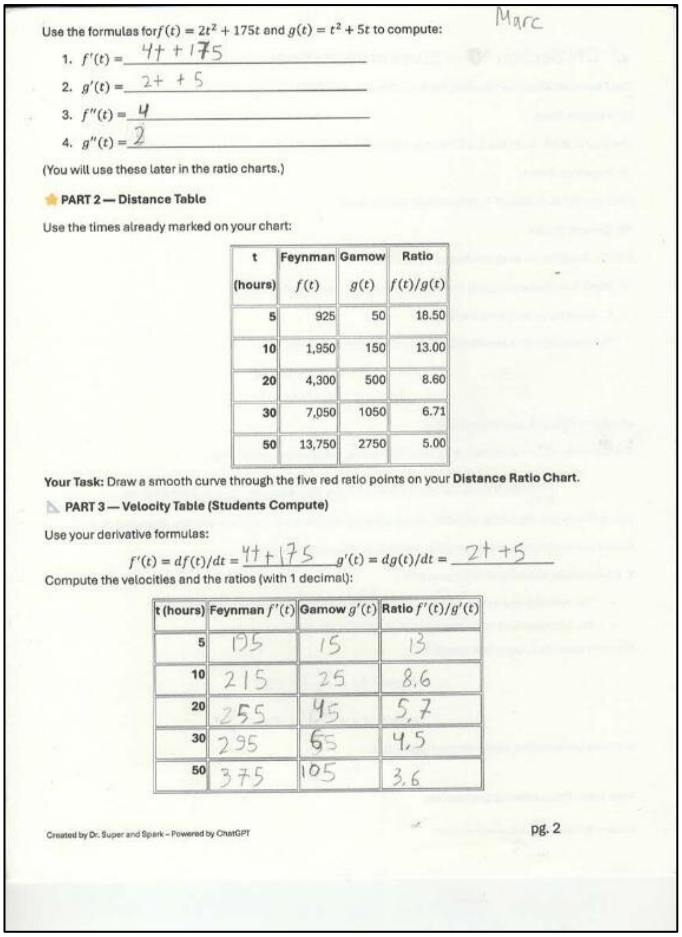

Marc

Use the formulas for $f(t) = 2t^2 + 175t$ and $g(t) = t^2 + 5t$ to compute:

1. $f'(t) =$ $4t + 175$
2. $g'(t) =$ $2t + 5$
3. $f''(t) =$ 4
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(You will use these later in the ratio charts.)

⭐ PART 2 — Distance Table

Use the times already marked on your chart:

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Your Task: Draw a smooth curve through the five red ratio points on your **Distance Ratio Chart.**

📐 PART 3 — Velocity Table (Students Compute)

Use your derivative formulas:

$$f'(t) = df(t)/dt = 4t + 175 \quad g'(t) = dg(t)/dt = 2t + 5$$

Compute the velocities and the ratios (with 1 decimal):

t (hours)	Feynman $f'(t)$	Gamow $g'(t)$	Ratio $f'(t)/g'(t)$
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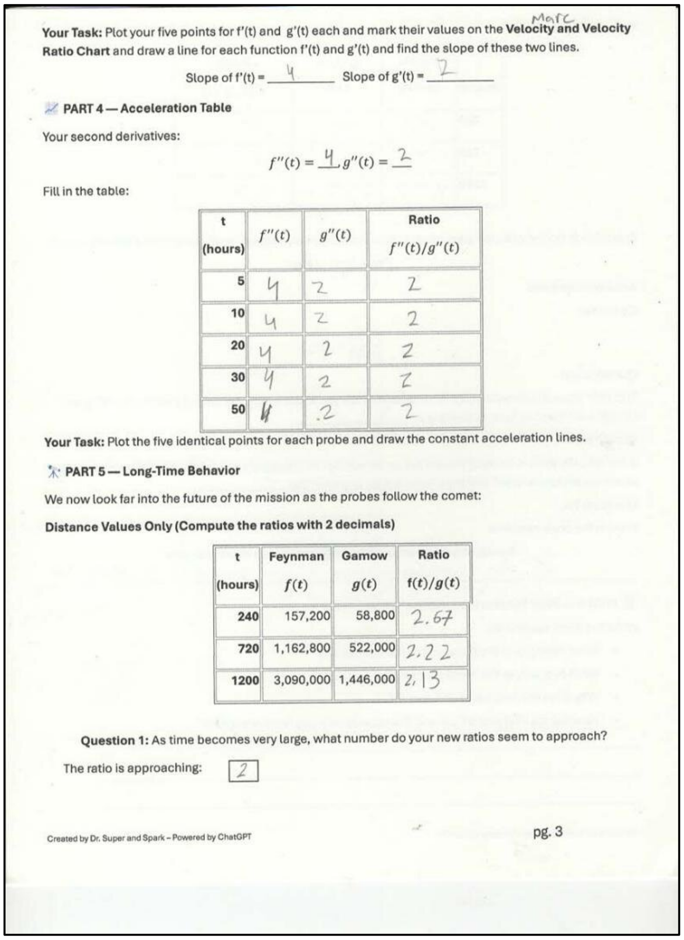

Marc

Your Task: Plot your five points for f'(t) and g'(t) each and mark their values on the **Velocity and Velocity Ratio Chart** and draw a line for each function f'(t) and g'(t) and find the slope of these two lines.

Slope of f'(t) = 4 Slope of g'(t) = 2

📈 PART 4 — Acceleration Table

Your second derivatives:

$$f''(t) = 4, g''(t) = 2$$

Fill in the table:

t (hours)	$f''(t)$	$g''(t)$	Ratio $f''(t)/g''(t)$
5	4	2	2
10	4	2	2
20	4	2	2
30	4	2	2
50	4	.2	2

Your Task: Plot the five identical points for each probe and draw the constant acceleration lines.

🔭 PART 5 — Long-Time Behavior

We now look far into the future of the mission as the probes follow the comet:

Distance Values Only (Compute the ratios with 2 decimals)

t (hours)	Feynman $f(t)$	Gamow $g(t)$	Ratio f(t)/g(t)
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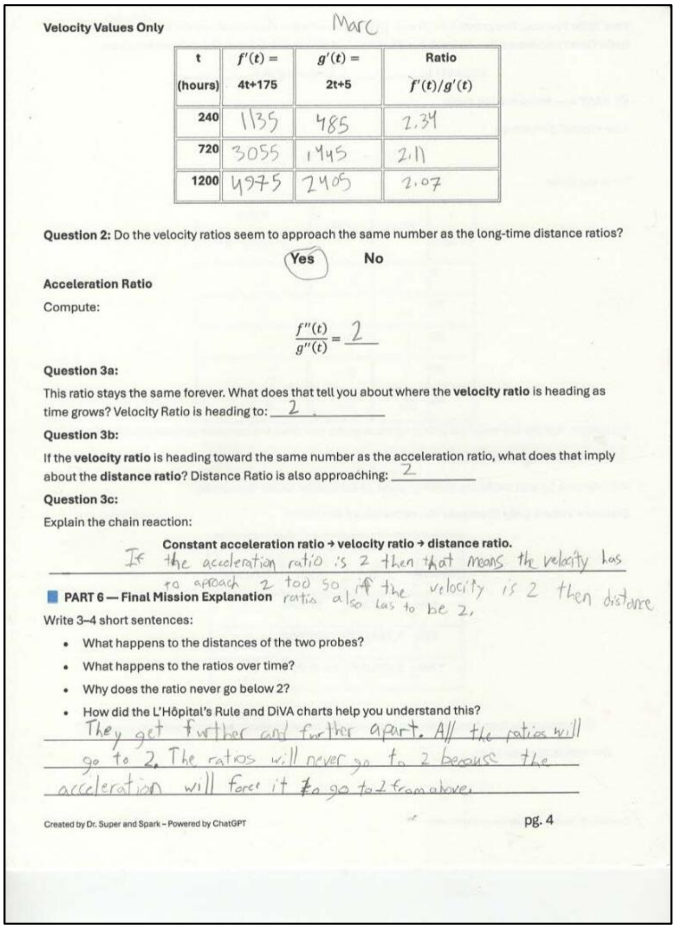
Question 1: As time becomes very large, what number do your new ratios seem to approach?

The ratio is approaching: $\boxed{2}$



Velocity Values Only

Marc

t (hours)	$f'(t) =$ 4t+175	$g'(t) =$ 2t+5	Ratio $f'(t)/g'(t)$
240	1135	485	2.34
720	3055	1445	2.11
1200	4975	2405	2.07

Question 2: Do the velocity ratios seem to approach the same number as the long-time distance ratios?

[✓ Yes] No

Acceleration Ratio

Compute:

$$\frac{f''(t)}{g''(t)} = 2$$

Question 3a:

This ratio stays the same forever. What does that tell you about where the **velocity ratio** is heading as time grows? Velocity Ratio is heading to: 2

Question 3b:

If the **velocity ratio** is heading toward the same number as the acceleration ratio, what does that imply about the **distance ratio**? Distance Ratio is also approaching: 2

Question 3c:

Explain the chain reaction:

Constant acceleration ratio → velocity ratio → distance ratio.

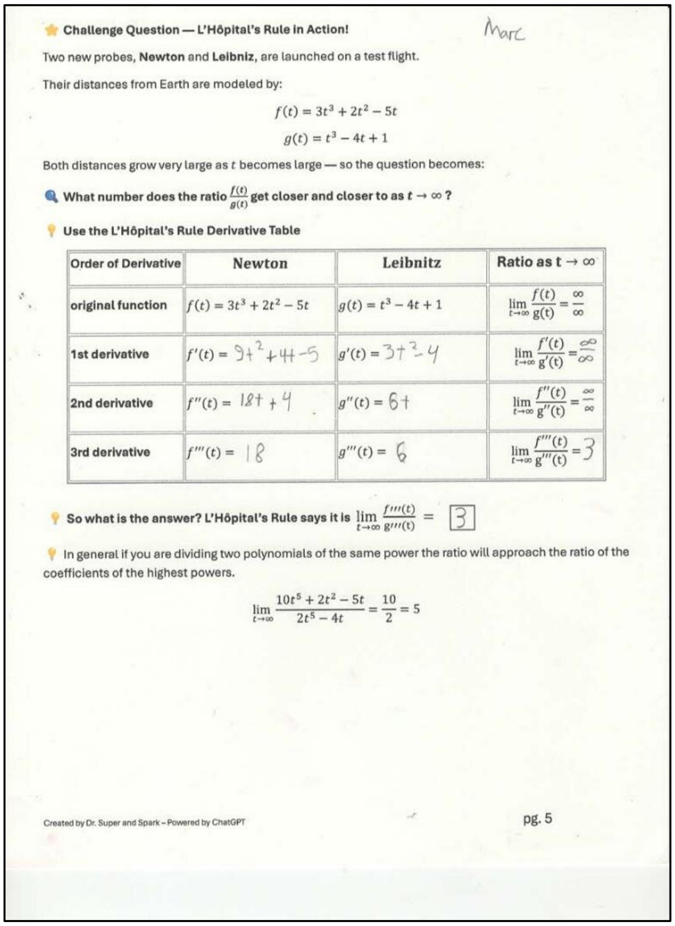
If the acceleration ratio is 2 then that means the velocity has to aproach 2 too so if the velocity is 2 then distance ratio also has to be 2.

PART 6 — Final Mission Explanation

Write 3–4 short sentences:

- What happens to the distances of the two probes?
- What happens to the ratios over time?
- Why does the ratio never go below 2?
- How did the L'Hôpital's Rule and DiVA charts help you understand this?

They get further and further apart. All the ratios will go to 2. The ratios will never go to 2 because the acceleration will force it to go to 2 from above.



⭐ Challenge Question — L'Hôpital's Rule in Action!

Marc

Two new probes, **Newton** and **Leibniz**, are launched on a test flight.

Their distances from Earth are modeled by:

$$f(t) = 3t^3 + 2t^2 - 5t$$
$$g(t) = t^3 - 4t + 1$$

Both distances grow very large as t becomes large — so the question becomes:

🔍 **What number does the ratio $\frac{f(t)}{g(t)}$ get closer and closer to as $t \to \infty$?**

💡 **Use the L'Hôpital's Rule Derivative Table**

Order of Derivative	Newton	Leibniz	Ratio as t → ∞
original function	$f(t) = 3t^3 + 2t^2 - 5t$	$g(t) = t^3 - 4t + 1$	$\lim_{t\to\infty} \frac{f(t)}{g(t)} = \frac{\infty}{\infty}$
1st derivative	$f'(t) = 9t^2 + 4t - 5$	$g'(t) = 3t^2 - 4$	$\lim_{t\to\infty} \frac{f'(t)}{g'(t)} = \frac{\infty}{\infty}$
2nd derivative	$f''(t) = 18t + 4$	$g''(t) = 6t$	$\lim_{t\to\infty} \frac{f''(t)}{g''(t)} = \frac{\infty}{\infty}$
3rd derivative	$f'''(t) = 18$	$g'''(t) = 6$	$\lim_{t\to\infty} \frac{f'''(t)}{g'''(t)} = 3$

💡 **So what is the answer? L'Hôpital's Rule says it is** $\lim_{t\to\infty} \frac{f'''(t)}{g'''(t)} = \boxed{3}$

💡 In general if you are dividing two polynomials of the same power the ratio will approach the ratio of the coefficients of the highest powers.

$$\lim_{t\to\infty} \frac{10t^5 + 2t^2 - 5t}{2t^5 - 4t} = \frac{10}{2} = 5$$

Adventure 11. –The Magic of Taylor Series

1. **Overview of the Session**

- Date: 12/17/2025
- Duration: 11/4 Hours 1:50-3:05 pm
- Students present: Cyrus, Rooz, Marc
- Summary of what the students did/discovered:
    - Review of Buffon’s Needle Problem and completing the Feynmann Gamow Space Probe Activity from last Math Circle.
    - Taylor Polynomial and Series and how they are computed.
    - Taylor Series for e^x, cos(x), and sin(x).
    - A Taylor Series builds a function from many tiny pieces.
    - By adding more terms, your approximation gets closer to the real curve.
    - Sine, cosine, and exponential all “grow” from the same idea.
    - And when we extend the exponential to imaginary numbers, the cosine and sine pieces combine perfectly to form a point on the unit circle.
    - How Taylor Polynomials for these functions can give good approximate values for any x even though they are calculated at 0.
    - How $\mathbf{e^{ix} = cos(x) + isin(x)}$ adding the Taylor series for cos(x) and isin(x).

2. Warm-Up Conversation / Storytelling/Videos

- I went over the Buffon’s needle problem and asked them why it was enough just to consider one half of one of the sections. Cyrus said because everywhere else we will have the same thing. I also asked Marc and Rooz the same question and eventually they came up with the answer also. Then I went over the integrals and Cyrus wanted to know about the case where the length of the needle was not equal to the distance between the lines and we covered that at the end also.
- We also discussed the questions at the end of Part 4 of the previous Math Circle, and I made sure that they saw that even though the constant acceleration ratio at 2 will force the ratio of velocities and distances to also go to 2, the values for both of them will get further apart. They also did Part 5 and saw how L’Hôpital’s Rule can be applied repeatedly to find the ratio that is infinity over infinity.

- Next, we read the story. And they reacted funny when I said they were emotional, particularly Cyrus. But they liked the story and this story is a good lead into Taylor Series.
- Then we watched the video: from 3Blue1Brown Chanelle on You Tube **Taylor series | Chapter 11, Essence of calculus** As usual we did stop it a few times and discussed it. It was a bit too long but as usual it covers the essentials well, and the animation is great.
- For a change today Rooz went faster than everybody and Marc was the slower person. He did seem to be in a dream state. Cyrus as usual was his meticulous self.

📊 3. Activities Completed

Part 1 — Rebuilding $e^x$from its Taylor Series and Computing e^1

They had no trouble putting values for e0. They did not know the meaning of cumulative value, so I explained it and still it was not clear for them exactly what to put in. Rooz then Marc then Cyrus completed the values and they all compared it with the real value of e^1 and marked it on the curve for e^x.

Part 2 — Rebuilding $\cos X$ from its Taylor Series

They also had no trouble putting in the values for the derivative of cos(x). Again, Rooz was going fast and did not need any help, and I could not even slow him down. They then compared the value they had found with the real value of cos(Л/6) and marked it on the curve of cos(x)

Part 3 — Rebuilding $\sin X$from its Taylor Series

Again, they had no trouble putting in the values for the derivative of sin(x). And again, Rooz was going fast and did not need any help and I could not even slow him down. They then compared the value they had found with the real value of sin(Л/6) and marked it on the curve of sin(x)

Part 4 — Bringing Everything Together: e^{iX}

This was a little trickier. Finding the values for powers of i that we had seen before was done quickly for Rooz and Cyrus, but Marc was a little hesitant. They all saw that if you add the Taylor Series for cos(x) and isin(x) you will get the Taylor Series for e^{ix}. I did go over how this will make e^{ix} sit on the unit circle in the complex plane. We had seen this (Euler's Formula) several times this year when we covered "Imaginary Numbers are Real"

🧑‍🏫 4. What Worked Well/Or Did Not

- They had no trouble with filling in the tables and finding the values on the graphs and enjoyed it.
- We could have done some more in this session.

🧠 5. Adjustments for Next Time

- I would make some more boxes in the tables for them to fill. And let them fill in all the values for the terms in the Taylor Polynomial and not just the cumulative values.

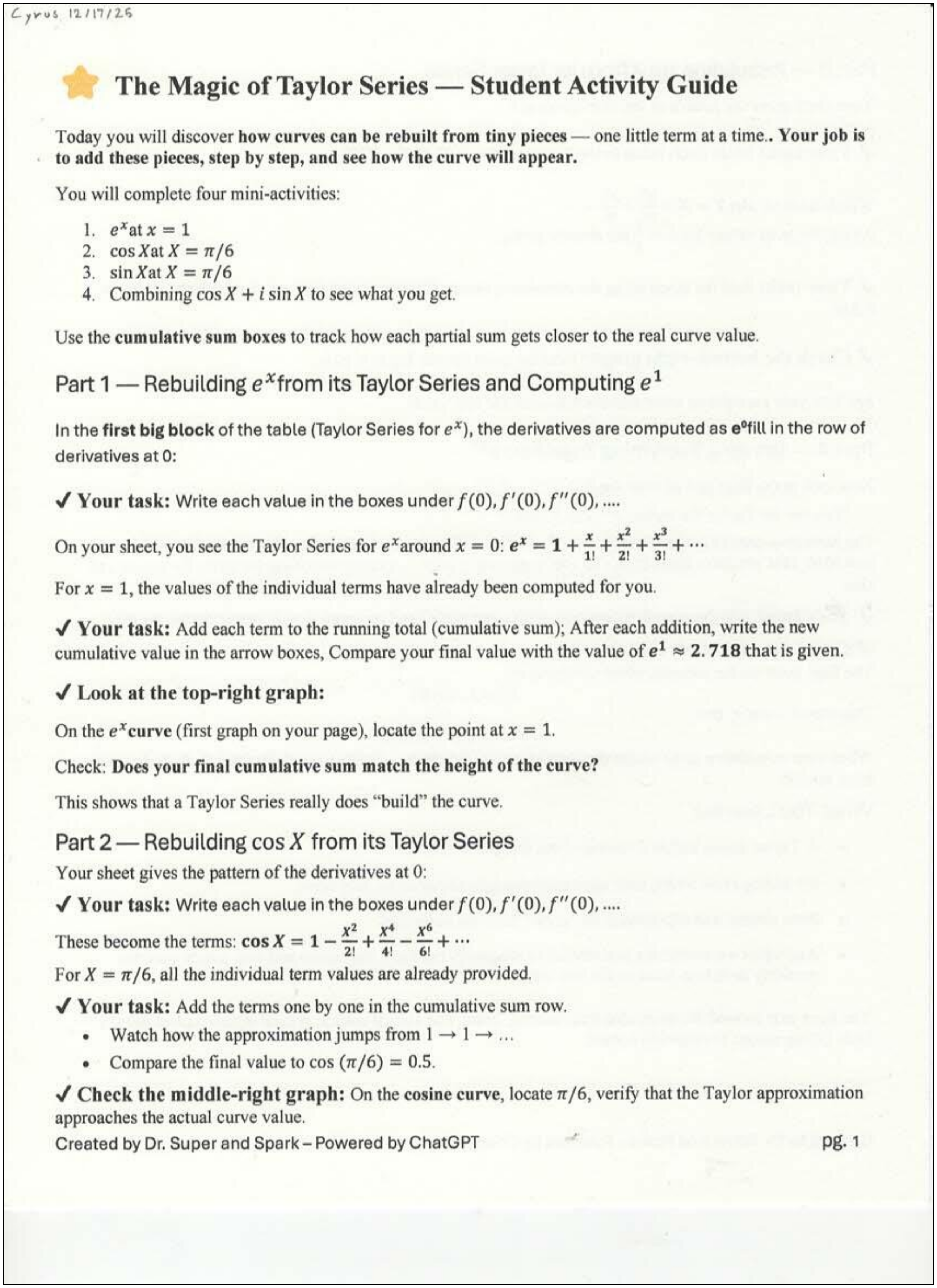
🪄 6. Student Quotes

- Cyrus asked at the end do we have another page to do and seemed disappointed that we were done! This is after an hour and 15 minutes!
- Rooz was going fast and I had a hard time slowing him down. It was great to see him on track and doing so well.
- As I said Marc was in a very quiet state and for a change going much slower than usual but seemed content.

🧱 7. Closing Reflection by Dr. Super

This was a very good session again. It was great to see Rooz do all the work on his own and even faster than Cyrus and Marc, **he was on fire!** Cyrus is asking more questions and three of them work together more and compare their results more. I think they all got the concepts well and enjoyed the activities. It seemed almost too easy for them and that is surprising as the Taylor Series concepts is not an easy one.

📘 9. Student Completed Worksheets



Cyrus 12/17/25

The Magic of Taylor Series — Student Activity Guide

Today you will discover **how curves can be rebuilt from tiny pieces** — one little term at a time.. **Your job is to add these pieces, step by step, and see how the curve will appear.**

You will complete four mini-activities:

1. e^xat $x = 1$
2. $\cos X$at $X = \pi/6$
3. $\sin X$at $X = \pi/6$
4. Combining $\cos X + i \sin X$ to see what you get.

Use the **cumulative sum boxes** to track how each partial sum gets closer to the real curve value.

Part 1 — Rebuilding $e^x$from its Taylor Series and Computing e^1

In the **first big block** of the table (Taylor Series for e^x), the derivatives are computed as $\mathbf{e^0}$fill in the row of derivatives at 0:

✓ **Your task:** Write each value in the boxes under $f(0), f'(0), f''(0), ...$

On your sheet, you see the Taylor Series for e^xaround $x = 0$: $\boldsymbol{e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots}$

For $x = 1$, the values of the individual terms have already been computed for you.

✓ **Your task:** Add each term to the running total (cumulative sum); After each addition, write the new cumulative value in the arrow boxes, Compare your final value with the value of $\mathbf{e^1 \approx 2.718}$ that is given.

✓ **Look at the top-right graph:**

On the e^x**curve** (first graph on your page), locate the point at $x = 1$.

Check: **Does your final cumulative sum match the height of the curve?**

This shows that a Taylor Series really does "build" the curve.

Part 2 — Rebuilding $\cos X$ from its Taylor Series

Your sheet gives the pattern of the derivatives at 0:

✓ **Your task:** Write each value in the boxes under $f(0), f'(0), f''(0), ...$

These become the terms: $\boldsymbol{\cos X = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots}$

For $X = \pi/6$, all the individual term values are already provided.

✓ **Your task:** Add the terms one by one in the cumulative sum row.

- Watch how the approximation jumps from 1 → 1 → …
- Compare the final value to $\cos(\pi/6) = 0.5$.

✓ **Check the middle-right graph:** On the **cosine curve**, locate $\pi/6$, verify that the Taylor approximation approaches the actual curve value.

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Part 3 — Rebuilding $\sin X$ from its Taylor Series

Your sheet gives the pattern of the derivatives at 0:

✓ **Your task:** Write each value in the boxes under $f(0), f'(0), f''(0), \ldots$

Which leads to: $\mathbf{\sin X = X - \frac{X^3}{3!} + \frac{X^5}{5!} - \cdots}$
Again, the term values for $X = \frac{\pi}{6}$ are already given.

✓ **Your task:** Add the terms using the cumulative boxes, Compare your final number with $\sin(\pi/6) = 0.866$.

✓ **Check the bottom-right graph:** Find the point for $\sin X$ at $X = \pi/6$.

See how your cumulative sums steadily approach the real value.

Part 4 — Bringing Everything Together: e^{iX}

Now look at the final part of your sheet.

You see the Taylor Series for: $e^{iX}, \cos X, i\sin X$

The derivative pattern cycles through: i, i^2, i^3, i^4, i^5, i^6, Fill in these values below them in the boxes that are provided. This produces alternating real and imaginary pieces — exactly matching the series for cosine and sine.

✓ **Your task:** Add the provided terms for cos(x) and isin(x) and compare it with Taylor Series for e^{iX}:

✓ **Check the unit circle diagram:**
The final point on the complex plane should sit at:

$$(\cos X, \sin X)$$

This shows visually that:

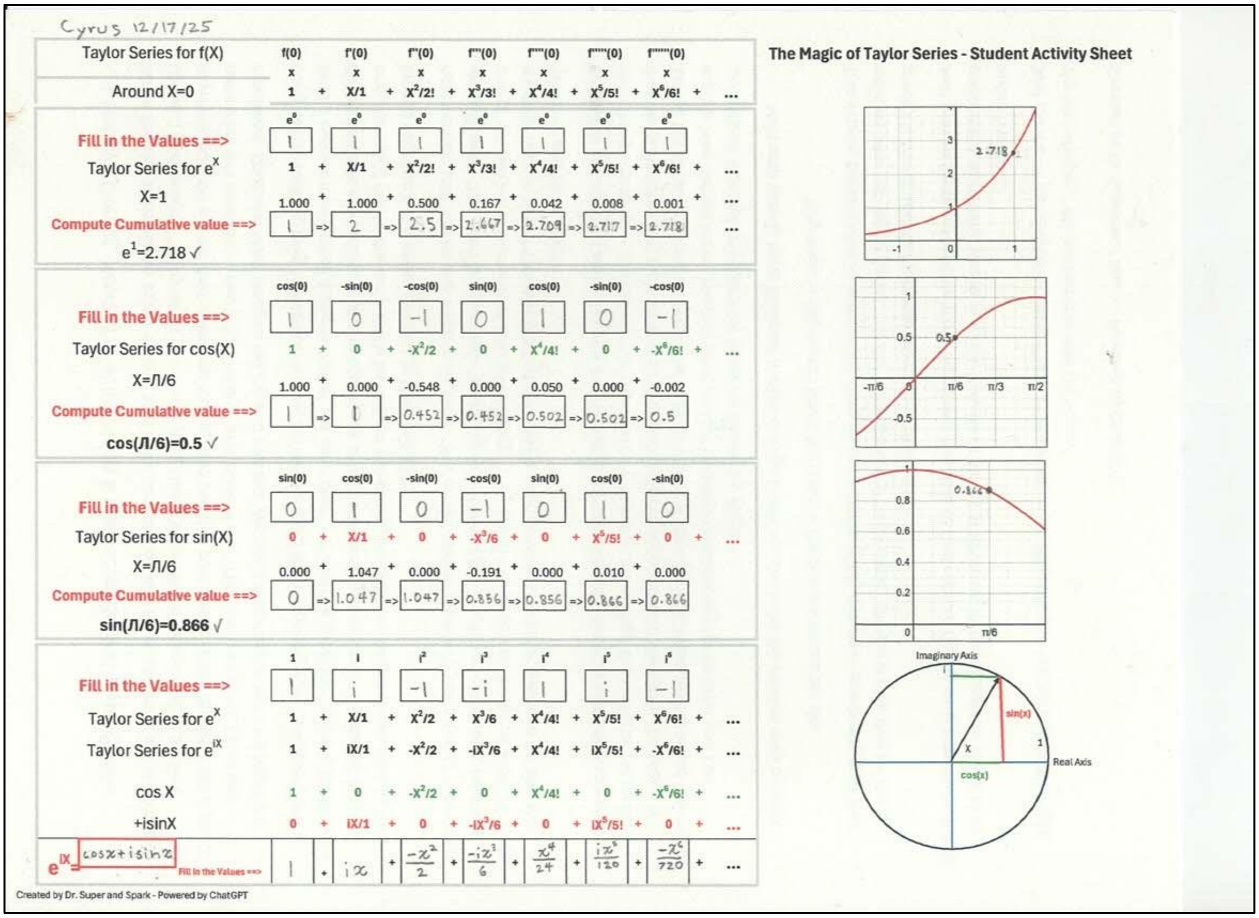
$$\mathbf{e^{iX} = \cos X + i\sin X}$$

When your cumulative sums match the coordinates on the circle, you have **reconstructed Euler's Formula** from scratch.

What You Learned

- A Taylor Series builds a function from many tiny pieces.
- By adding more terms, your approximation gets closer to the real curve.
- Sine, cosine, and exponential all "grow" from the same idea.
- And when we extend the exponential to imaginary numbers, the cosine and sine pieces combine perfectly to form a point on the unit circle.

You have now learned the same idea that Newton, Euler, and Halley used to predict motion across the sky — from falling moons to returning comets.



Cyrus 12/17/25

The Magic of Taylor Series - Student Activity Sheet

Taylor Series for f(X)	f(0)	f'(0)	f''(0)	f'''(0)	f''''(0)	f'''''(0)	f''''''(0)	
	x	x	x	x	x	x	x	
Around X=0	1 +	X/1 +	$X^2/2!$ +	$X^3/3!$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
	e^0	e^0	e^0	e^0	e^0	e^0	e^0	
Fill in the Values ==>	1	1	1	1	1	1	1	
Taylor Series for e^X	1 +	X/1 +	$X^2/2!$ +	$X^3/3!$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
X=1	1.000 +	1.000 +	0.500 +	0.167 +	0.042 +	0.008 +	0.001 +	...
Compute Cumulative value ==>	1 =>	2 =>	2.5 =>	2.667 =>	2.709 =>	2.717 =>	2.718	...
e^1=2.718 ✓								
	cos(0)	-sin(0)	-cos(0)	sin(0)	cos(0)	-sin(0)	-cos(0)	
Fill in the Values ==>	1	0	-1	0	1	0	-1	
Taylor Series for cos(X)	1 +	0 +	$-X^2/2$ +	0 +	$X^4/4!$ +	0 +	$-X^6/6!$ +	...
X=Л/6	1.000 +	0.000 +	-0.548 +	0.000 +	0.050 +	0.000 +	-0.002	
Compute Cumulative value ==>	1 =>	1 =>	0.452 =>	0.452 =>	0.502 =>	0.502 =>	0.5	
cos(Л/6)=0.5 ✓								
	sin(0)	cos(0)	-sin(0)	-cos(0)	sin(0)	cos(0)	-sin(0)	
Fill in the Values ==>	0	1	0	-1	0	1	0	
Taylor Series for sin(X)	0 +	X/1 +	0 +	$-X^3/6$ +	0 +	$X^5/5!$ +	0 +	...
X=Л/6	0.000 +	1.047 +	0.000 +	-0.191 +	0.000 +	0.010 +	0.000	
Compute Cumulative value ==>	0 =>	1.047 =>	1.047 =>	0.856 =>	0.856 =>	0.866 =>	0.866	
sin(Л/6)=0.866 ✓								
	1	i	i^2	i^3	i^4	i^5	i^6	
Fill in the Values ==>	1	i	-1	-i	1	i	-1	
Taylor Series for e^X	1 +	X/1 +	$X^2/2$ +	$X^3/6$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
Taylor Series for e^{iX}	1 +	iX/1 +	$-X^2/2$ +	$-iX^3/6$ +	$X^4/4!$ +	$iX^5/5!$ +	$-X^6/6!$ +	...
cos X	1 +	0 +	$-X^2/2$ +	0 +	$X^4/4!$ +	0 +	$-X^6/6!$ +	...
+isinX	0 +	iX/1 +	0 +	$-iX^3/6$ +	0 +	$iX^5/5!$ +	0 +	...
e^{iX}= $\boxed{\cos x + i\sin x}$ Fill in the Values ==>	1 +	ix +	$\frac{-x^2}{2}$ +	$\frac{-ix^3}{6}$ +	$\frac{x^4}{24}$ +	$\frac{ix^5}{120}$ +	$\frac{-x^6}{720}$ +	...

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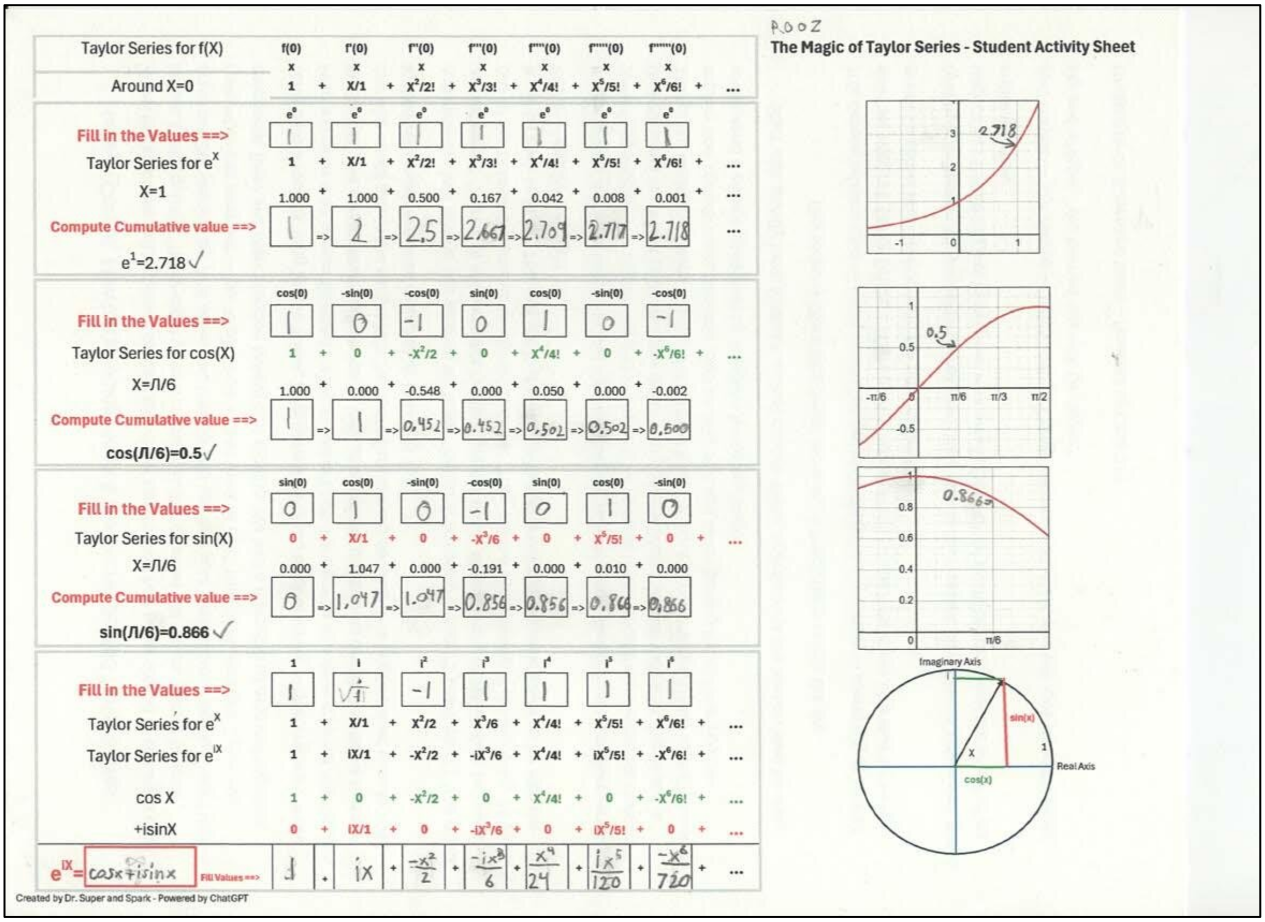

ROOZ

The Magic of Taylor Series - Student Activity Sheet

Taylor Series for f(X)	f(0)	f'(0)	f''(0)	f'''(0)	f''''(0)	f'''''(0)	f''''''(0)	
	x	x	x	x	x	x	x	
Around X=0	1 +	X/1 +	$X^2/2!$ +	$X^3/3!$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
	e^0	e^0	e^0	e^0	e^0	e^0	e^0	
Fill in the Values ==>	1	1	1	1	1	1	1	
Taylor Series for e^X	1 +	X/1 +	$X^2/2!$ +	$X^3/3!$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
X=1	1.000 +	1.000 +	0.500 +	0.167 +	0.042 +	0.008 +	0.001 +	...
Compute Cumulative value ==>	1 =>	2 =>	2.5 =>	2.667 =>	2.709 =>	2.717 =>	2.718	...
e^1=2.718 ✓								
	cos(0)	-sin(0)	-cos(0)	sin(0)	cos(0)	-sin(0)	-cos(0)	
Fill in the Values ==>	1	0	-1	0	1	0	-1	
Taylor Series for cos(X)	1 +	0 +	$-X^2/2$ +	0 +	$X^4/4!$ +	0 +	$-X^6/6!$ +	...
X=Л/6	1.000 +	0.000 +	-0.548 +	0.000 +	0.050 +	0.000 +	-0.002	
Compute Cumulative value ==>	1 =>	1 =>	0.452 =>	0.452 =>	0.502 =>	0.502 =>	0.500	
cos(Л/6)=0.5 ✓								
	sin(0)	cos(0)	-sin(0)	-cos(0)	sin(0)	cos(0)	-sin(0)	
Fill in the Values ==>	0	1	0	-1	0	1	0	
Taylor Series for sin(X)	0 +	X/1 +	0 +	$-X^3/6$ +	0 +	$X^5/5!$ +	0 +	...
X=Л/6	0.000 +	1.047 +	0.000 +	-0.191 +	0.000 +	0.010 +	0.000	
Compute Cumulative value ==>	0 =>	1.047 =>	1.047 =>	0.856 =>	0.856 =>	0.866 =>	0.866	
sin(Л/6)=0.866 ✓								
	1	i	i^2	i^3	i^4	i^5	i^6	
Fill in the Values ==>	1	$\sqrt{-1}$	-1	1	1	1	1	
Taylor Series for e^X	1 +	X/1 +	$X^2/2$ +	$X^3/6$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
Taylor Series for e^{iX}	1 +	iX/1 +	$-X^2/2$ +	$-iX^3/6$ +	$X^4/4!$ +	$iX^5/5!$ +	$-X^6/6!$ +	...
cos X	1 +	0 +	$-X^2/2$ +	0 +	$X^4/4!$ +	0 +	$-X^6/6!$ +	...
+isinX	0 +	iX/1 +	0 +	$-iX^3/6$ +	0 +	$iX^5/5!$ +	0 +	...
e^{iX}= $\boxed{\cos x + i\sin x}$ Fill Values ==>	1 +	ix +	$\frac{-x^2}{2}$ +	$\frac{-ix^3}{6}$ +	$\frac{x^4}{24}$ +	$\frac{ix^5}{120}$ +	$\frac{-x^6}{720}$ +	...

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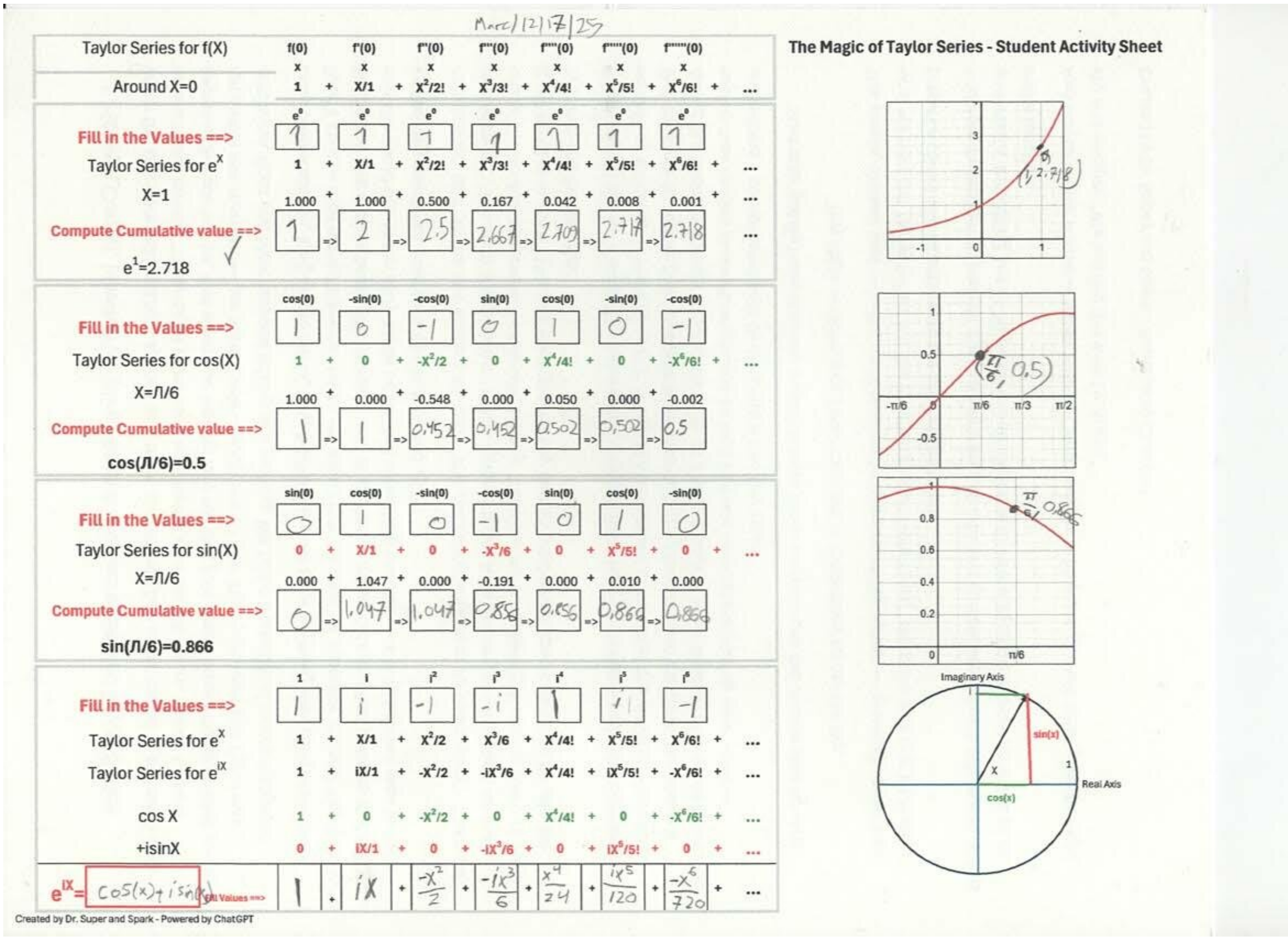

Marc/12/17/25

The Magic of Taylor Series - Student Activity Sheet

Taylor Series for f(X)	f(0)	f'(0)	f''(0)	f'''(0)	f''''(0)	f'''''(0)	f''''''(0)	
	x	x	x	x	x	x	x	
Around X=0	1 +	X/1 +	$X^2/2!$ +	$X^3/3!$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
	e^0	e^0	e^0	e^0	e^0	e^0	e^0	
Fill in the Values ==>	1	1	1	1	1	1	1	
Taylor Series for e^X	1 +	X/1 +	$X^2/2!$ +	$X^3/3!$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
X=1	1.000 +	1.000 +	0.500 +	0.167 +	0.042 +	0.008 +	0.001 +	...
Compute Cumulative value ==> ✓	1 =>	2 =>	2.5 =>	2.667 =>	2.709 =>	2.717 =>	2.718	...
$e^1=2.718$								
	cos(0)	-sin(0)	-cos(0)	sin(0)	cos(0)	-sin(0)	-cos(0)	
Fill in the Values ==>	1	0	-1	0	1	0	-1	
Taylor Series for cos(X)	1 +	0 +	$-X^2/2$ +	0 +	$X^4/4!$ +	0 +	$-X^6/6!$ +	...
X=Л/6	1.000 +	0.000 +	-0.548 +	0.000 +	0.050 +	0.000 +	-0.002	
Compute Cumulative value ==>	1 =>	1 =>	0.452 =>	0.452 =>	0.502 =>	0.502 =>	0.5	
cos(Л/6)=0.5								
	sin(0)	cos(0)	-sin(0)	-cos(0)	sin(0)	cos(0)	-sin(0)	
Fill in the Values ==>	0	1	0	-1	0	1	0	
Taylor Series for sin(X)	0 +	X/1 +	0 +	$-X^3/6$ +	0 +	$X^5/5!$ +	0 +	...
X=Л/6	0.000 +	1.047 +	0.000 +	-0.191 +	0.000 +	0.010 +	0.000	
Compute Cumulative value ==>	0 =>	1.047 =>	1.047 =>	0.858 =>	0.856 =>	0.866 =>	0.866	
sin(Л/6)=0.866								
	1	i	i^2	i^3	i^4	i^5	i^6	
Fill in the Values ==>	1	i	-1	-i	1	i	-1	
Taylor Series for e^X	1 +	X/1 +	$X^2/2$ +	$X^3/6$ +	$X^4/4!$ +	$X^5/5!$ +	$X^6/6!$ +	...
Taylor Series for e^{iX}	1 +	iX/1 +	$-X^2/2$ +	$-iX^3/6$ +	$X^4/4!$ +	$iX^5/5!$ +	$-X^6/6!$ +	...
cos X	1 +	0 +	$-X^2/2$ +	0 +	$X^4/4!$ +	0 +	$-X^6/6!$ +	...
+isinX	0 +	iX/1 +	0 +	$-iX^3/6$ +	0 +	$iX^5/5!$ +	0 +	...
$e^{iX}=$ $\boxed{\cos(x)+i\sin(x)}$ Fill Values ==>	1 +	iX +	$\frac{-x^2}{2}$ +	$\frac{-ix^3}{6}$ +	$\frac{x^4}{24}$ +	$\frac{ix^5}{120}$ +	$\frac{-x^6}{720}$ +	...

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🧑‍🏫 Adventure 12. – Maximum Height, Critical Points, and Vertical Motion

🧩 1. Overview of the Session

- Date: 01/07/2026
- Duration: 1:50–2:55 PM
- Students Present: Cyrus, Rooz (Marc absent)

Section 12 introduced students to one of the most important ideas in both physics and calculus: a maximum height occurs when the velocity becomes zero, even though acceleration is still acting.

Students explored this idea through three related motions:

- A ball thrown straight upward
- A cannonball fired vertically
- A cannonball fired at an angle

Although these motions look very different, students discovered that their vertical motion is identical whenever the initial vertical velocity is the same.

🎥 2. Video Discussion

We began by watching the projectile motion video by Sabins.

Before viewing, students were asked:

What happens to the vertical velocity at the highest point of the motion?

Rather than focusing on formulas, students were encouraged to observe what the projectile was doing as it rose, paused, and fell.

After the video we discussed:

- Why every projectile eventually stops rising.
- Whether gravity disappears at the top.
- Why the projectile begins falling immediately after reaching its highest point.
- How horizontal motion and vertical motion appear to behave differently.

Several students initially believed that the acceleration should become zero at the highest point. The video helped establish that it is the velocity, not the acceleration, that becomes zero.

📜 3. Story Discussion

Students next read *The Cannonball and the Moment That Changed Motion*.

The discussion focused on Galileo's realization that motion could be studied one direction at a time and Newton's later interpretation of maximum height as a special point where the slope becomes zero.

Important questions included:

- Why did Galileo focus on vertical motion?
- What is special about the brief pause at the top of the flight?
- How could mathematicians use that pause to predict motion?

Students connected the historical story to the video and began anticipating what the DiVA charts would reveal.

📊 4. Activity Discussion and Discoveries

Using the DiVA charts, students observed that:

- The ball reaches a maximum height of 45 meters.
- The maximum occurs at 3 seconds.
- The velocity becomes zero at exactly the same instant.
- Acceleration remains constant at -10 m/s^2 throughout the motion.

Students then compared a ball thrown upward and a cannonball fired vertically. They discovered that both motions have identical height, velocity, and acceleration graphs.

The most surprising result came when comparing a vertically thrown ball with a cannonball fired at an angle. Although the paths through space were completely different, the vertical motion remained exactly the same.

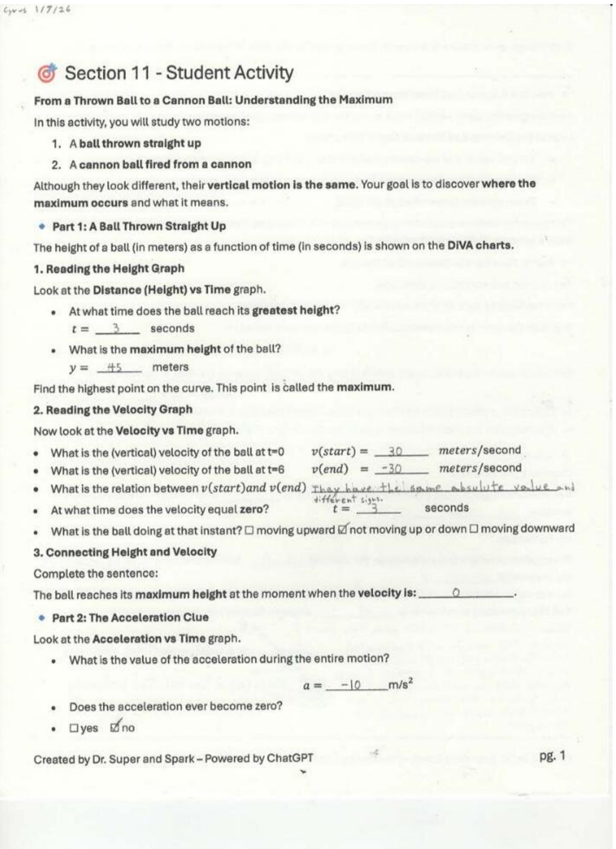
⭐ 5. Main Takeaway

By the end of the session, students were able to state:

A maximum height occurs when the vertical velocity is zero, even though the acceleration remains constant.

This was their first physical example of a critical point, laying the groundwork for future discussions of derivatives, optimization, and projectile motion.

9. Student Completed Worksheets



Cyrus 1/7/26

Section 11 - Student Activity

From a Thrown Ball to a Cannon Ball: Understanding the Maximum

In this activity, you will study two motions:

1. A **ball thrown straight up**
2. A **cannon ball fired from a cannon**

Although they look different, their **vertical motion is the same.** Your goal is to discover **where the maximum occurs** and what it means.

Part 1: A Ball Thrown Straight Up

The height of a ball (in meters) as a function of time (in seconds) is shown on the **DiVA charts.**

1. Reading the Height Graph

Look at the **Distance (Height) vs Time** graph.

- At what time does the ball reach its **greatest height?**
  $t =$ 3 seconds
- What is the **maximum height** of the ball?
  $y =$ 45 meters

Find the highest point on the curve. This point is called the **maximum.**

2. Reading the Velocity Graph

Now look at the **Velocity vs Time** graph.

- What is the (vertical) velocity of the ball at t=0 $v(start) =$ 30 $meters/$second
- What is the (vertical) velocity of the ball at t=6 $v(end) =$ -30 $meters/$second
- What is the relation between $v(start)$and $v(end)$ They have the same absulute value and different signs.
- At what time does the velocity equal **zero?** $t =$ 3 seconds
- What is the ball doing at that instant? ☐ moving upward ☑ not moving up or down ☐ moving downward

3. Connecting Height and Velocity

Complete the sentence:

The ball reaches its **maximum height** at the moment when the **velocity is:** 0.

Part 2: The Acceleration Clue

Look at the **Acceleration vs Time** graph.

- What is the value of the acceleration during the entire motion?
  $a =$ -10 m/s^2
- Does the acceleration ever become zero?
- ☐ yes ☑ no

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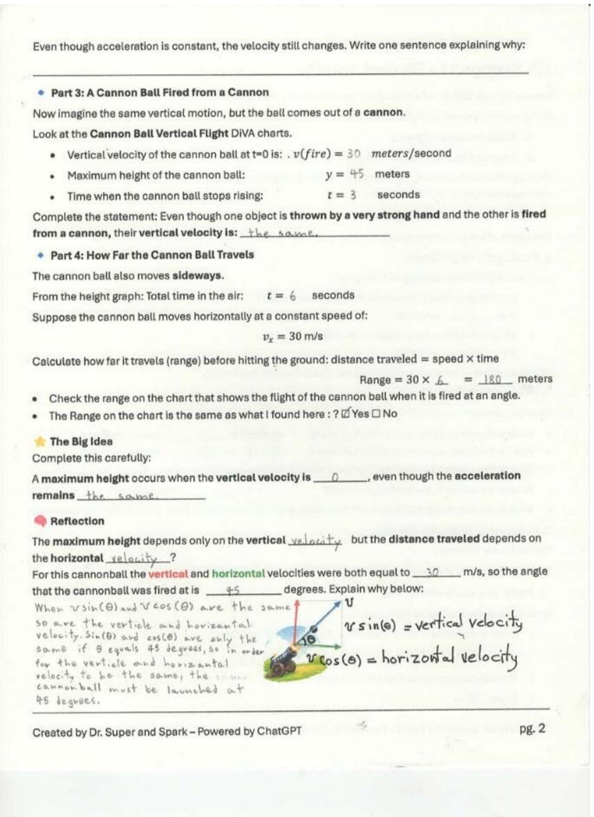
pg. 1



Even though acceleration is constant, the velocity still changes. Write one sentence explaining why:

Part 3: A Cannon Ball Fired from a Cannon

Now imagine the same vertical motion, but the ball comes out of a **cannon.**

Look at the **Cannon Ball Vertical Flight** DiVA charts.

- Vertical velocity of the cannon ball at t=0 is: . $v(fire) =$ 30 $meters/$second
- Maximum height of the cannon ball: $y =$ 45 meters
- Time when the cannon ball stops rising: $t =$ 3 seconds

Complete the statement: Even though one object is **thrown by a very strong hand** and the other is **fired from a cannon,** their **vertical velocity is:** the same.

Part 4: How Far the Cannon Ball Travels

The cannon ball also moves **sideways.**

From the height graph: Total time in the air: $t =$ 6 seconds

Suppose the cannon ball moves horizontally at a constant speed of:

$$v_x = 30 \text{ m/s}$$

Calculate how far it travels (range) before hitting the ground: distance traveled = speed × time

Range = 30 × 6 = 180 meters

- Check the range on the chart that shows the flight of the cannon ball when it is fired at an angle.
- The Range on the chart is the same as what I found here : ? ☑ Yes ☐ No

The Big Idea

Complete this carefully:

A **maximum height** occurs when the **vertical velocity is** 0, even though the **acceleration remains** the same.

Reflection

The **maximum height** depends only on the **vertical** velocity but the **distance traveled** depends on the **horizontal** velocity?

For this cannonball the vertical and horizontal velocities were both equal to 30 m/s, so the angle that the cannonball was fired at is 45 degrees. Explain why below:

When v sin(θ) and v cos(θ) are the same so are the verticle and horizantal velocity. Sin(θ) and cos(θ) are only the same if θ equals 45 degrees, so in order for the verticle and horizantal velocity to be the same, the cannonball must be launched at 45 degrees.

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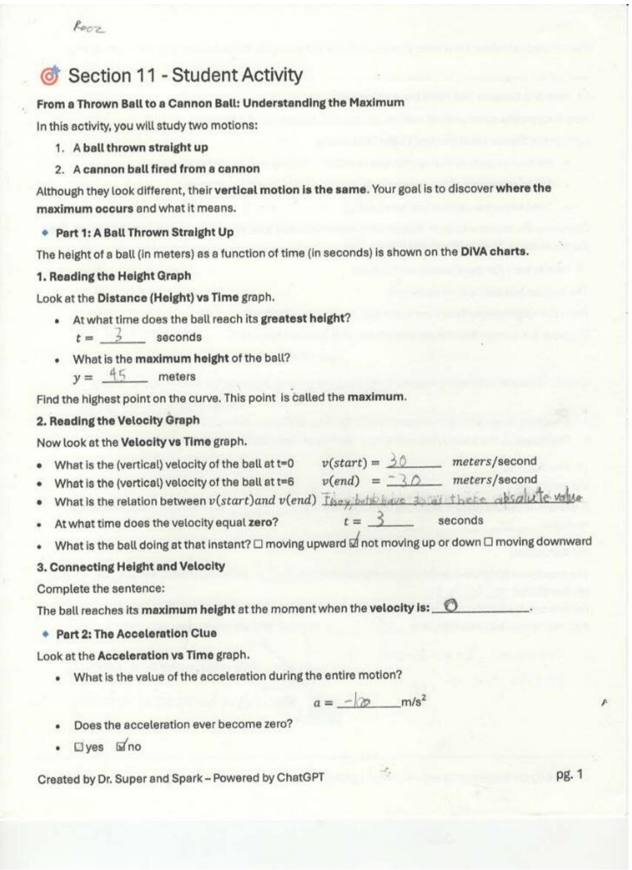
pg. 2



Rooz

Section 11 - Student Activity

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- At what time does the velocity equal **zero?** $t =$ 3 seconds
- What is the ball doing at that instant? ☐ moving upward ☑ not moving up or down ☐ moving downward

3. Connecting Height and Velocity

Complete the sentence:

The ball reaches its **maximum height** at the moment when the **velocity is:** 0.

Part 2: The Acceleration Clue

Look at the **Acceleration vs Time** graph.

- What is the value of the acceleration during the entire motion?
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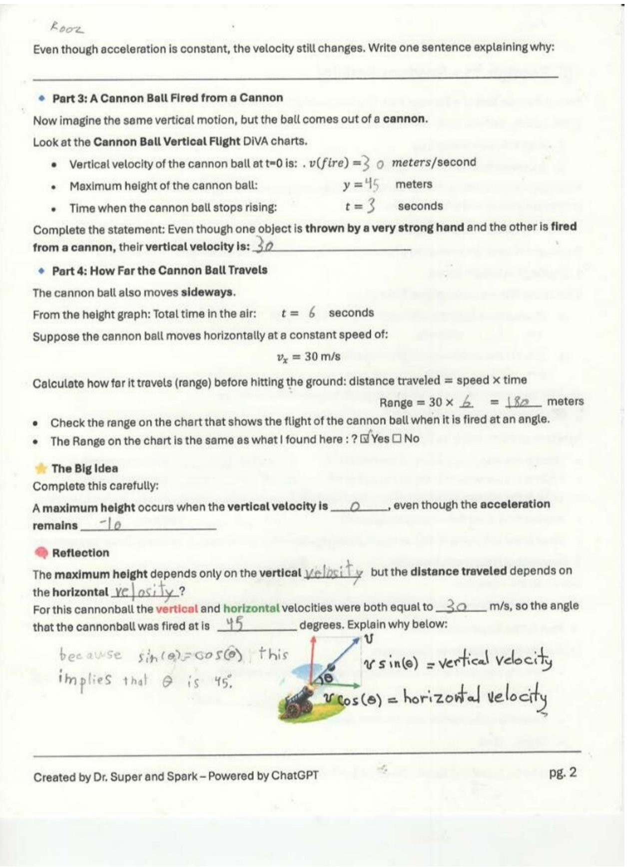
pg. 1



Rooz

Even though acceleration is constant, the velocity still changes. Write one sentence explaining why:

Part 3: A Cannon Ball Fired from a Cannon

Now imagine the same vertical motion, but the ball comes out of a **cannon.**

Look at the **Cannon Ball Vertical Flight** DiVA charts.

- Vertical velocity of the cannon ball at t=0 is: . $v(fire) =$ 30 $meters/$second
- Maximum height of the cannon ball: $y =$ 45 meters
- Time when the cannon ball stops rising: $t =$ 3 seconds

Complete the statement: Even though one object is **thrown by a very strong hand** and the other is **fired from a cannon,** their **vertical velocity is:** 30

Part 4: How Far the Cannon Ball Travels

The cannon ball also moves **sideways.**

From the height graph: Total time in the air: $t =$ 6 seconds

Suppose the cannon ball moves horizontally at a constant speed of:

$$v_x = 30 \text{ m/s}$$

Calculate how far it travels (range) before hitting the ground: distance traveled = speed × time

Range = 30 × 6 = 180 meters

- Check the range on the chart that shows the flight of the cannon ball when it is fired at an angle.
- The Range on the chart is the same as what I found here : ? ☑ Yes ☐ No

The Big Idea

Complete this carefully:

A **maximum height** occurs when the **vertical velocity is** 0, even though the **acceleration remains** -10

Reflection

The **maximum height** depends only on the **vertical** velosity but the **distance traveled** depends on the **horizontal** velosity?

For this cannonball the vertical and horizontal velocities were both equal to 30 m/s, so the angle that the cannonball was fired at is 45 degrees. Explain why below:

because sin(θ) = cos(θ) this implies that θ is 45°.

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pg. 2

Adventure 13 – Drawing Curves – Maximum, Minimum and Inflection Points

1. Overview of the Sessions

Dates:

01/21/2026

02/11/2026

Students Present:

Cyrus, Rooz (Session 1)

Cyrus, Rooz, Marc (Session 2)

Duration:

Approximately 1 hour per session

This Math Circle session explored how derivatives determine the shape of a curve using an ATP availability model from biology. The central idea was simple but powerful: You can reconstruct a curve using only critical points and signs.

2. Warm-Up Conversation / Storytelling

We began with the ATP story and the biological motivation:

- What is ATP?
- Why does it change over time?
- Why must its availability form a curve?

We connected the idea of flow in mechanics to flow in metabolism.

Students were asked: If ATP changes over time, what must its graph look like? This led naturally to derivatives.

At the start of the second session we discussed what happens when you turn on the light. We covered direct vs alternating currents and talked about Edison and Tesla and their roles and also Batteries and generators.

3. Activities Completed

Students:

- Reviewed $x^2, -x^2, x^3, -x^3$
- Identified max, min, and inflection points
- Computed first and second derivatives of the ATP model
- Drew A(t) first
- Drew V(t) second

- Built a sign table
- Reconstructed D(t) from calculus alone

In Activity 3, students analyzed a modified ATP model representing early and late energy boosts.

4. What Worked Well

- Drawing acceleration first clarified everything.
- The sign table provided strong visual structure.
- Students recognized that zero slope does not automatically mean maximum or minimum.
- Biological framing increased engagement.

5. Adjustments for Next Time

- Slow down during sign table construction.
- Emphasize second derivative test more explicitly.
- Encourage verbal explanation before drawing.

6. Student Quotes

“This is like predicting the graph before you see it.”

“We didn’t need all the points.”

“That’s kind of scary… but cool.”

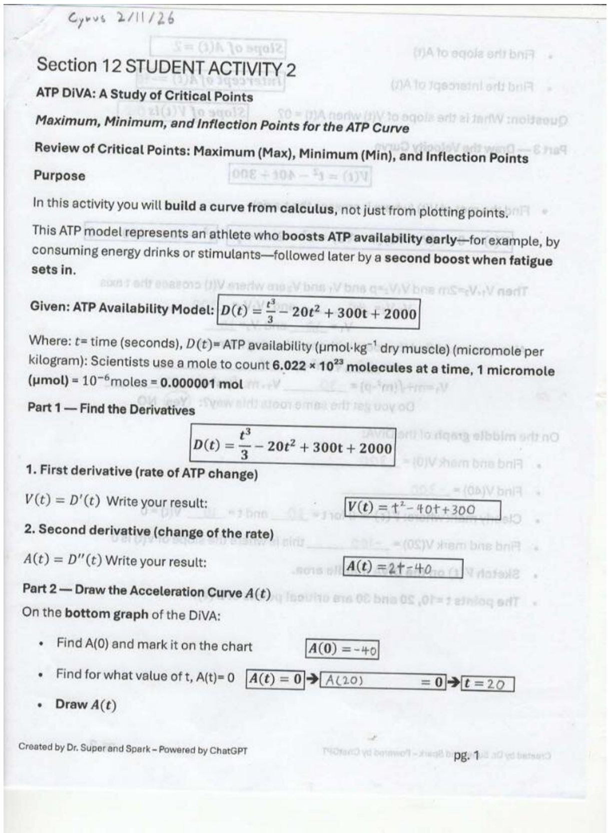
7. Closing Reflection by Dr. Super

Adventure 13 showed that derivatives are not mechanical tools — they are structural detectors.

Students moved from calculating to reasoning.

The “Parting Shot” idea — sketching from critical points alone — was clearly understood.

Student Completed Worksheets



Cyrus 2/11/26

Section 12 STUDENT ACTIVITY 2

ATP DiVA: A Study of Critical Points

Maximum, Minimum, and Inflection Points for the ATP Curve

Review of Critical Points: Maximum (Max), Minimum (Min), and Inflection Points

Purpose

In this activity you will **build a curve from calculus**, not just from plotting points.

This ATP model represents an athlete who **boosts ATP availability early**—for example, by consuming energy drinks or stimulants—followed later by a **second boost when fatigue sets in.**

Given: ATP Availability Model: $\boxed{D(t) = \frac{t^3}{3} - 20t^2 + 300t + 2000}$

Where: t= time (seconds), $D(t)$= ATP availability (μmol·kg^{-1} dry muscle) (micromole per kilogram): Scientists use a mole to count **6.022 × 10^{23} molecules at a time, 1 micromole** **(μmol)** = 10^{-6}moles = **0.000001 mol**

Part 1 — Find the Derivatives

$$\boxed{D(t) = \frac{t^3}{3} - 20t^2 + 300t + 2000}$$

1. First derivative (rate of ATP change)

$V(t) = D'(t)$ Write your result: $\boxed{V(t) = t^2 - 40t + 300}$

2. Second derivative (change of the rate)

$A(t) = D''(t)$ Write your result: $\boxed{A(t) = 2t - 40}$

Part 2 — Draw the Acceleration Curve $A(t)$

On the **bottom graph** of the DiVA:

- Find A(0) and mark it on the chart $\boxed{A(0) = -40}$
- Find for what value of t, A(t)= 0 $\boxed{A(t) = 0} \rightarrow \boxed{A(20) \quad = 0} \rightarrow \boxed{t = 20}$
- **Draw $A(t)$**

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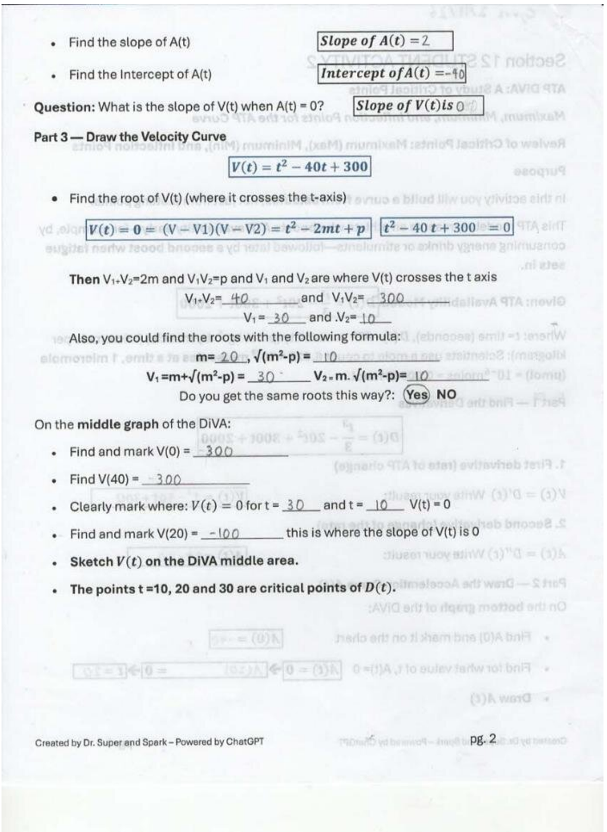

- Find the slope of A(t) $\boxed{\textit{Slope of } A(t) = 2}$
- Find the Intercept of A(t) $\boxed{\textit{Intercept of } A(t) = -40}$

Question: What is the slope of V(t) when A(t) = 0? $\boxed{\textit{Slope of } V(t) \textit{ is } 0}$

Part 3 — Draw the Velocity Curve

$$\boxed{V(t) = t^2 - 40t + 300}$$

- Find the root of V(t) (where it crosses the t-axis)

$\boxed{V(t) = 0 = (V - V1)(V - V2) = t^2 - 2mt + p}$ $\boxed{t^2 - 40\,t + 300 = 0}$

Then $V_1 + V_2$=2m and V_1V_2=p and V_1 and V_2 are where V(t) crosses the t axis

$V_1 + V_2$= 40 and V_1V_2= 300

V_1 = 30 and V_2= 10

Also, you could find the roots with the following formula:

m= 20 **, √(m²-p) =** 10

V_1 =m+√(m²-p) = 30 **V_2 = m. √(m²-p)=** 10

Do you get the same roots this way?: [✓ **Yes**] **NO**

On the **middle graph** of the DiVA:

- Find and mark V(0) = 300
- Find V(40) = 300
- Clearly mark where: $V(t) = 0$ for t = 30 and t = 10 V(t) = 0
- Find and mark V(20) = -100 this is where the slope of V(t) is 0
- **Sketch $V(t)$ on the DiVA middle area.**
- **The points t =10, 20 and 30 are critical points of $D(t)$.**

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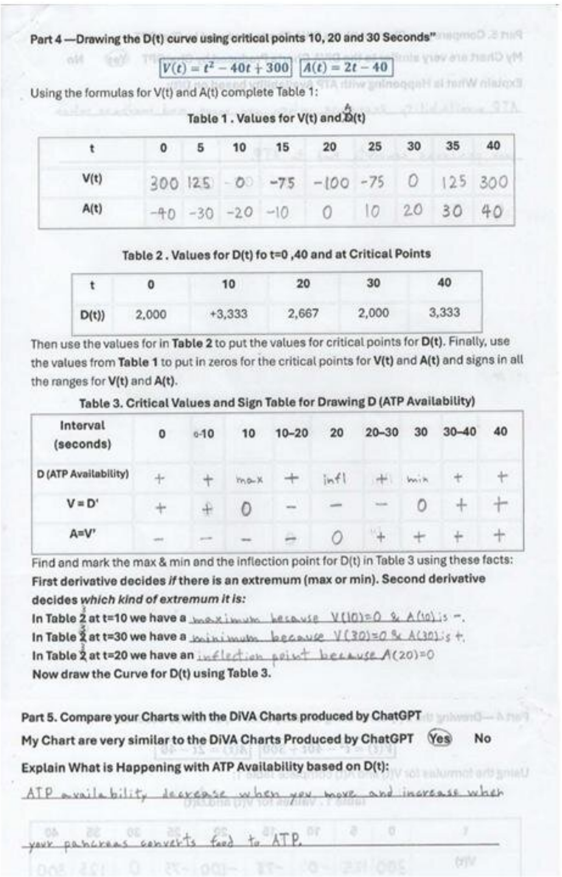


Part 4 —Drawing the D(t) curve using critical points 10, 20 and 30 Seconds"

$\boxed{V(t) = t^2 - 40t + 300}$ $\boxed{A(t) = 2t - 40}$

Using the formulas for V(t) and A(t) complete Table 1:

Table 1 . Values for V(t) and $\overset{A}{D}$(t)

t	0	5	10	15	20	25	30	35	40
V(t)	300	125	0	-75	-100	-75	0	125	300
A(t)	-40	-30	-20	-10	0	10	20	30	40

Table 2 . Values for D(t) fo t=0 ,40 and at Critical Points

t	0	10	20	30	40
D(t))	2,000	+3,333	2,667	2,000	3,333

Then use the values for in **Table 2** to put the values for critical points for **D(t)**. Finally, use the values from **Table 1** to put in zeros for the critical points for **V(t)** and **A(t)** and signs in all the ranges for **V(t)** and **A(t)**.

Table 3. Critical Values and Sign Table for Drawing D (ATP Availability)

Interval (seconds)	0	0-10	10	10–20	20	20–30	30	30–40	40
D (ATP Availability)	+	+	max	+	infl	+	min	+	+
V = D'	+	+	0	−	−	−	0	+	+
A=V'	−	−	−	−	0	+	+	+	+

Find and mark the max & min and the inflection point for D(t) in Table 3 using these facts: **First derivative decides *if* there is an extremum (max or min). Second derivative decides *which kind of extremum it is:***

In Table $\overset{3}{2}$ at t=10 we have a maximum because V(10)=0 & A(10) is −.

In Table $\overset{3}{2}$ at t=30 we have a minimum because V(30)=0 & A(30) is +.

In Table $\overset{3}{2}$ at t=20 we have an inflection point because A(20)=0

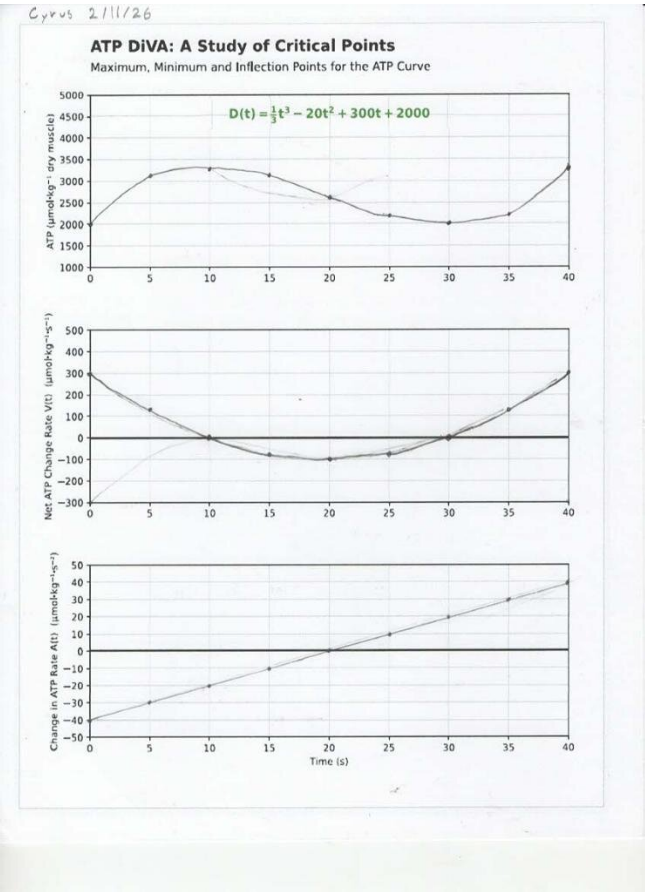
Now draw the Curve for D(t) using Table 3.

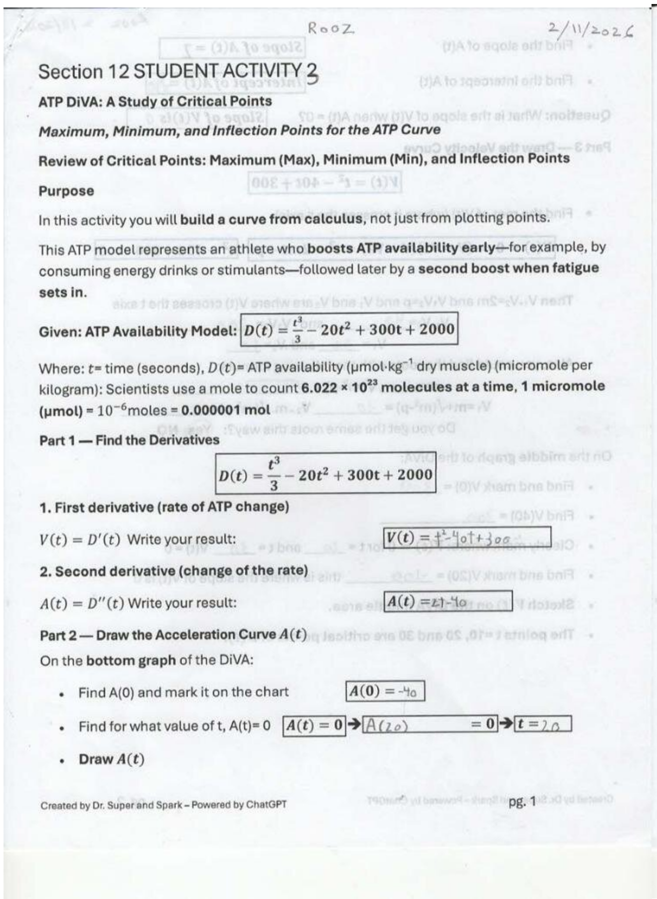
Part 5. Compare your Charts with the DiVA Charts produced by ChatGPT

My Chart are very similar to the DiVA Charts Produced by ChatGPT [✓ **Yes**] **No**

Explain What is Happening with ATP Availability based on D(t):

ATP availability decrease when you move and increase when your pancreas converts food to ATP.





Rooz 2/11/2026

Section 12 STUDENT ACTIVITY 3

ATP DiVA: A Study of Critical Points

Maximum, Minimum, and Inflection Points for the ATP Curve

Review of Critical Points: Maximum (Max), Minimum (Min), and Inflection Points

Purpose

In this activity you will **build a curve from calculus**, not just from plotting points.

This ATP model represents an athlete who **boosts ATP availability early**—for example, by consuming energy drinks or stimulants—followed later by a **second boost when fatigue sets in.**

Given: ATP Availability Model: $\boxed{D(t) = \frac{t^3}{3} - 20t^2 + 300t + 2000}$

Where: t= time (seconds), $D(t)$= ATP availability (μmol·kg^{-1} dry muscle) (micromole per kilogram): Scientists use a mole to count **6.022 × 10^{23} molecules at a time, 1 micromole** (μmol) = 10^{-6}moles = **0.000001 mol**

Part 1 — Find the Derivatives

$$\boxed{D(t) = \frac{t^3}{3} - 20t^2 + 300t + 2000}$$

1. First derivative (rate of ATP change)

$V(t) = D'(t)$ Write your result: $\boxed{V(t) = t^2-40t+300}$

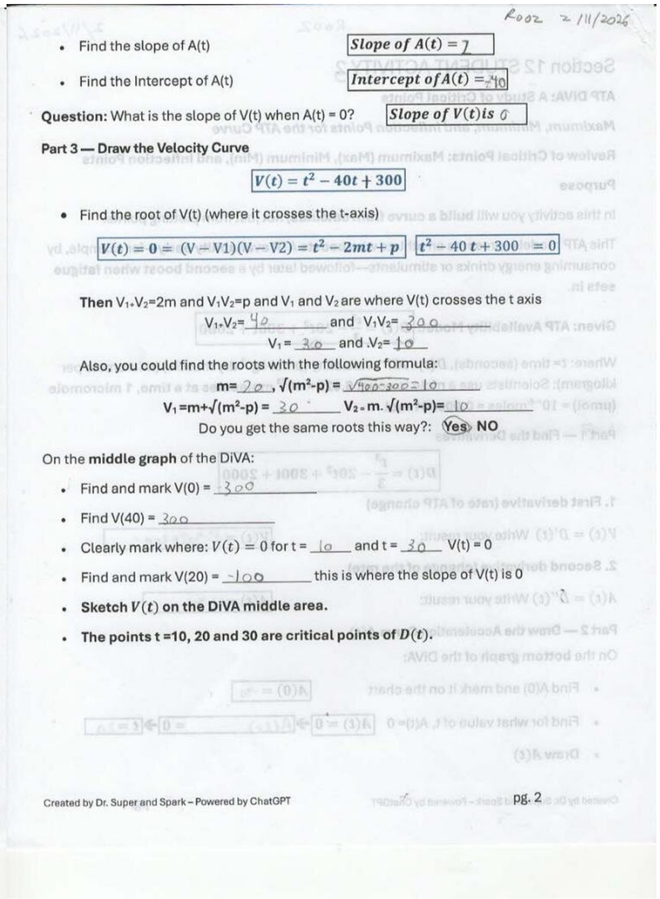
2. Second derivative (change of the rate)

$A(t) = D''(t)$ Write your result: $\boxed{A(t) = 2t-40}$

Part 2 — Draw the Acceleration Curve $A(t)$

On the **bottom graph** of the DiVA:

- Find A(0) and mark it on the chart $\boxed{A(0) = -40}$
- Find for what value of t, A(t)= 0 $\boxed{A(t) = 0} \rightarrow \boxed{A(20) \quad = 0} \rightarrow \boxed{t = 20}$
- **Draw $A(t)$**



Rooz 2/11/2026

- Find the slope of A(t) $\boxed{Slope\ of\ A(t) = 2}$
- Find the Intercept of A(t) $\boxed{Intercept\ of\ A(t) = -40}$

Question: What is the slope of V(t) when A(t) = 0? $\boxed{Slope\ of\ V(t) is\ 0}$

Part 3 — Draw the Velocity Curve

$$\boxed{V(t) = t^2 - 40t + 300}$$

- Find the root of V(t) (where it crosses the t-axis)

$\boxed{V(t) = 0 = (V - V1)(V - V2) = t^2 - 2mt + p}$ $\boxed{t^2 - 40\,t + 300 = 0}$

Then V_1+V_2=2m and V_1V_2=p and V_1 and V_2 are where V(t) crosses the t axis

V_1+V_2= 40 and V_1V_2= 300

V_1 = 30 and V_2= 10

Also, you could find the roots with the following formula:

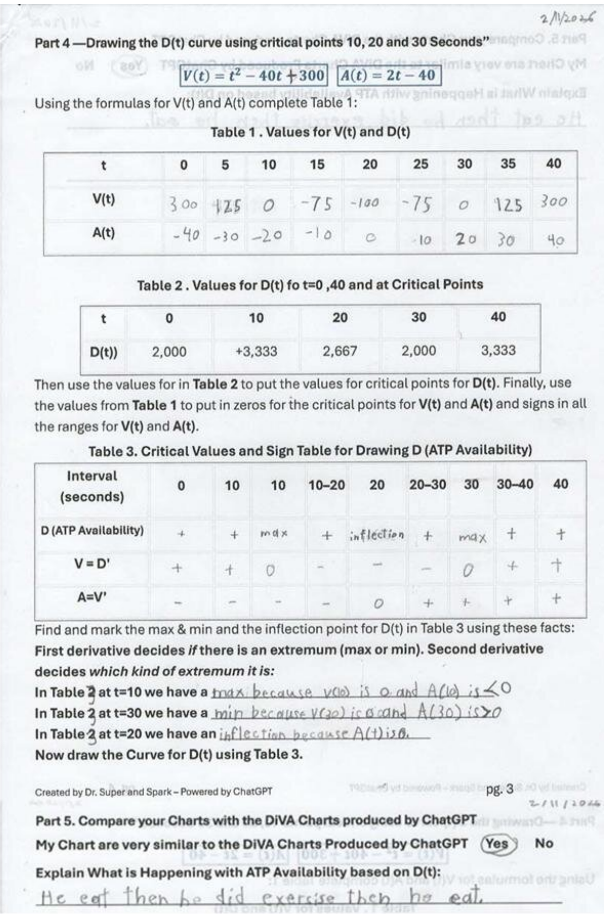
m= 20 , **√(m²-p) =** $\sqrt{400-300} = 10$

V_1 =m+√(m²-p) = 30 **V_2 = m. √(m²-p)=** 10

Do you get the same roots this way?: [✓ **Yes**] **NO**

On the **middle graph** of the DiVA:

- Find and mark V(0) = 300
- Find V(40) = 300
- Clearly mark where: $V(t) = 0$ for t = 10 and t = 30 V(t) = 0
- Find and mark V(20) = -100 this is where the slope of V(t) is 0
- **Sketch $V(t)$ on the DiVA middle area.**
- **The points t =10, 20 and 30 are critical points of $D(t)$.**



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Part 4 —Drawing the D(t) curve using critical points 10, 20 and 30 Seconds"

$\boxed{V(t) = t^2 - 40t + 300}$ $\boxed{A(t) = 2t - 40}$

Using the formulas for V(t) and A(t) complete Table 1:

Table 1 . Values for V(t) and D(t)

t	0	5	10	15	20	25	30	35	40
V(t)	300	125	0	-75	-100	-75	0	125	300
A(t)	-40	-30	-20	-10	0	10	20	30	40

Table 2 . Values for D(t) fo t=0 ,40 and at Critical Points

t	0	10	20	30	40
D(t))	2,000	+3,333	2,667	2,000	3,333

Then use the values for in **Table 2** to put the values for critical points for **D(t)**. Finally, use the values from **Table 1** to put in zeros for the critical points for **V(t)** and **A(t)** and signs in all the ranges for **V(t)** and **A(t)**.

Table 3. Critical Values and Sign Table for Drawing D (ATP Availability)

Interval (seconds)	0	10	10	10–20	20	20–30	30	30–40	40
D (ATP Availability)	+	+	max	+	inflection	+	max	+	+
V = D'	+	+	0	-	-	-	0	+	+
A=V'	-	-	-	-	0	+	+	+	+

Find and mark the max & min and the inflection point for D(t) in Table 3 using these facts:
First derivative decides *if* there is an extremum (max or min). Second derivative decides *which kind of extremum it is:*

In Table 3 at t=10 we have a max because V(10) is 0 and A(10) is <0
In Table 3 at t=30 we have a min because V(30) is 0 and A(30) is >0
In Table 3 at t=20 we have an inflection because A(t) is 0.
Now draw the Curve for D(t) using Table 3.

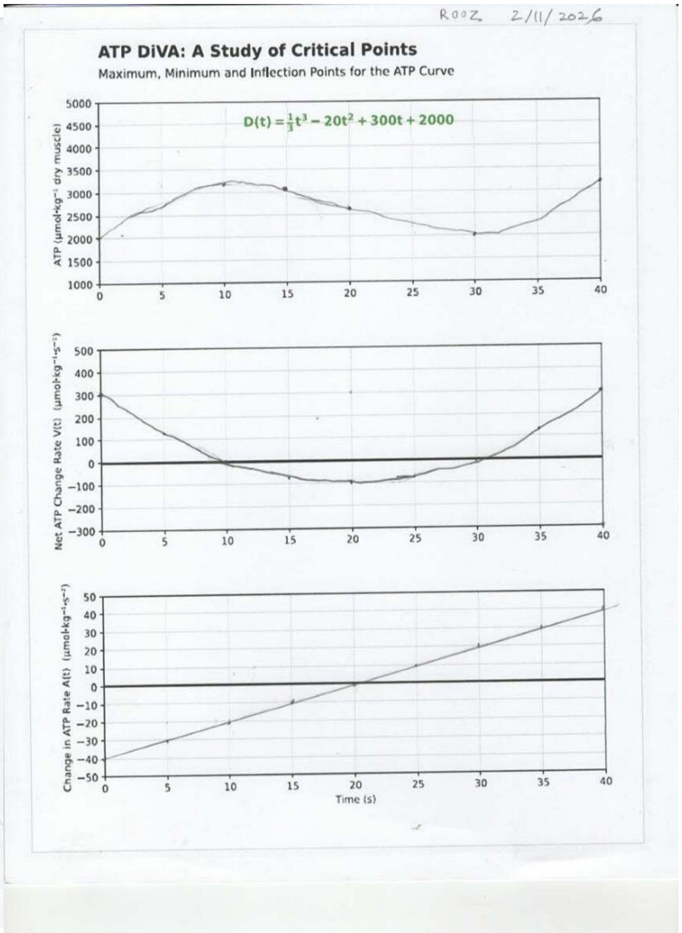
2/11/2026

Part 5. Compare your Charts with the DiVA Charts produced by ChatGPT

My Chart are very similar to the DiVA Charts Produced by ChatGPT [✓ **Yes**] **No**

Explain What is Happening with ATP Availability based on D(t):

He eat then he did exercise then he eat.



Rooz 2/11/2026

ATP DiVA: A Study of Critical Points

Maximum, Minimum and Inflection Points for the ATP Curve

Adventure 14 – How the Mathematics behind how Moon Circles the Earth

Date: February 15, 2026

Participants: Cyrus and Rooz

1️⃣ *Review of Magnetism.*

We watched the video **Why Do Magnets Work? The One Question Feynman Refused to Explain**. We began by how magnetism and the electromagnetic force are really the same force. We watched approximately 10 minutes of the video *“Why Do Magnets Work? The One Question Feynman Refused to Explain.”*

Both Cyrus and Rooz reacted strongly to the tone of the AI-generated “Feynman.” They felt it was condescending and evasive. This actually became a productive teaching moment:

- Why do physicists sometimes say “we don’t know why”?
- What does science really mean when it explains something?
- We clarified an important idea:
- Science describes *how nature behaves* using mathematical laws.
- It does not necessarily explain the ultimate “why.”

This led naturally to a broader discussion.

2️⃣ Key Ideas Discussed

🔹 The Four Fundamental Forces

Gravitational

Electromagnetic

Strong nuclear

Weak nuclear

We emphasized that:

Magnetism and electricity are not separate forces.

They are different aspects of the **electromagnetic force**.

What we experience as “solid contact” is largely electromagnetic repulsion between atoms.

This shifted the students’ thinking from:

“Objects touching”

To “Fields interacting in space.”

That conceptual transition — from things pushing to fields interacting — is central to modern physics.

3 Transition to the Moon

We then asked:

Is magnetism is a force field in space?

What about gravity?

This brought us to the Moon.

Why doesn’t the Moon fall down?

Why doesn’t it fly away?

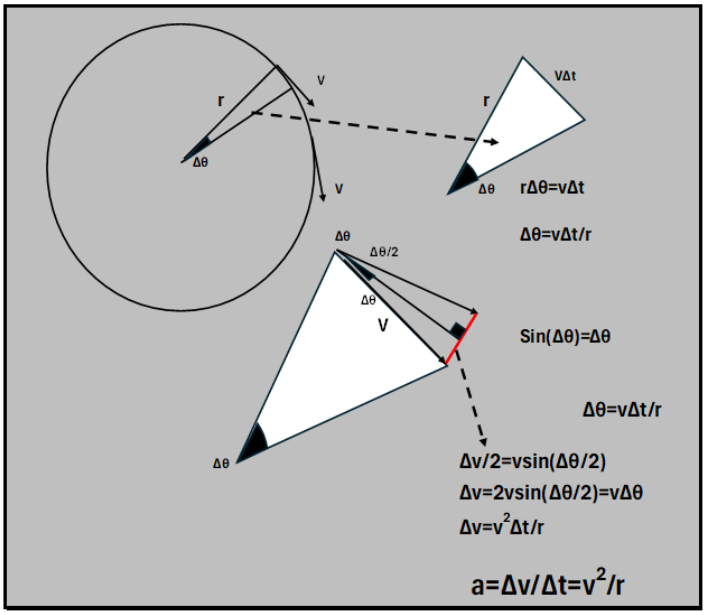
What keeps it going in a circle?

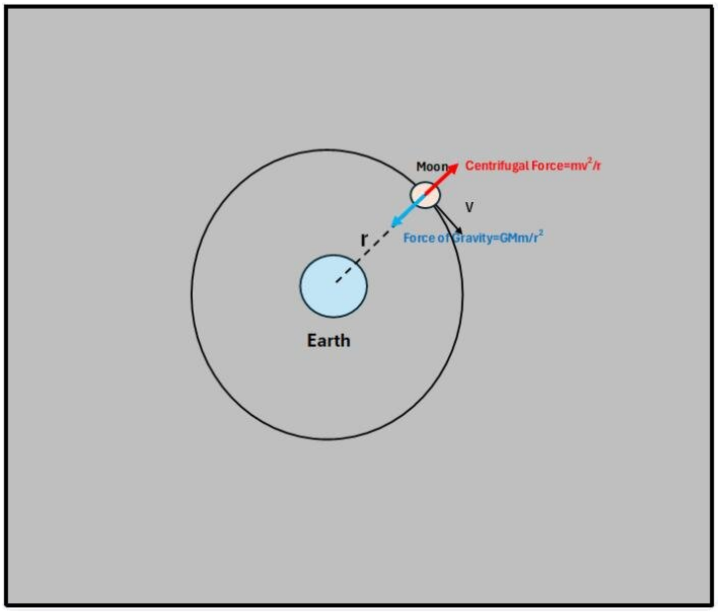
4 Centripetal Force and Acceleration

This was the most challenging conceptual section.

We developed:

$$a = \frac{v^2}{R}$$





and explained that circular motion requires **continuous inward acceleration**.

I think Cyrus understood approximately 50% independently. Rooz required guided assistance.

We emphasized:

Acceleration does not mean speeding up. It can mean changing direction. The Moon is constantly accelerating toward Earth. This was a breakthrough idea.

5 Calculating the Moon’s Orbital Speed

Using Newton’s Law of Gravitation:

$$F = \frac{GMm}{R^2}$$

and the circular motion requirement:

$$F = \frac{mv^2}{R}$$

We set them equal and derived:

$$v = \sqrt{\frac{GM}{R}}$$

Cyrus handled calculator computations very well. I helped Rooz a little here.

They both obtained:

Orbital speed ≈ **1.02 km/s**

Orbital period ≈ **27.4 days**

The students were visibly impressed that such a simple equation produced the correct lunar month.

That moment — when mathematics predicts reality — was powerful.

Cyrus said this is 3,600 km/hour!

6️⃣ Deeper Insight

We concluded with the beautiful idea: The Moon is constantly falling toward Earth… but its sideways velocity keeps missing it. This helped unify: Fields; Forces; Acceleration; Motion;Geometry

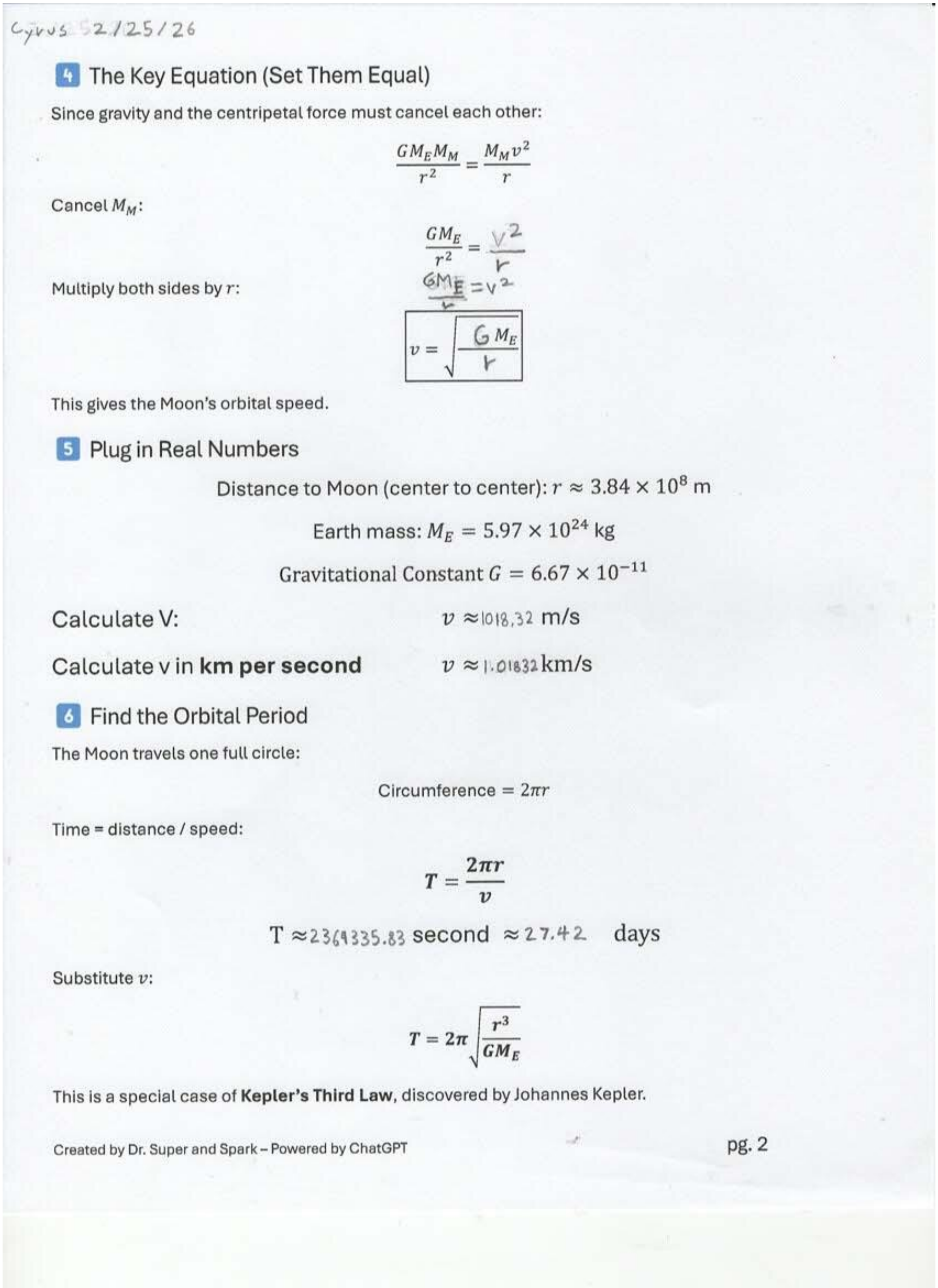
🎯 Reflections

Tone matters in teaching delivery. What may be good for adults may not work for younger students. Conceptual acceleration remains difficult at this age. Calculator confidence is growing. Predictive power of mathematics made a strong impression. Overall: Productive session with solid conceptual growth. The focus was on reviewing deep misconceptions in magnetic force and how this is the same force as the electromagnetic force. We also examined how the moon circles the earth and fond it's velocity and period.

🎯 Final words

The session went well in spite of their objections to Feynman's (AI) tone. I think in the future I will talk instead of Feynman! Explaining the centripetal force and acceleration was not easy (for me either) but I got through it and Cyrus probably go 50% of it for Rooz I am not sure. The exercise for calculating the orbital speed and period went well but Rooz needed help. They were impressed by the simplicity of the results, in particular that we found 27.4 Days for each lunar month! Cyrus was really good on the calculator.

Students completed Worksheets



Cyrus 2/25/26

4 The Key Equation (Set Them Equal)

Since gravity and the centripetal force must cancel each other:

$$\frac{GM_EM_M}{r^2} = \frac{M_Mv^2}{r}$$

Cancel M_M:

$$\frac{GM_E}{r^2} = \frac{v^2}{r}$$

Multiply both sides by r:

$$\frac{GM_E}{r} = v^2$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

This gives the Moon's orbital speed.

5 Plug in Real Numbers

Distance to Moon (center to center): $r \approx 3.84 \times 10^8$ m

Earth mass: $M_E = 5.97 \times 10^{24}$ kg

Gravitational Constant $G = 6.67 \times 10^{-11}$

Calculate V: $v \approx 1018.32$ m/s

Calculate v in **km per second** $v \approx 1.01832$ km/s

6 Find the Orbital Period

The Moon travels one full circle:

$$\text{Circumference} = 2\pi r$$

Time = distance / speed:

$$T = \frac{2\pi r}{v}$$

T ≈ 2369335.83 second ≈ 27.42 days

Substitute v:

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

This is a special case of **Kepler's Third Law**, discovered by Johannes Kepler.

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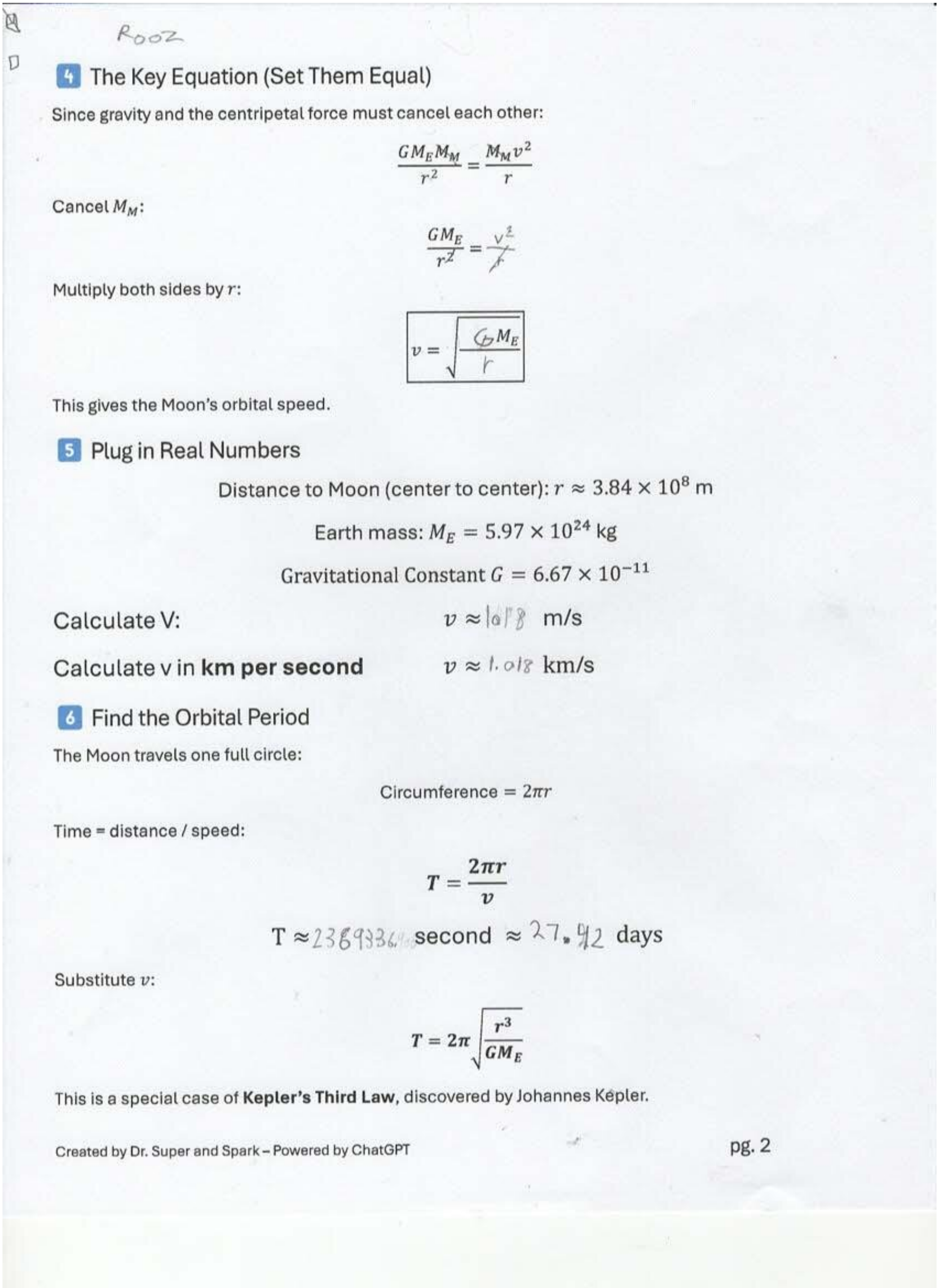

Rooz

4 The Key Equation (Set Them Equal)

Since gravity and the centripetal force must cancel each other:

$$\frac{GM_EM_M}{r^2}=\frac{M_Mv^2}{r}$$

Cancel M_M:

$$\frac{GM_E}{r^2}=\frac{v^2}{\cancel{r}}$$

Multiply both sides by r:

$$\boxed{v=\sqrt{\frac{GM_E}{r}}}$$

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Earth mass: $M_E = 5.97 \times 10^{24}$ kg

Gravitational Constant $G = 6.67 \times 10^{-11}$

Calculate V: $v \approx 1018$ m/s

Calculate v in **km per second** $v \approx 1.018$ km/s

6 Find the Orbital Period

The Moon travels one full circle:

$$\text{Circumference} = 2\pi r$$

Time = distance / speed:

$$T=\frac{2\pi r}{v}$$

T ≈ 2369336 second ≈ 27.42 days

Substitute v:

$$T=2\pi\sqrt{\frac{r^3}{GM_E}}$$

This is a special case of **Kepler's Third Law**, discovered by Johannes Kepler.

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Adventure 15 – The Sun Controls the Solar System

Date: February 15, 2026

Participants: Cyrus and Rooz

1️⃣ Review of Fire

We began the session with a long lecture and discussion about fire, based loosely on the video *"Why Does Fire Burn? – Feynman's Answer."*

The discussion focused on the idea that fire is not a mysterious substance but a chemical reaction between oxygen and carbon-based materials. When substances like wood burn, oxygen combines with carbon in the material and releases energy in the form of heat and light.

We also discussed cellulose, the primary structural material in plants. Wood, paper, and many plant fibers are largely made of cellulose molecules. When these molecules react with oxygen during combustion, energy stored in chemical bonds is released.

An interesting connection was made to biology: our bodies also perform a kind of controlled combustion. Inside our cells, oxygen reacts with sugars to release energy, though the process is much slower and more controlled than an open flame.

The discussion lasted about 20–25 minutes and worked well as a warm-up conversation about energy and chemical reactions.

2️⃣ Transition to Planetary Motion

After discussing fire and energy, the conversation shifted to a very different but related topic: motion in space. We asked a fundamental question: What keeps the planets moving around the Sun? Unlike a car moving in a straight line, planets are constantly changing direction as they travel around the Sun. This change in direction means that some form of acceleration must be present. This led to the concept of centripetal acceleration.

3️⃣ Centripetal Motion

We reviewed the geometric argument showing why an object moving in a circle must experience acceleration toward the center. That we had covered in the previous Math Circle. Even when the speed remains constant, the velocity vector changes direction.If two velocity vectors separated by a small time interval Δt are compared, the difference between them is: Δv,By examining the geometry of these vectors, we showed that $\Delta v \approx v\Delta\theta$ and because $r\Delta\theta = v\Delta t$ we obtain $\Delta v = v^2\Delta t / r$ Dividing by Δt gives the famous result $\mathbf{a = v^2 / r}$

This is the centripetal acceleration required to keep an object moving in a circular path.

4️⃣ Gravity and Planetary Motion

Next we asked:

What provides this centripetal acceleration for planets? The answer is gravity. Newton's law of gravitation states that the force between two masses is $F = GMm / r^2$ When this force acts on a planet, it produces the centripetal acceleration needed to maintain its orbit. From this we obtain the orbital velocity formula

$$\mathbf{v = \sqrt{}(GM / r)}$$

This means that once the value of GM for the Sun is known, the orbital speed of any planet can be calculated from its distance from the Sun.

5️⃣ Computing GM of the Sun

Using known values

$G = 6.67 \times 10^{-11}$

$M\_sun = 2 \times 10^{30}$ kg

we compute: $GM\_sun \approx 1.33 \times 10^{20}\ m^3/s^2$

This single number controls the motion of the entire solar system.

6️⃣ Exploring Planetary Orbits

Students then worked through the Adventure 15 activity sheet, computing orbital quantities for:

- Mercury
- Earth
- Neptune

Using planetary distance data they calculated:

- orbital velocity
- orbit size
- orbital period

After completing the calculations they examined the broader planetary data tables to look for patterns.

7️⃣ Deeper Insight: Kepler's Law

From the data, an important relationship emerges:

$$T \propto r^3/r^2$$

or equivalently

$$T^2 \propto r^3$$

This is Kepler's Third Law, which describes how the orbital period of a planet depends on its distance from the Sun.

Students were encouraged to explain how this relationship follows directly from the formulas for orbital velocity and orbital period.

🎯 Reflections

The activity highlighted a remarkable fact: A single constant (GM of the Sun) determines the motion of every planet in the solar system. Once that value is known, the orbital speed and period of any planet can be predicted from its distance alone. This provides a powerful illustration of how Newton's laws unify motion on Earth with motion in space.

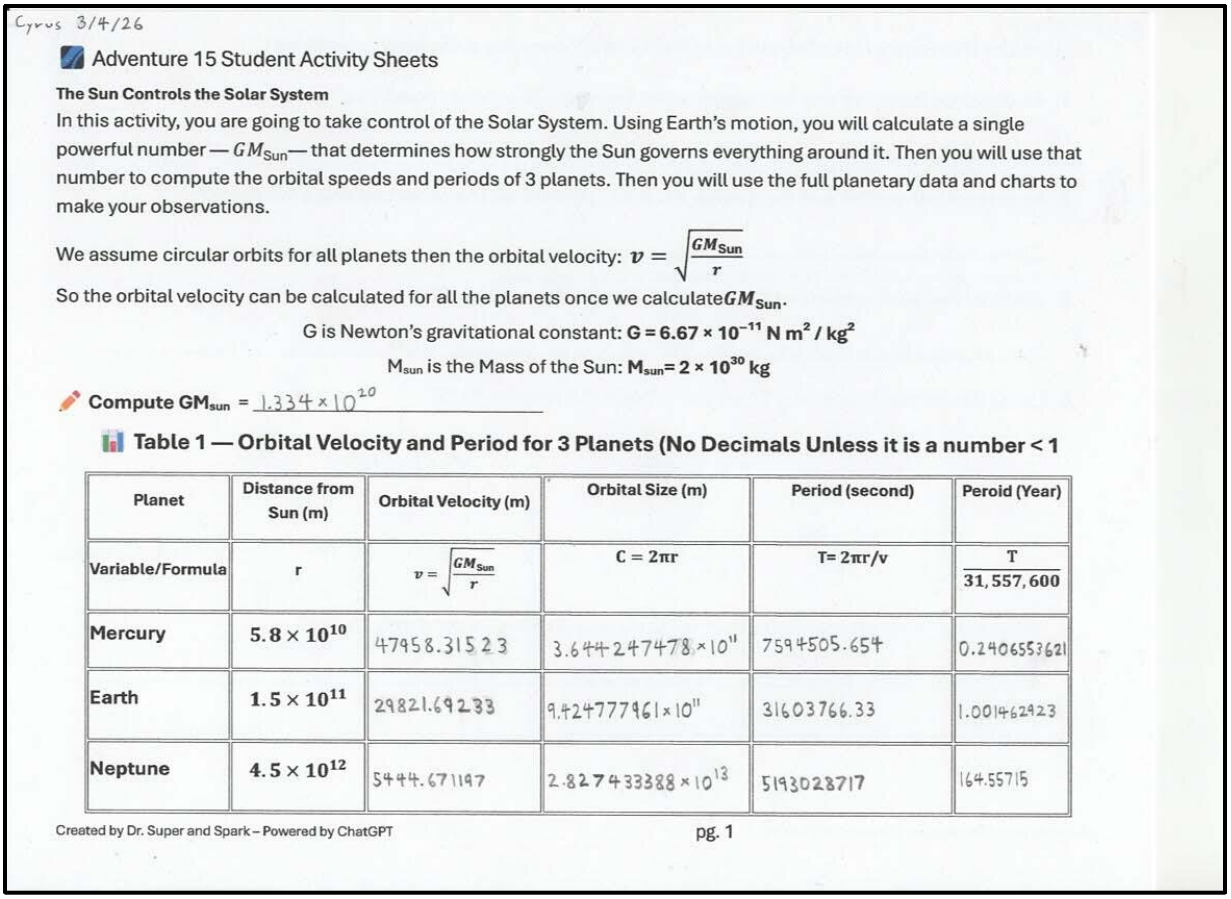
🎯 Final Words

The session connected several major ideas: energy and combustion; circular motion; centripetal acceleration; Newton's law of gravitation; planetary motion and Kepler's laws

Through a combination of geometry, physics, and data analysis, the students were able to see how the structure of the solar system follows directly from a few simple physical principles.

One remarkable incident when we were discussing fire was that, I asked the punch line question "**So what is fire**" and Rooz almost immediately said **The Sun**. Cyrus smiled and it was a great moment as indeed Fire is the sun rays that are stored in the wood and released when we have fire!

Completed Student Activity Sheets



Cyrus 3/4/26

Adventure 15 Student Activity Sheets

The Sun Controls the Solar System

In this activity, you are going to take control of the Solar System. Using Earth's motion, you will calculate a single powerful number — GM_{Sun} — that determines how strongly the Sun governs everything around it. Then you will use that number to compute the orbital speeds and periods of 3 planets. Then you will use the full planetary data and charts to make your observations.

We assume circular orbits for all planets then the orbital velocity: $v = \sqrt{\frac{GM_{\text{Sun}}}{r}}$

So the orbital velocity can be calculated for all the planets once we calculate GM_{Sun}.

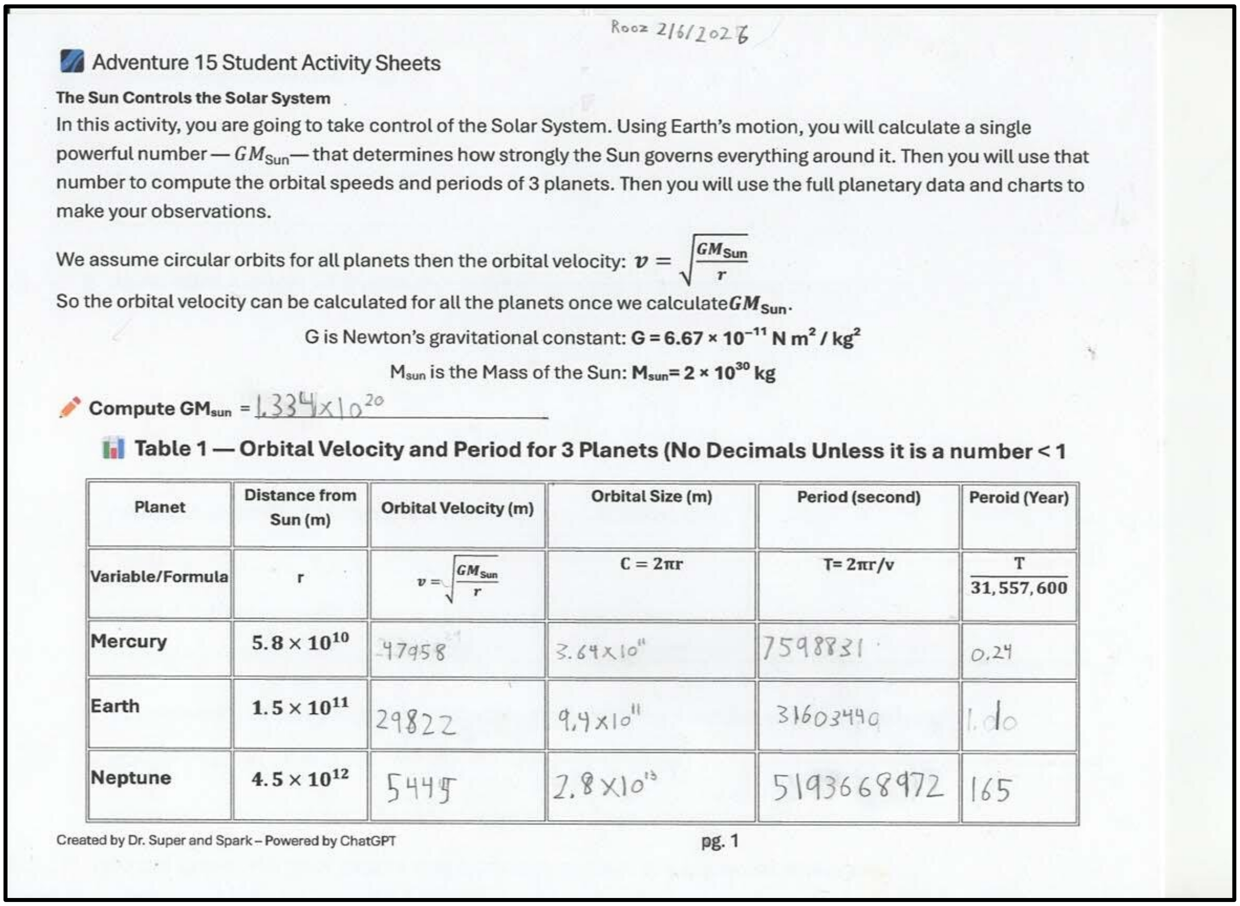
G is Newton's gravitational constant: **$G = 6.67 \times 10^{-11}$ N m^2 / kg^2**

M_{sun} is the Mass of the Sun: **$M_{\text{sun}} = 2 \times 10^{30}$ kg**

Compute GM_{sun} = 1.334×10^{20}

Table 1 — Orbital Velocity and Period for 3 Planets (No Decimals Unless it is a number < 1

Planet	Distance from Sun (m)	Orbital Velocity (m)	Orbital Size (m)	Period (second)	Peroid (Year)
Variable/Formula	r	$v = \sqrt{\frac{GM_{\text{Sun}}}{r}}$	$C = 2\pi r$	$T = 2\pi r/v$	$\frac{T}{31{,}557{,}600}$
Mercury	5.8×10^{10}	47958.31523	$3.644247478 \times 10^{11}$	7594505.654	0.2406553621
Earth	1.5×10^{11}	29821.69233	$9.424777961 \times 10^{11}$	31603766.33	1.001462923
Neptune	4.5×10^{12}	5444.671197	$2.827433388 \times 10^{13}$	5193028717	164.55715



Rooz 2/6/2026

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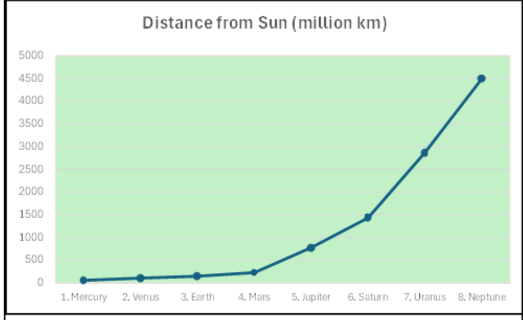
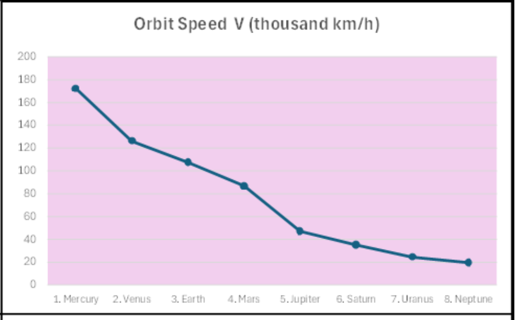
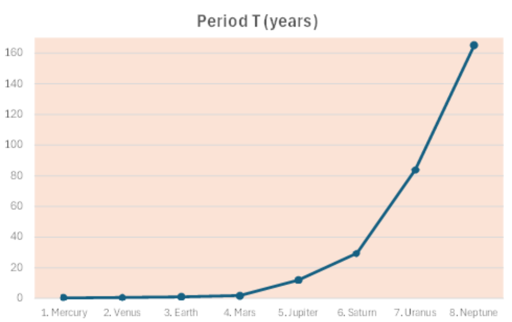
Mercury	5.8×10^{10}	47958	3.64×10^{11}	7598831	0.24
Earth	1.5×10^{11}	29822	9.4×10^{11}	31603440	1.00
Neptune	4.5×10^{12}	5445	2.8×10^{13}	5193668972	165

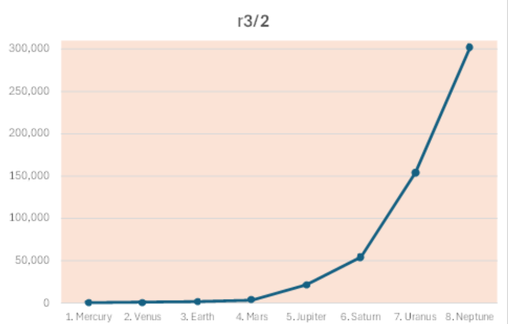
Planetary Data Comparisons

Planet	Distance from Sun (million km)	Distance from Sun (light-min)	Orbit Size (million km)	Orbit Speed (km/s)	Orbit Speed V (thousand km/h)	Period T (years)	$r^{3/2}$	$T/r^{3/2}$
1. Mercury	57.9	3.2	364	47.9	172	0.24	441	1,836
2. Venus	108.2	6	680	35	126	0.62	1,125	1,815
3. Earth	149.6	8.3	940	29.8	107	1	1,830	1,830
4. Mars	227.9	12.7	1,432	24.1	87	2	3,440	1,830
5. Jupiter	778.6	43.3	4,891	13.1	47	12	21,726	1,832
6. Saturn	1,433	79.6	9,005	9.7	35	29	54,246	1,841
7. Uranus	2,872	159.6	18,048	6.8	24	84	153,913	1,832
8. Neptune	4,495	249.7	28,254	5.4	19	165	301,366	1,829


Distance from Sun (million km)


Orbit Speed V (thousand km/h)


Period T (years)


r3/2

Planetary Data Comparisons

Planet	Distance from Sun (million km)	Distance from Sun (light-min)	Orbit Size (million km)	Orbit Speed (km/s)	Speed V (thousand km/h)	Period T (years)	$r^{3/2}$	$T/r^{3/2}$
1. Mercury	57.9	3.2	364	47.9	172	0.24	441	1,836
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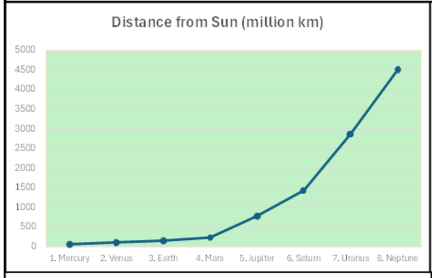
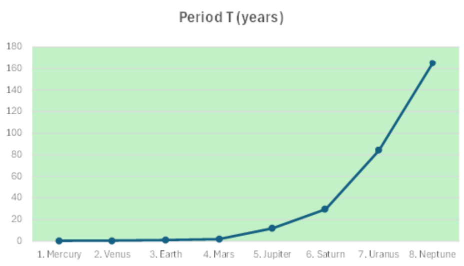
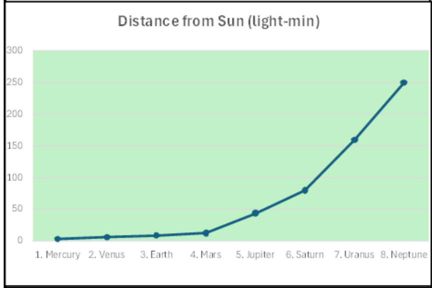
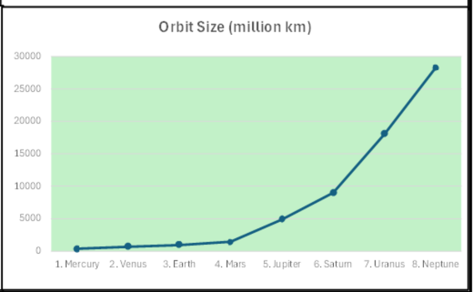
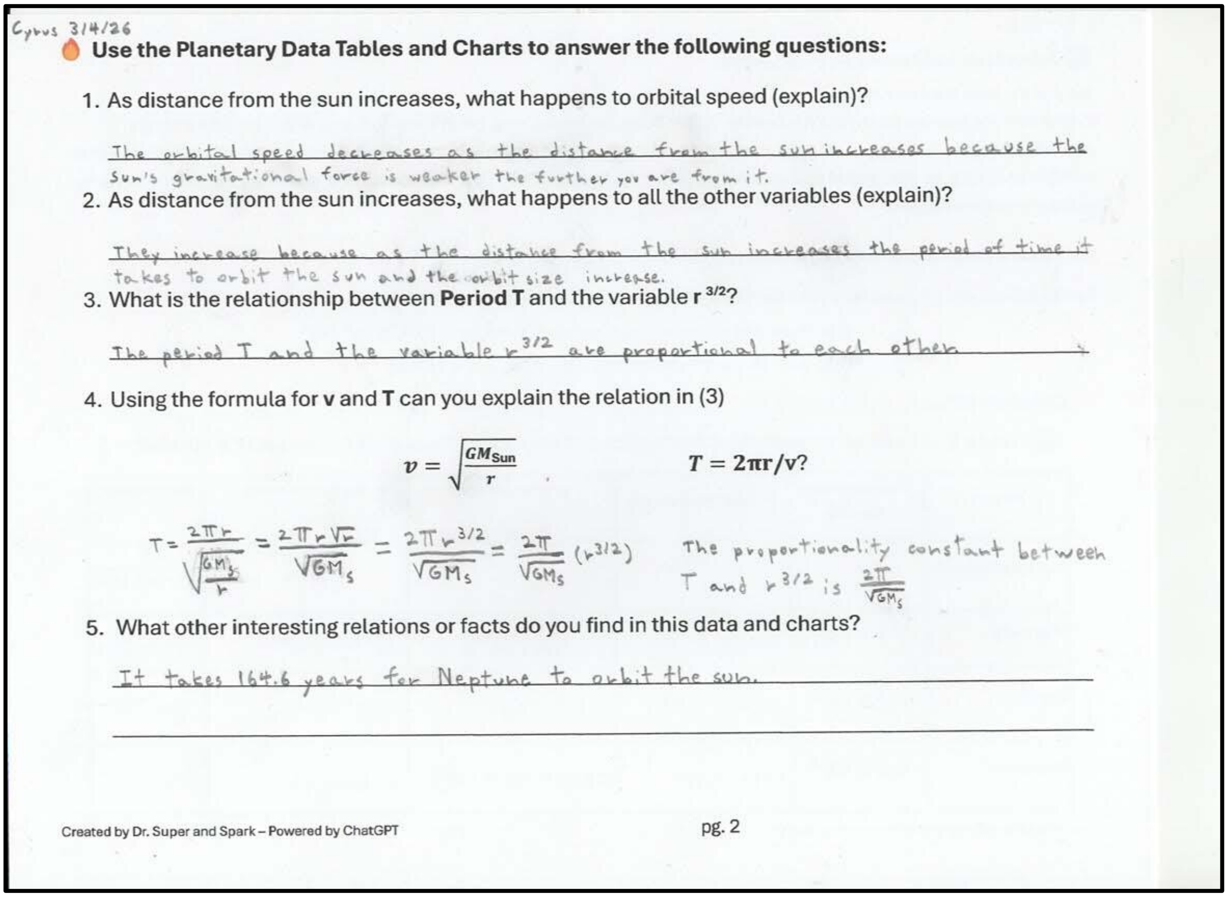

Distance from Sun (million km)


Period T (years)


Distance from Sun (light-min)


Orbit Size (million km)



Cyrus 3/14/26

Use the Planetary Data Tables and Charts to answer the following questions:

1. As distance from the sun increases, what happens to orbital speed (explain)?

The orbital speed decreases as the distance from the sun increases because the Sun's gravitational force is weaker the further you are from it.

2. As distance from the sun increases, what happens to all the other variables (explain)?

They increase because as the distance from the sun increases the period of time it takes to orbit the sun and the orbit size increase.

3. What is the relationship between **Period T** and the variable **r** $^{3/2}$?

The period T and the variable $r^{3/2}$ are proportional to each other.

4. Using the formula for **v** and **T** can you explain the relation in (3)

$$v = \sqrt{\frac{GM_{Sun}}{r}} \qquad T = 2\pi r/v?$$

$$T = \frac{2\pi r}{\sqrt{\frac{GM_s}{r}}} = \frac{2\pi r\sqrt{r}}{\sqrt{GM_s}} = \frac{2\pi r^{3/2}}{\sqrt{GM_s}} = \frac{2\pi}{\sqrt{GM_s}}(r^{3/2})$$

The proportionality constant between T and $r^{3/2}$ is $\frac{2\pi}{\sqrt{GM_s}}$

5. What other interesting relations or facts do you find in this data and charts?

It takes 164.6 years for Neptune to orbit the sun.

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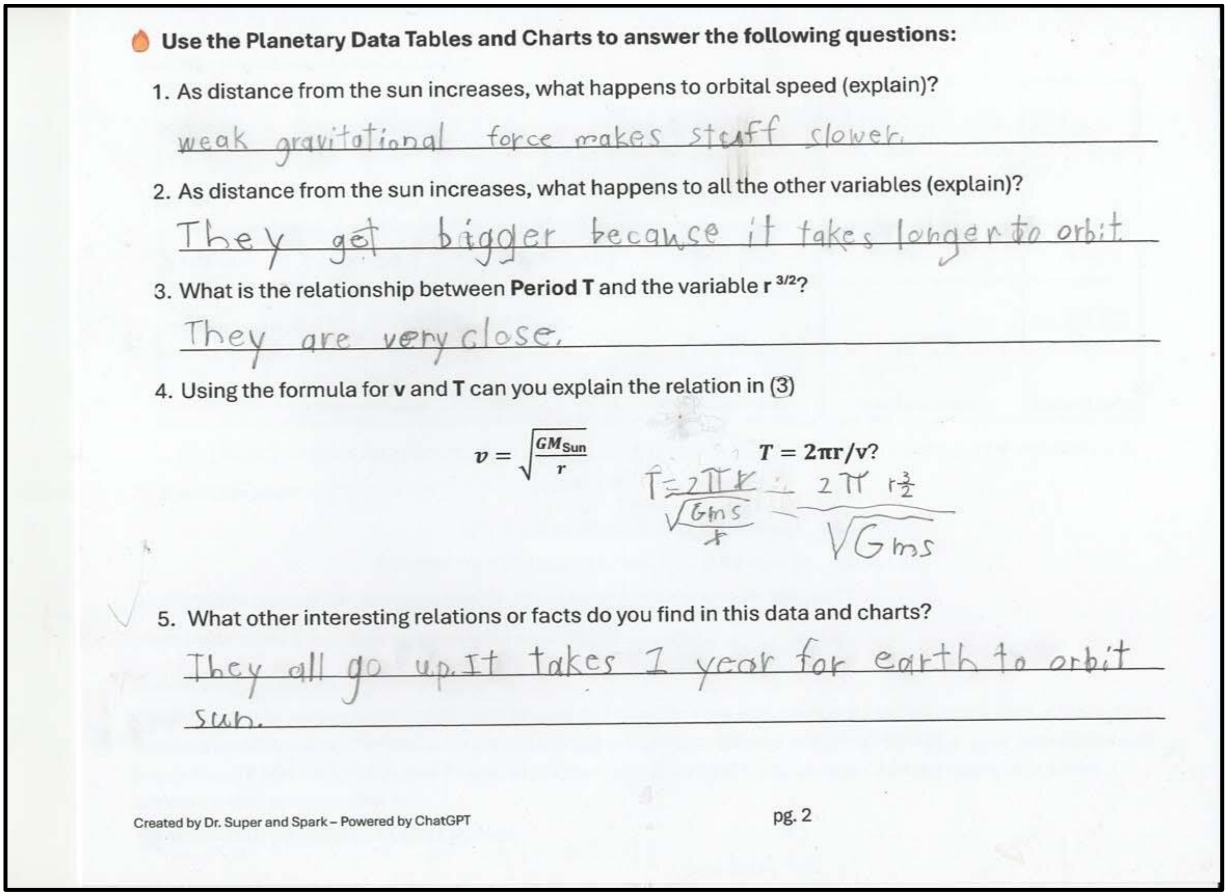
pg. 2



Use the Planetary Data Tables and Charts to answer the following questions:

1. As distance from the sun increases, what happens to orbital speed (explain)?

weak gravitational force makes stuff slower.

2. As distance from the sun increases, what happens to all the other variables (explain)?

They get bigger because it takes longer to orbit

3. What is the relationship between **Period T** and the variable **r** $^{3/2}$?

They are very close.

4. Using the formula for **v** and **T** can you explain the relation in (3)

$$v = \sqrt{\frac{GM_{Sun}}{r}} \qquad T = 2\pi r/v?$$

$$T = \frac{2\pi r}{\sqrt{\frac{Gms}{r}}} = \frac{2\pi\ r^{\frac{3}{2}}}{\sqrt{Gms}}$$

5. What other interesting relations or facts do you find in this data and charts?

They all go up. It takes 1 year for earth to orbit sun.

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pg. 2

The original explorers of these Adventures:

Rooz (9), Cyrus (11), and Marc (11)

Every lesson, activity, chart, and discussion in this book was tested during weekly Wednesday Math Circles. Their questions, discoveries, and insights helped shape the Adventures and demonstrated that young students can understand the fundamental ideas of calculus long before they encounter its formal techniques.

About the Author

Behrouz B. Aghevli, PhD (Dr. Super) earned degrees in Physics, Mathematics, and Statistics from Occidental College and Northwestern University. Over a career spanning university teaching, educational publishing, software development and Business Intelligence, he also authored mathematics materials, developed educational software, and presented at (NCTM) conferences.

Beginning in the early 1990s, while exploring mathematics with his daughter, Dr. Super developed a lifelong passion for creating hands-on, visual approaches to mathematics. Over the next three decades, he worked closely with teachers and students, developing classroom activities, educational software, and mathematics programs spanning kindergarten through high school.

A pioneer in Virtual Manipulatives and hands-on mathematics education, he holds four U.S. patents, including Virtual Manipulatives, Factor Blocks, Terrific Triangles, and the Fractal Mathematics Kit.

In 2022, inspired by his grandchildren Cyrus and Rooz, he returned to teaching through weekly Math Circles. These explorations ultimately led to the creation of the Adventures series and the DiVA Distance–velocity–Acceleration) charts and approach to learning calculus through visualization, motion, and discovery.